Chapter 8

Linear Programming Applications

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. **Model a wide variety of medium to large LP problems.**
2. **Understand major application areas, including marketing, production, labor scheduling, fuel blending, transportation, and finance.**
3. **Gain experience in solving LP problems with QM for Windows and Excel Solver software.**

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Chapter Outline

8.1 Introduction
8.2 Marketing Applications
8.3 Manufacturing Applications
8.4 Employee Scheduling Applications
8.5 Financial Applications
8.6 Ingredient Blending Applications
8.7 Transportation Applications

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Introduction

- The graphical method of LP is useful for understanding how to formulate and solve small LP problems.
- There are many types of problems that can be solved using LP.
- The principles developed here are applicable to larger problems.

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Marketing Applications

- Linear programming models have been used in the advertising field as a decision aid in selecting an effective media mix.
- Media selection problems can be approached with LP from two perspectives:
  - Maximize audience exposure.
  - Minimize advertising costs.

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Win Big Gambling Club

- The Win Big Gambling Club promotes gambling junkets to the Bahamas.
- It has $8,000 per week to spend on advertising.
- Its goal is to reach the largest possible high-potential audience.
- Media types and audience figures are shown in the following table.
- It needs to place at least five radio spots per week.
- No more than $1,800 can be spent on radio advertising each week.

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Win Big Gambling Club

Advertising options

MEDIUM	AUDIENCE REACHED PER AD	COST PER AD ($)	MAXIMUM ADS PER WEEK
TV spot (1 minute)	5,000	800	12
Daily newspaper (full-page ad)	8,500	925	5
Radio spot (30 seconds, prime time)	2,400	290	25
Radio spot (1 minute, afternoon)	2,800	380	20

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Win Big Gambling Club

The problem formulation is

X_1 = number of 1-minute TV spots each week
X_2 = number of daily paper ads each week
X_3 = number of 30-second radio spots each week
X_4 = number of 1-minute radio spots each week

Objective:

Maximize audience coverage $= 5{,}000X_1 + 8{,}500X_2 + 2{,}400X_3 + 2{,}800X_4$

Subject to

$X_1 \leq 12$ (max TV spots/wk)
$X_2 \leq 5$ (max newspaper ads/wk)
$X_3 \leq 25$ (max 30-sec radio spots ads/wk)
$X_4 \leq 20$ (max newspaper ads/wk)
$800X_1 + 925X_2 + 290X_3 + 380X_4 \leq \$8{,}000$ (weekly advertising budget)
$X_3 + X_4 \geq 5$ (min radio spots contracted)
$290X_3 + 380X_4 \leq \$1{,}800$ (max dollars spent on radio)
$X_1, X_2, X_3, X_4 \geq 0$

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Win Big Gambling Club Solution in Excel 2010

	A	B	C	D	E	F	G	H
1	Win Big Gambling Club							
2				Radio	Radio			
3		TV	Newspaper	30 sec.	1 min.			
4	Variables	X1	X2	X3	X4			
5	Solution	1.9688	5	6.2069	0	Total Audience		
6	Audience per ad	5000	8500	2400	2800	67240.3017		
7								
8	Constraints					LHS		RHS
9	Max. TV	1				1.9688	≤	12
10	Max. Newspaper		1			5	≤	5
11	Max. 30-sec. radio			1		6.2069	≤	25
12	Max. 1 min. radio				1	0	≤	20
13	Cost	800	925	290	380	8000	≤	8000
14	Radio dollars			290	380	1800	≤	1800
15	Radio spots			1	1	6.2069	≥	5

Program 8.1

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Marketing Research

- Linear programming has also been applied to marketing research problems and the area of consumer research.
- Statistical pollsters can use LP to help make strategy decisions.

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Management Sciences Association

- Management Sciences Associates (MSA) is a marketing research firm.
- MSA determines that it must fulfill several requirements in order to draw statistically valid conclusions:
  - Survey at least 2,300 U.S. households.
  - Survey at least 1,000 households whose heads are 30 years of age or younger.
  - Survey at least 600 households whose heads are between 31 and 50 years of age.
  - Ensure that at least 15% of those surveyed live in a state that borders on Mexico.
  - Ensure that no more than 20% of those surveyed who are 51 years of age or over live in a state that borders on Mexico.

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Management Sciences Association

- MSA decides that all surveys should be conducted in person.
- It estimates the costs of reaching people in each age and region category are as follows:

	COST PER PERSON SURVEYED ($)		
REGION	AGE ≤ 30	AGE 31-50	AGE ≥ 51
State bordering Mexico	$7.50	$6.80	$5.50
State not bordering Mexico	$6.90	$7.25	$6.10

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Management Sciences Association

- MSA's goal is to meet the sampling requirements at the least possible cost.
- The decision variables are:

X_1 = number of 30 or younger and in a border state
X_2 = number of 31-50 and in a border state
X_3 = number 51 or older and in a border state
X_4 = number 30 or younger and not in a border state
X_5 = number of 31-50 and not in a border state
X_6 = number 51 or older and not in a border state

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Management Sciences Association

Objective function

Minimize total interview costs $= \$7.50X_1 + \$6.80X_2 + \$5.50X_3 + \$6.90X_4 + \$7.25X_5 + \$6.10X_6$

subject to

$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \geq 2{,}300$ (total households)
$X_1 + X_4 \geq 1{,}000$ (households 30 or younger)
$X_2 + X_5 \geq 600$ (households 31-50)
$X_1 + X_2 + X_3 \geq 0.15(X_1 + X_2 + X_3 + X_4 + X_5 + X_6)$ (border states)
$X_3 \leq 0.20(X_3 + X_6)$ (limit on age group 51+ who can live in border state)
$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$

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MSA Solution in Excel 2010

	A	B	C	D	E	F	G	H	I	J
1	Management Science Associates									
2										
3	Variable	X1	X2	X3	X4	X5	X6			
4	Solution	0	600	140	1000	0	560	Total Cost		
5	Min. Cost	7.5	6.8	5.5	6.9	7.25	6.1	15166		
6										
7	Constraints							LHS		RHS
8	Total Households	1	1	1	1	1	1	2300	≥	2,300
9	30 and Younger	1	0	0	1	0	0	1000	≥	1,000
10	31-50	0	1	0	0	1	0	600	≥	600
11	Border States	0.85	0.85	0.85	-0.15	-0.15	-0.15	395	≥	0
12	51+ Border States	0	0	0.8	0	0	-0.2	0	≤	0

Program 8.2

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Management Sciences Association

- The following table summarizes the results of the MSA analysis.
- It will cost MSA $15,166 to conduct this research.

REGION	AGE ≤ 30	AGE 31-50	AGE ≥ 51
State bordering Mexico	0	600	140
State not bordering Mexico	1,000	0	560

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Manufacturing Applications

- Production Mix
  - LP can be used to plan the optimal mix of products to manufacture.
  - Company must meet a myriad of constraints, ranging from financial concerns to sales demand to material contracts to union labor demands.
  - Its primary goal is to generate the largest profit possible.

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Fifth Avenue Industries

- Fifth Avenue Industries produces four varieties of ties:
  - One is expensive all-silk
  - One is all-polyester
  - Two are polyester and cotton blends
- The table on the below shows the cost and availability of the three materials used in the production process:

MATERIAL	COST PER YARD ($)	MATERIAL AVAILABLE PER MONTH (YARDS)
Silk	24	1,200
Polyester	6	3,000
Cotton	9	1,600

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Fifth Avenue Industries

- The firm has contracts with several major department store chains to supply ties.
- Contracts require a minimum number of ties but may be increased if demand increases.
- Fifth Avenue's goal is to maximize monthly profit given the following decision variables.

X_1 = number of all-silk ties produced per month
X_2 = number all-polyester ties
X_3 = number of blend 1 polyester-cotton ties
X_4 = number of blend 2 silk-cotton ties

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Fifth Avenue Industries Data

VARIETY OF TIE	SELLING PRICE PER TIE ($)	MONTHLY CONTRACT MINIMUM	MONTHLY DEMAND	MATERIAL REQUIRED PER TIE (YARDS)	MATERIAL REQUIREMENTS
All silk	19.24	5,000	7,000	0.125	100% silk
All polyester	8.70	10,000	14,000	0.08	100% polyester
Poly – cotton blend 1	9.52	13,000	16,000	0.10	50% polyester – 50% cotton
Silk-cotton blend 2	10.64	5,000	8,500	0.11	60% silk - 40% cotton

Table 8.1

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Fifth Avenue Industries

- Fifth Avenue also has to calculate profit per tie for the objective function.

VARIETY OF TIE	SELLING PRICE PER TIE ($)	MATERIAL REQUIRED PER TIE (YARDS)	MATERIAL COST PER YARD ($)	COST PER TIE ($)	PROFIT PER TIE ($)
All silk	$19.24	0.125	$24	$3.00	$16.24
All polyester	$8.70	0.08	$6	$0.48	$8.22
Poly-cotton blend 1	$9.52	0.05	$6	$0.30	
		0.05	$9	$0.45	$8.77
Silk – cotton blend 2	$10.64	0.06	$24	$1.44	
		0.06	$9	$0.54	$8.66

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Fifth Avenue Industries

The complete Fifth Avenue Industries Model

Objective function

Maximize profit = $\$16.24X_1 + \$8.22X_2 + \$8.77X_3 + \$8.66X_4$

Subject to

$$
\begin{aligned}
0.125X_1 + 0.066X_4 &\le 1200 && \text{(yds of silk)}\\
0.08X_2 + 0.05X_3 &\le 3{,}000 && \text{(yds of polyester)}\\
0.05X_3 + 0.44X_4 &\le 1{,}600 && \text{(yds of cotton)}\\
X_1 &\ge 5{,}000 && \text{(contract min for silk)}\\
X_1 &\le 7{,}000 && \text{(contract min)}\\
X_2 &\ge 10{,}000 && \text{(contract min for all polyester)}\\
X_2 &\le 14{,}000 && \text{(contract max)}\\
X_3 &\ge 13{,}000 && \text{(contract mini for blend 1)}\\
X_3 &\le 16{,}000 && \text{(contract max)}\\
X_4 &\ge 5{,}000 && \text{(contract mini for blend 2)}\\
X_4 &\le 8{,}500 && \text{(contract max)}\\
X_1, X_2, X_3, X_4 &\ge 0
\end{aligned}
$$

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Fifth Avenue Solution in Excel 2010

	A	B	C	D	E	F	G	H
1	Fifth Avenue Industries							
2								
3		All silk	All poly.	Blend 1	Blend 2			
4	Variables	X1	X2	X3	X4			
5	Values	5112	14000	16000	8500	Total Profit		
6	Profit	16.24	8.22	8.77	8.66	412028.88		
7								
8	Constraints					LHS		RHS
9	Silk available	0.125			0.066	1200	≤	1200
10	Polyester available		0.08	0.05		1920	≤	3000
11	Cotton available			0.05	0.044	1174	≤	1600
12	Maximum silk	1				5112	≤	7000
13	Maximum polyester		1			14000	≤	14000
14	Maximum blend 1			1		16000	≤	16000
15	Maximum blend 2				1	8500	≤	8500
16	Minimum silk	1				5112	≥	5000
17	Minimum polyester		1			14000	≥	10000
18	Minimum blend 1			1		16000	≥	13000
19	Minimum blend 2				1	8500	≥	5000

Program 8.3

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Manufacturing Applications

- **Production Scheduling**
  - Setting a low-cost production schedule over a period of weeks or months is a difficult and important management task.
  - Important factors include labor capacity, inventory and storage costs, space limitations, product demand, and labor relations.
  - When more than one product is produced, the scheduling process can be quite complex.
  - The problem resembles the product mix model for each time period in the future.

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Greenberg Motors

- Greenberg Motors, Inc. manufactures two different electric motors for sale under contract to Drexel Corp.
- Drexel places orders three times a year for four months at a time.
- Demand varies month to month as shown below.
- Greenberg wants to develop its production plan for the next four months.

MODEL	JANUARY	FEBRUARY	MARCH	APRIL
GM3A	800	700	1,000	1,100
GM3B	1,000	1,200	1,400	1,400

Table 8.2

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Greenberg Motors

- Production planning at Greenberg must consider four factors:
  - Desirability of producing the same number of motors each month to simplify planning and scheduling.
  - Necessity to keep inventory carrying costs down.
  - Warehouse limitations.
  - Its no-lay-off policy.
- LP is a useful tool for creating a minimum total cost schedule the resolves conflicts between these factors.

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Greenberg Motors

A_i = Number of model GM3A motors produced in month i (i = 1, 2, 3, 4 for January – April)

B_i = Number of model GM3B motors produced in month i

- It costs \$20 to produce a GM3A and \$15 to produce a GM3B
- Both costs increase by 10% on March 1, thus

Cost of production = $\$20A_1 + \$20A_2 + \$22A_3 + \$22A_4 + \$15B_1 + \$15B_2 + \$16.50B_3 + \$16.50B_4$

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Greenberg Motors

- We can use the same approach to create the portion of the objective function dealing with inventory carrying costs.

IA_i= Units of GM3A left in inventory at the end of month i (i = 1, 2, 3, 4 for January – April)

IB_i= Units of GM3B left in inventory at the end of month i (i = 1, 2, 3, 4 for January – April)

- The carrying cost for GM3A motors is \$0.36 per unit per month and the GM3B costs \$0.26 per unit per month.
- Monthly ending inventory levels are used for the average inventory level.

Cost of carrying inventory = $\$0.36A_1 + \$0.36A_2 + \$0.36A_3 + 0.36A_4 + \$0.26B_1 + \$0.26B_2 + \$0.26B_3 + \$0.26B_4$

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Greenberg Motors

We combine these two for the objective function:

$$\text{Minimize total cost} = \$20A_1 + \$20A_2 + \$22A_3 + 22A_4 + \$15B_1 + \$15B_2 + \$16.50B_3 + \$16.50B_4 + \$0.36IA_1 + \$0.36IA_2 + \$0.36IA_3 + 0.36IA_4 + \$0.26IB_1 + \$0.26IB_2 + \$0.26IB_3 + \$0.26IB_4$$

End of month inventory is calculated using this relationship:

$$\begin{pmatrix}\text{Inventory}\\ \text{at the end}\\ \text{of last}\\ \text{month}\end{pmatrix} + \begin{pmatrix}\text{Current}\\ \text{month's}\\ \text{production}\end{pmatrix} - \begin{pmatrix}\text{Inventory at}\\ \text{the end of}\\ \text{this month}\end{pmatrix} = \begin{pmatrix}\text{Sales to}\\ \text{Drexel this}\\ \text{month}\end{pmatrix}$$

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Greenberg Motors

- Greenberg is starting a new four-month production cycle with a change in design specification that left no old motors in stock on January 1.
- Given January demand for both motors:

$$IA_1 = 0 + A_1 - 800$$
$$IB_1 = 0 + B_1 - 1{,}000$$

- Rewritten as January's constraints:

$$A_1 - IA_1 = 800$$
$$B_1 - IB_1 = 1{,}000$$

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Greenberg Motors

Constraints for February, March, and April:

$A_2 + IA_1 - IA_2 =$	700	February GM3A demand
$B_2 + IB_1 - IB_2 =$	1,200	February GM3B demand
$A_3 + IA_2 - IA_3 =$	1,000	March GM3A demand
$B_3 + IB_2 - IB_3 =$	1,400	March GM3B demand
$A_4 + IA_3 - IA_4 =$	1,100	April GM3A demand
$B_4 + IB_3 - IB_4 =$	1,400	April GM3B demand

And constraints for April's ending inventory:

$$IA_4 = 450$$
$$IB_4 = 300$$

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Greenberg Motors

- **We also need constraints for warehouse space:**

$$IA_1 + IB_1 \leq 3{,}300$$
$$IA_2 + IB_2 \leq 3{,}300$$
$$IA_3 + IB_3 \leq 3{,}300$$
$$IA_4 + IB_4 \leq 3{,}300$$

- **No worker is ever laid off so Greenberg has a base employment level of 2,240 labor hours per month.**
- **By adding temporary workers, available labor hours can be increased to 2,560 hours per month.**
- **Each GM3A motor requires 1.3 labor hours and each GM3B requires 0.9 hours.**

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Greenberg Motors

Labor hour constraints:

$1.3A_1 + 0.9B_1$	$\geq 2{,}240$	(January min hrs/month)
$1.3A_1 + 0.9B_1$	$\leq 2{,}560$	(January max hrs/month)
$1.3A_2 + 0.9B_2$	$\geq 2{,}240$	(February labor min)
$1.3A_2 + 0.9B_2$	$\leq 2{,}560$	(February labor max)
$1.3A_3 + 0.9B_3$	$\geq 2{,}240$	(March labor min)
$1.3A_3 + 0.9B_3$	$\leq 2{,}560$	(March labor max)
$1.3A_4 + 0.9B_4$	$\geq 2{,}240$	(April labor min)
$1.3A_4 + 0.9B_4$	$\leq 2{,}560$	(April labor max)
All variables	≥ 0	Nonnegativity constraints

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Greenberg Motors Solution in Excel 2010

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	Greenberg Motors																			
2																				
3	Variable	A1	A2	A3	A4	B1	B2	B3	B4	IA1	IA2	IA3	IA4	IB1	IB2	IB3	IB4			
4	Solution	1276.9	223.1	1757.7	792.3	1000	2522.2	77.8	1700	476.9	0	757.7	450	0	1322.2	0	300	Total Cost		
5	Min. Cost	20	20	22	22	15	15	16.5	16.5	0.36	0.36	0.36	0.36	0.26	0.26	0.26	0.26	169294.9		
6																				
7	Demand Constraints																	LHS	Sign	RHS
8	Jan. GM3A	1								-1								800	=	800
9	Feb. GM3A		1							1	-1							700	=	700
10	Mar. GM3A			1							1	-1						1000	=	1000
11	Apr. GM3A				1							1	-1					1100	=	1100
12	Jan. GM3B					1								-1				1000	=	1000
13	Feb. GM3B						1							1	-1			1200	=	1200
14	Mar. GM3B							1							1	-1		1400	=	1400
15	Apr. GM3B								1							1	-1	1400	=	1400
16	Inv.GM3A Apr.												1					450	=	450
17	Inv.GM3B Apr.																1	300	=	300
18	Labor Hour Constraints																			
19	Hrs Min. Jan.	1.3				0.9												2560	≥	2240
20	Hrs Min. Feb.		1.3				0.9											2560	≥	2240
21	Hrs Min. Mar.			1.3				0.9										2355	≥	2240
22	Hrs Min. Apr.				1.3				0.9									2560	≥	2240
23	Hrs Max. Jan.	1.3				0.9												2560	≤	2560
24	Hrs Max. Feb.		1.3				0.9											2560	≤	2560
25	Hrs Max.Mar.			1.3				0.9										2355	≤	2560
26	Hrs Max. Apr.				1.3				0.9									2560	≤	2560
27	Storage Constraints																			
28	Jan. Inv. Limit									1				1				476.92	≤	3300
29	Feb. Inv. Limit										1				1			1322.22	≤	3300
30	Mar. Inv. Limit											1				1		757.69	≤	3300
31	Apr. Inv. Limit												1				1	750	≤	3300

Program 8.4

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Greenberg Motors

Solution to Greenberg Motors Problem

PRODUCTION SCHEDULE	JANUARY	FEBRUARY	MARCH	APRIL
Units GM3A produced	1,277	223	1,758	792
Units GM3B produced	1,000	2,522	78	1,700
Inventory GM3A carried	477	0	758	450
Inventory GM3B carried	0	1,322	0	300
Labor hours required	2,560	2,560	2,355	2,560

Table 8.3

- Total cost for this four month period is $169,294.90.
- Complete model has 16 variables and 22 constraints.

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Employee Scheduling Applications

- Labor Planning
  - These problems address staffing needs over a particular time.
  - They are especially useful when there is some flexibility in assigning workers that require overlapping or interchangeable talents.

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Hong Kong Bank of Commerce and Industry

- Hong Kong Bank of Commerce and Industry has requirements for between 10 and 18 tellers depending on the time of day.
- Lunch time from noon to 2 pm is generally the busiest.
- The bank employs 12 full-time tellers but has many part-time workers available.
- Part-time workers must put in exactly four hours per day, can start anytime between 9 am and 1 pm, and are inexpensive.
- Full-time workers work from 9 am to 3 pm and have 1 hour for lunch.

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Hong Kong Bank of Commerce and Industry

Labor requirements for Hong Kong Bank of Commerce and Industry

TIME PERIOD	NUMBER OF TELLERS REQUIRED
9 am – 10 am	10
10 am – 11 am	12
11 am – Noon	14
Noon – 1 pm	16
1 pm – 2 pm	18
2 pm – 3 pm	17
3 pm – 4 pm	15
4 pm – 5 pm	10

Table 8.4

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Hong Kong Bank of Commerce and Industry

- Part-time hours are limited to a maximum of 50% of the day's total requirements.
- Part-timers earn $8 per hour on average.
- Full-timers earn $100 per day on average.
- The bank wants a schedule that will minimize total personnel costs.
- It will release one or more of its part-time tellers if it is profitable to do so.

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Hong Kong Bank of Commerce and Industry

Let

F = full-time tellers
P_1 = part-timers starting at 9 am (leaving at 1 pm)
P_2 = part-timers starting at 10 am (leaving at 2 pm)
P_3 = part-timers starting at 11 am (leaving at 3 pm)
P_4 = part-timers starting at noon (leaving at 4 pm)
P_5 = part-timers starting at 1 pm (leaving at 5 pm)

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Hong Kong Bank of Commerce and Industry

Objective:

Minimize total daily personnel cost $= \$100F + \$32(P_1 + P_2 + P_3 + P_4 + P_5)$

subject to:

F	$+ P_1$					≥ 10	(9 am – 10 am needs)
F	$+ P_1$	$+ P_2$				≥ 12	(10 am – 11 am needs)
$0.5F$	$+ P_1$	$+ P_2$	$+ P_3$			≥ 14	(11 am – noon needs)
$0.5F$	$+ P_1$	$+ P_2$	$+ P_3$	$+ P_4$		≥ 16	(noon – 1 pm needs)
F		$+ P_2$	$+ P_3$	$+ P_4$	$+ P_5$	≥ 18	(1 pm – 2 pm needs)
F			$+ P_3$	$+ P_4$	$+ P_5$	≥ 17	(2 pm – 3 pm needs)
F				$+ P_4$	$+ P_5$	≥ 15	(3 pm – 4 pm needs)
F					$+ P_5$	≥ 10	(4 pm – 5 pm needs)
F						≤ 12	(12 full-time tellers)

$$4P_1 + 4P_2 + 4P_3 + 4P_4 + 4P_5 \leq 0.50(112) \quad \text{(max 50\% part-timers)}$$

$$P_1, P_2, P_3, P_4, P_5 \geq 0$$

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Hong Kong Bank of Commerce and Industry

- There are several alternate optimal schedules Hong Kong Bank can follow:
- $F = 10$, $P_2 = 2$, $P_3 = 7$, $P_4 = 5$, P_1, $P_5 = 0$
- $F = 10$, $P_1 = 6$, $P_2 = 1$, $P_3 = 2$, $P_4 = 5$, $P_5 = 0$
- The cost of either of these two policies is \$1,448 per day.

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Labor Planning Solution in Excel 2010

	A	B	C	D	E	F	G	H	I	J
1	Labor Planning Example									
2										
3										
4	Variables	F	P1	P2	P3	P4	P5			
5	Values	10	0	7	2	5	0	Total Cost		
6	Cost	100	32	32	32	32	32	1448		
7										
8	Constraints							LHS	Sign	RHS
9	9 a.m. - 10 a.m.	1	1					10	≥	10
10	10 a.m. - 11 a.m.	1	1	1				17	≥	12
11	11 a.m. - noon	0.5	1	1	1			14	≥	14
12	noon - 1 p.m.	0.5	1	1	1	1		19	≥	16
13	1 p.m. - 2 p.m.	1		1	1	1	1	24	≥	18
14	2 p.m. - 3 p.m.	1			1	1	1	17	≥	17
15	3 p.m. - 4 p.m.	1				1	1	15	≥	15
16	4 p.m. - 5 p.m.	1					1	10	≥	10
17	Max. Full time	1						10	≤	12
18	Total PT hours		4	4	4	4	4	56	≤	56

Program 8.5

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Financial Applications

- **Portfolio Selection**
  - **Bank, investment funds, and insurance companies often have to select specific investments from a variety of alternatives.**
  - **The manager's overall objective is generally to maximize the potential return on the investment given a set of legal, policy, or risk restraints.**

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International City Trust

- International City Trust (ICT) invests in short-term trade credits, corporate bonds, gold stocks, and construction loans.
- The board of directors has placed limits on how much can be invested in each area:

INVESTMENT	INTEREST EARNED (%)	MAXIMUM INVESTMENT ($ MILLIONS)
Trade credit	7	1.0
Corporate bonds	11	2.5
Gold stocks	19	1.5
Construction loans	15	1.8

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International City Trust

- ICT has $5 million to invest and wants to accomplish two things:
  - Maximize the return on investment over the next six months.
  - Satisfy the diversification requirements set by the board.
- The board has also decided that at least 55% of the funds must be invested in gold stocks and construction loans and no less than 15% be invested in trade credit.

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International City Trust

The variables in the model are:

X_1 = dollars invested in trade credit
X_2 = dollars invested in corporate bonds
X_3 = dollars invested in gold stocks
X_4 = dollars invested in construction loans

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International City Trust

Objective:

Maximize dollars of interest earned $= 0.07X_1 + 0.11X_2 + 0.19X_3 + 0.15X_4$

subject to:

$$
\begin{aligned}
X_1 &\le 1{,}000{,}000 \\
X_2 &\le 2{,}500{,}000 \\
X_3 &\le 1{,}500{,}000 \\
X_4 &\le 1{,}800{,}000 \\
X_3 + X_4 &\ge 0.55(X_1 + X_2 + X_3 + X_4) \\
X_1 &\ge 0.15(X_1 + X_2 + X_3 + X_4) \\
X_1 + X_2 + X_3 + X_4 &\le 5{,}000{,}000 \\
X_1, X_2, X_3, X_4 &\ge 0
\end{aligned}
$$

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International City Trust

- The optimal solution to the ICT is to make the following investments:

  X_1 = \$750,000

  X_2 = \$950,000

  X_3 = \$1,500,000

  X_4 = \$1,800,000

- The total interest earned with this plan is \$712,000.

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ICT Portfolio Solution in Excel 2010

	A	B	C	D	E	F	G	H
1	ICT Portfolio Selection							
2								
3	Variable	X1	X2	X3	X4			
4	Solution	750000	950000	1500000	1800000	Total Return		
5	Max. Return	0.07	0.11	0.19	0.15	712000		
6								
7						LHS		RHS
8	Trade	1				750000	≤	1,000,000
9	Bonds		1			950000	≤	2,500,000
10	Gold			1		1500000	≤	1,500,000
11	Construction				1	1800000	≤	1,800,000
12	Min. Gold+Const	-0.55	-0.55	0.45	0.45	550000	≥	0
13	Min. Trade	0.85	-0.15	-0.15	-0.15	0	≥	0
14	Total Invested	1	1	1	1	5000000	≤	5000000

Program 8.6

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Truck Loading Problem

- Truck Loading Problem
  - The truck loading problem involves deciding which items to load on a truck so as to maximize the value of a load shipped.
  - Goodman Shipping has to ship the following six items:

ITEM	VALUE ($)	WEIGHT (POUNDS)
1	22,500	7,500
2	24,000	7,500
3	8,000	3,000
4	9,500	3,500
5	11,500	4,000
6	9,750	3,500

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Goodman Shipping

- The objective is to maximize the value of items loaded into the truck.
- The truck has a capacity of 10,000 pounds.
- The decision variable is:

  X_i = proportion of each item i loaded on the truck

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Goodman Shipping

Objective:

$$\text{Maximize load value} = \$22{,}500X_1 + \$24{,}000X_2 + \$8{,}000X_3 + \$9{,}500X_4 + \$11{,}500X_5 + \$9{,}750X_6$$

subject to

$$7{,}500X_1 + 7{,}500X_2 + 3{,}000X_3 + 3{,}500X_4 + 4{,}000X_5 + 3{,}500X_6 \leq 10{,}000 \text{ lb capacity}$$

$$X_1 \leq 1$$
$$X_2 \leq 1$$
$$X_3 \leq 1$$
$$X_4 \leq 1$$
$$X_5 \leq 1$$
$$X_6 \leq 1$$
$$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$$

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Goodman Truck Loading Solution in Excel

	A	B	C	D	E	F	G	H	I	J
1	Goodman Shipping									
2										
3	Variables	X1	X2	X3	X4	X5	X6			
4	Values	0.3333	1	0	0	0	0	Total Value		
5	Load Value $	22500	24000	8000	9500	11500	9750	31500		
6										
7	Constraints							LHS	Sign	RHS
8	Total weight	7500	7500	3000	3500	4000	3500	10000	≤	10000
9	% Item 1	1						0.3333333	≤	1
10	% Item 2		1					1	≤	1
11	% Item 3			1				0	≤	1
12	% Item 4				1			0	≤	1
13	% Item 5					1		0	≤	1
14	% Item 6						1	0	≤	1

Program 8.7

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Goodman Shipping

- The Goodman Shipping problem raises an interesting issue:
  - The solution calls for one third of Item 1 to be loaded on the truck.
  - What if Item 1 cannot be divided into smaller pieces?
- Rounding down leaves unused capacity on the truck and results in a value of $24,000.
- Rounding up is not possible since this would exceed the capacity of the truck.
- Using *integer programming*, in which the solution is required to contain only integers, the solution is to load one unit of Items 3, 4, and 6 for a value of $27,250.

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Ingredient Blending Applications

- Diet Problems
  - This is one of the earliest LP applications, and is used to determine the most economical diet for hospital patients.
  - This is also known as the feed mix problem.

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Whole Food Nutrition Center

- The Whole Food Nutrition Center uses three bulk grains to blend a natural cereal.
- It advertises that the cereal meets the U.S. Recommended Daily Allowance (USRDA) for four key nutrients.
- It wants to select the blend that will meet the requirements at the minimum cost.

NUTRIENT	USRDA
Protein	3 units
Riboflavin	2 units
Phosphorus	1 unit
Magnesium	0.425 unit

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Whole Food Nutrition Center

Let

X_A = pounds of grain A in one 2-ounce serving of cereal
X_B = pounds of grain B in one 2-ounce serving of cereal
X_C = pounds of grain C in one 2-ounce serving of cereal

Whole Food's Natural Cereal requirements:

GRAIN	COST PER POUND (CENTS)	PROTEIN (UNITS/LB)	RIBOFLAVIN (UNITS/LB)	PHOSPHOROUS (UNITS/LB)	MAGNESIUM (UNITS/LB)
A	33	22	16	8	5
B	47	28	14	7	0
C	38	21	25	9	6

Table 8.5

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Whole Food Nutrition Center

The objective is:

Minimize total cost of mixing a 2-ounce serving $= \$0.33X_A + \$0.47X_B + \$0.38X_C$

subject to

$$
\begin{aligned}
22X_A + 28X_B + 21X_C &\geq 3 && \text{(protein units)}\\
16X_A + 14X_B + 25X_C &\geq 2 && \text{(riboflavin units)}\\
8X_A + 7X_B + 9X_C &\geq 1 && \text{(phosphorous units)}\\
5X_A + 0X_B + 6X_C &\geq 0.425 && \text{(magnesium units)}\\
X_A + X_B + X_C &= 0.125 && \text{(total mix)}\\
X_A, X_B, X_C &\geq 0
\end{aligned}
$$

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Whole Food Diet Solution in Excel 2010

	A	B	C	D	E	F	G
1	Whole Foods Nutrition Problem						
2							
3		Grain A	Grain B	Grain C			
4	Variable	Xa	Xb	Xc			
5	Solution	0.025	0.05	0.05	Total Cost		
6	Minimize	0.33	0.47	0.38	0.05075		
7							
8	Constraints				LHS	Sign	RHS
9	Protein	22	28	21	3	≥	3
10	Riboflavin	16	14	25	2.35	≥	2
11	Phosphorus	8	7	9	1	≥	1
12	Magnesium	5	0	6	0.425	≥	0.425
13	Total Weight	1	1	1	0.125	=	0.125

Program 8.8

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Ingredient Blending Applications

- Ingredient Mix and Blending Problems
  - Diet and feed mix problems are special cases of a more general class of problems known as *ingredient* or *blending problems.*
  - Blending problems arise when decisions must be made regarding the blending of two or more resources to produce one or more product.
  - Resources may contain essential ingredients that must be blended so that a specified percentage is in the final mix.

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Low Knock Oil Company

- The Low Knock Oil Company produces two grades of cut-rate gasoline for industrial distribution.
- The two grades, regular and economy, are created by blending two different types of crude oil.
- The crude oil differs in cost and in its content of crucial ingredients.

CRUDE OIL TYPE	INGREDIENT A (%)	INGREDIENT B (%)	COST/BARREL ($)
X100	35	55	30.00
X220	60	25	34.80

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Low Knock Oil Company

The firm lets

X_1 = barrels of crude X100 blended to produce the refined regular

X_2 = barrels of crude X100 blended to produce the refined economy

X_3 = barrels of crude X220 blended to produce the refined regular

X_4 = barrels of crude X220 blended to produce the refined economy

The objective function is

$$\text{Minimize cost} = \$30X_1 + \$30X_2 + \$34.80X_3 + \$34.80X_4$$

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Low Knock Oil Company

Problem formulation

At least 45% of each barrel of regular must be ingredient A

$(X_1 + X_3)$ = total amount of crude blended to produce the refined regular gasoline demand

Thus,

$$0.45(X_1 + X_3) = \text{amount of ingredient A required}$$

But:

$0.35X_1 + 0.60X_3$ = amount of ingredient A in refined regular gas

So

$$0.35X_1 + 0.60X_3 \geq 0.45X_1 + 0.45X_3$$

or

$-0.10X_1 + 0.15X_3 \geq 0$ (ingredient A in regular constraint)

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Low Knock Oil Company

Problem formulation

$$\text{Minimize cost} = 30X_1 + 30X_2 + 34.80X_3 + 34.80X_4$$

subject to

$$\begin{aligned}
X_1 + X_3 &\geq 25{,}000 \\
X_2 + X_4 &\geq 32{,}000 \\
-0.10X_1 + 0.15X_3 &\geq 0 \\
0.05X_2 - 0.25X_4 &\leq 0 \\
X_1, X_2, X_3, X_4 &\geq 0
\end{aligned}$$

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Low Knock Oil Solution in Excel 2010

	A	B	C	D	E	F	G	H
1	Low Knock Oil Company							
2								
3		X100 Reg	X100 Econ	X220 Reg	X220 Econ			
4	Variable	X1	X2	X3	X4			
5	Solution	15000	26666.67	10000	5333.33	Total Cost		
6	Cost	30	30	34.8	34.8	1783600		
7								
8	Constraints					LHS	Sign	RHS
9	Demand Regular	1		1		25000	≥	25000
10	Demand Economy		1		1	32000	≥	32000
11	Ing. A in Regular	-0.1		0.15		0	≥	0
12	Ing. B in Economy		0.05		-0.25	0	≤	0

Program 8.9

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Transportation Applications

- Shipping Problem
  - The transportation or shipping problem involves determining the amount of goods or items to be transported from a number of origins to a number of destinations.
  - The objective usually is to minimize total shipping costs or distances.
  - This is a specific case of LP and a special algorithm has been developed to solve it.

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Top Speed Bicycle Company

- The Top Speed Bicycle Co. manufactures and markets a line of 10-speed bicycles.
- The firm has final assembly plants in two cities where labor costs are low.
- It has three major warehouses near large markets.
- The sales requirements for the next year are:
  - New York – 10,000 bicycles
  - Chicago – 8,000 bicycles
  - Los Angeles – 15,000 bicycles
- The factory capacities are:
  - New Orleans – 20,000 bicycles
  - Omaha – 15,000 bicycles

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Top Speed Bicycle Company

The cost of shipping bicycles from the plants to the warehouses is different for each plant and warehouse:

FROM \ TO	NEW YORK	CHICAGO	LOS ANGELES
New Orleans	$2	$3	$5
Omaha	$3	$1	$4

The company wants to develop a shipping schedule that will minimize its total annual cost.

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Top Speed Bicycle Company

Network Representation of the Transportation Problem with Costs, Demands, and Supplies

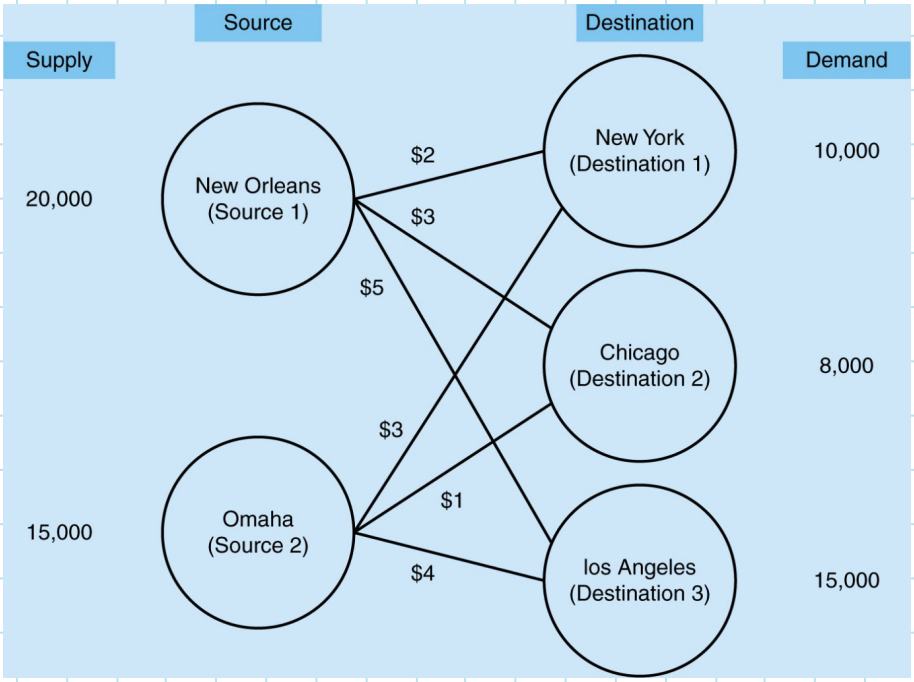

Figure 8.1

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Top Speed Bicycle Company

The double subscript variables will represent the origin factory and the destination warehouse:

X_{ij} = bicycles shipped from factory i to warehouse j

So:

X_{11} = number of bicycles shipped from New Orleans to New York
X_{12} = number of bicycles shipped from New Orleans to Chicago
X_{13} = number of bicycles shipped from New Orleans to Los Angeles
X_{21} = number of bicycles shipped from Omaha to New York
X_{22} = number of bicycles shipped from Omaha to Chicago
X_{23} = number of bicycles shipped from Omaha to Los Angeles

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Top Speed Bicycle Company

Objective:

Minimize total shipping costs $= 2X_{11} + 3X_{12} + 5X_{13} + 3X_{21} + 1X_{22} + 4X_{23}$

subject to

$$
\begin{aligned}
X_{11} + X_{21} &= 10{,}000 && \text{(New York demand)} \\
X_{12} + X_{22} &= 8{,}000 && \text{(Chicago demand)} \\
X_{13} + X_{23} &= 15{,}000 && \text{(Los Angeles demand)} \\
X_{11} + X_{12} + X_{13} &\leq 20{,}000 && \text{(New Orleans factory supply)} \\
X_{21} + X_{22} + X_{23} &\leq 15{,}000 && \text{(Omaha factory supply)} \\
\text{All variables} &\geq 0
\end{aligned}
$$

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Top Speed Bicycle Company Solution in Excel 2010

	A	B	C	D	E	F	G	H	I	J
1	Top Speed Bicycle Company									
2		N.O. to	N.O. to	N.O. to	Omaha to	Omaha to	Omaha to			
3		NY	Chicago	LA	NY	Chicago	LA			
4	**Variables**	X11	X12	X13	X21	X22	X23			
5	**Values**	10000	0	8000	0	8000	7000	Total Cost		
6	**Cost**	2	3	5	3	1	4	96000		
7										
8	Constraints							LHS	Sign	RHS
9	NY Demand	1			1			10000	=	10000
10	Chi. Demand		1			1		8000	=	8000
11	LA Demand			1			1	15000	=	15000
12	N.O. Supply	1	1	1				18000	≤	20000
13	Omaha Supply				1	1	1	15000	≤	15000

Program 8.10

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Top Speed Bicycle Company

Top Speed Bicycle solution:

FROM \ TO	NEW YORK	CHICAGO	LOS ANGELES
New Orleans	10,000	0	8,000
Omaha	0	8,000	7,000

- **Total shipping cost equals $96,000.**
- **Transportation problems are a special case of LP as the coefficients for every variable in the constraint equations equal 1.**

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Chapter 9

Transportation and Assignment Models

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. Structure LP problems using the transportation, transshipment and assignment models.
2. Use the northwest corner and stepping-stone methods.
3. Solve facility location and other application problems with transportation models.
4. Solve assignment problems with the Hungarian (matrix reduction) method.

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Chapter Outline

9.1 Introduction
9.2 The Transportation Problem
9.3 The Assignment Problem
9.4 The Transshipment Problem
9.5 The Transportation Algorithm
9.6 Special Situations with the Transportation Algorithm
9.7 Facility Location Analysis
9.8 The Assignment Algorithm
9.9 Special Situations with the Assignment Algorithm

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Introduction

- In this chapter we will explore three special linear programming models:
  - The transportation problem.
  - The assignment problem.
  - The transshipment problem.
- These problems are members of a category of LP techniques called *network flow problems.*

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The Transportation Problem

- The *transportation problem* deals with the distribution of goods from several points of supply (*sources*) to a number of points of demand (*destinations*).
- Usually we are given the capacity of goods at each source and the requirements at each destination.
- Typically the objective is to minimize total transportation and production costs.

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The Transportation Problem

- The Executive Furniture Corporation manufactures office desks at three locations: Des Moines, Evansville, and Fort Lauderdale.
- The firm distributes the desks through regional warehouses located in Boston, Albuquerque, and Cleveland.

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The Transportation Problem

Network Representation of a Transportation Problem, with Costs, Demands and Supplies

Executive Furniture Company

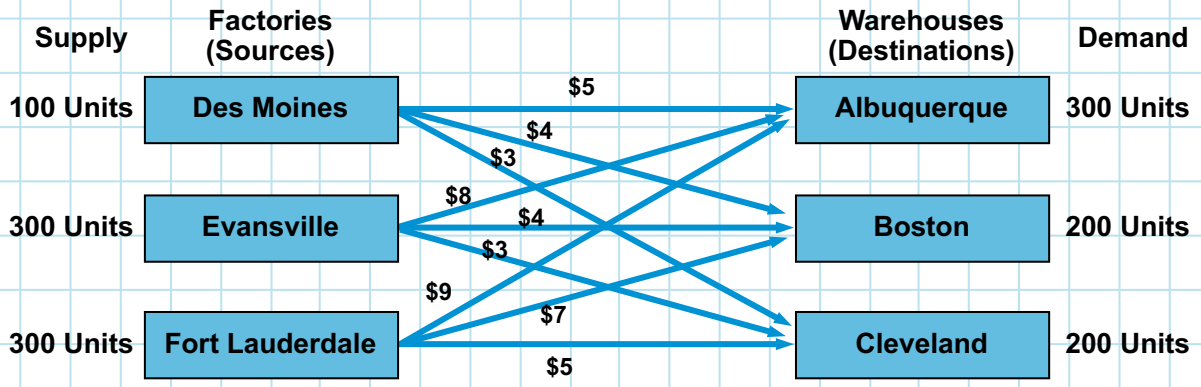

Figure 9.1

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Linear Programming for the Transportation Example

- Let X_{ij} = number of units shipped from source *i* to destination *j*,
  - Where:
    - *i* = 1, 2, 3, with 1 = Des Moines, 2 = Evansville, and 3 = Fort Lauderdale
    - *j* = 1, 2, 3, with 1 = Albuquerque, 2 = Boston, and 3 = Cleveland.

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Linear Programming for the Transportation Example

- Minimize total cost = $5X_{11} + 4X_{12} + 3X_{13} + 8X_{21} + 4X_{22} + 3X_{23} + 9X_{31} + 7X_{32} + 5X_{33}$
- Subject to:
  - $X_{11} + X_{12} + X_{13} \leq 100$ (Des Moines supply)
  - $X_{21} + X_{22} + X_{23} \leq 300$ (Evansville supply)
  - $X_{31} + X_{32} + X_{33} \leq 300$ (Fort Lauderdale supply)
  - $X_{11} + X_{21} + X_{31} = 300$ (Albuquerque demand)
  - $X_{12} + X_{22} + X_{32} = 200$ (Boston demand)
  - $X_{13} + X_{23} + X_{33} = 200$ (Cleveland demand)
  - $X_{ij} \geq 0$ for all *i* and *j*.

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Executive Furniture Corporation Solution in Excel 2010

	A	B	C	D	E	F
1		Shipping Cost Per Unit				
2	From\To	Albuquerque	Boston	Cleveland		
3	Des Moines	5	4	3		
4	Evansville	8	4	3		
5	Fort Lauderdale	9	7	5		
6						
7						
8		Solution - Number of units shipped				
9		Albuquerque	Boston	Cleveland	Total shipped	Supply
10	Des Moines	100	0	0	100	100
11	Evansville	0	200	100	300	300
12	Fort Lauderdale	200	0	100	300	300
13	Total received	300	200	200		
14	Demand	300	200	200		
15						
16	Total cost =	3900				

	E
9	Total shipped
10	=SUM(B10:D10)

	B
13	=SUM(B10:B12)

Program 9.1

	B
16	=SUMPRODUCT(B3:D5,B10:D12)

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A General LP Model for Transportation Problems

- **Let:**
  - X_{ij} = number of units shipped from source *i* to destination *j*.
  - c_{ij} = cost of one unit from source *i* to destination *j*.
  - s_i = supply at source *i*.
  - d_j = demand at destination *j*.

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A General LP Model for Transportation Problems

Minimize cost = $\sum_{j=1}^{n}\sum_{i=1}^{m} c_{ij}x_{ij}$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \le s_i \qquad i = 1, 2, \ldots, m.$$

$$\sum_{i=1}^{m} x_{ij} \le d_j \qquad j = 1, 2, \ldots, n.$$

$$x_{ij} \ge 0 \qquad \text{for all } i \text{ and } j.$$

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The Assignment Problem

- This type of problem determines the most efficient assignment of people to particular tasks, etc.
- Objective is typically to minimize total cost or total task time.

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Linear Program for Assignment Example

- The Fix-it Shop has just received three new repair projects that must be repaired quickly: a radio, a toaster oven, and a coffee table.
- Three workers with different talents are able to do the jobs.
- The owner estimates the cost in wages if the workers are assigned to each of the three jobs.
- Objective: minimize total cost.

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Example of an Assignment Problem in a Transportation Network Format

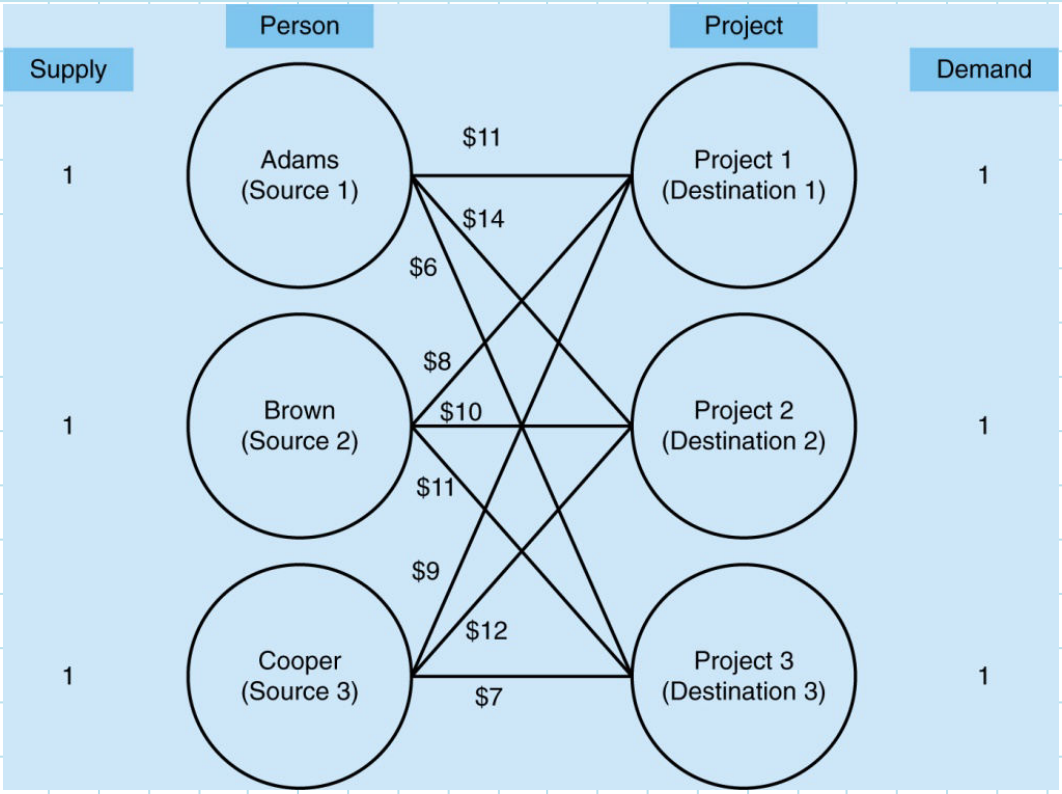

Figure 9.2

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Linear Program for Assignment Example

Let:

- X_{ij} = 1 if person *i* is assigned to project *j*, or 0 otherwise.

Where:

- *i* = 1,2,3 with 1 = Adams, 2 = Brown, and 3 = Cooper
- *j* = 1,2,3, with 1 = Project 1, 2 = Project 2, and 3 = Project 3.

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Linear Program for Assignment Example

Minimize total cost $= 11X_{11} + 14X_{12} + 6X_{13} + 8X_{21} + 10X_{22} + 11X_{23} + 9X_{31} + 12X_{32} + 7X_{33}$

Subject to:

- $X_{11} + X_{12} + X_{13} \leq 1$
- $X_{21} + X_{22} + X_{23} \leq 1$
- $X_{31} + X_{32} + X_{33} \leq 1$
- $X_{11} + X_{21} + X_{31} = 1$
- $X_{12} + X_{22} + X_{32} = 1$
- $X_{13} + X_{23} + X_{33} = 1$
- $X_{ij} = 0$ or 1 for all *i* and *j*

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Fix-it Shop Solution in Excel 2010

	A	B	C	D	E	F
1		Cost for Assignments				
2	Person\Project	Project 1	Project 2	Project 3		
3	Adams	11	14	6		
4	Brown	8	10	11		
5	Cooper	9	12	7		
6						
7						
8			Made			
9		Project 1	Project 2	Project 3	Total projects	Supply
10	Adams	0	0	1	1	1
11	Brown	0	1	0	1	1
12	Cooper	1	0	0	1	1
13	Total assigned	1	1	1		
14	Total workers	1	1	1		
15						
16	Total cost =	25				

	E
10	=SUM(B10:D10)

	B
13	=SUM(B10:B12)

Program 9.2

	B
16	=SUMPRODUCT(B3:D5,B10:D12)

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Linear Program for Assignment Example

- X_{13} = 1, so Adams is assigned to project 3.
- X_{22} = 1, so Brown is assigned to project 2.
- X_{31} = 1, so Cooper is assigned to project 3.
- Total cost of the repairs is $25.

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Transshipment Applications

When the items are being moved from a source to a destination through an intermediate point (a *transshipment point*), the problem is called a *transshipment problem.*

Distribution Centers

- **Frosty Machines manufactures snow blowers in Toronto and Detroit.**
- **These are shipped to regional distribution centers in Chicago and Buffalo.**
- **From there they are shipped to supply houses in New York, Philadelphia, and St Louis.**
- **Shipping costs vary by location and destination.**
- **Snow blowers cannot be shipped directly from the factories to the supply houses.**

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Network Representation of Transshipment Example

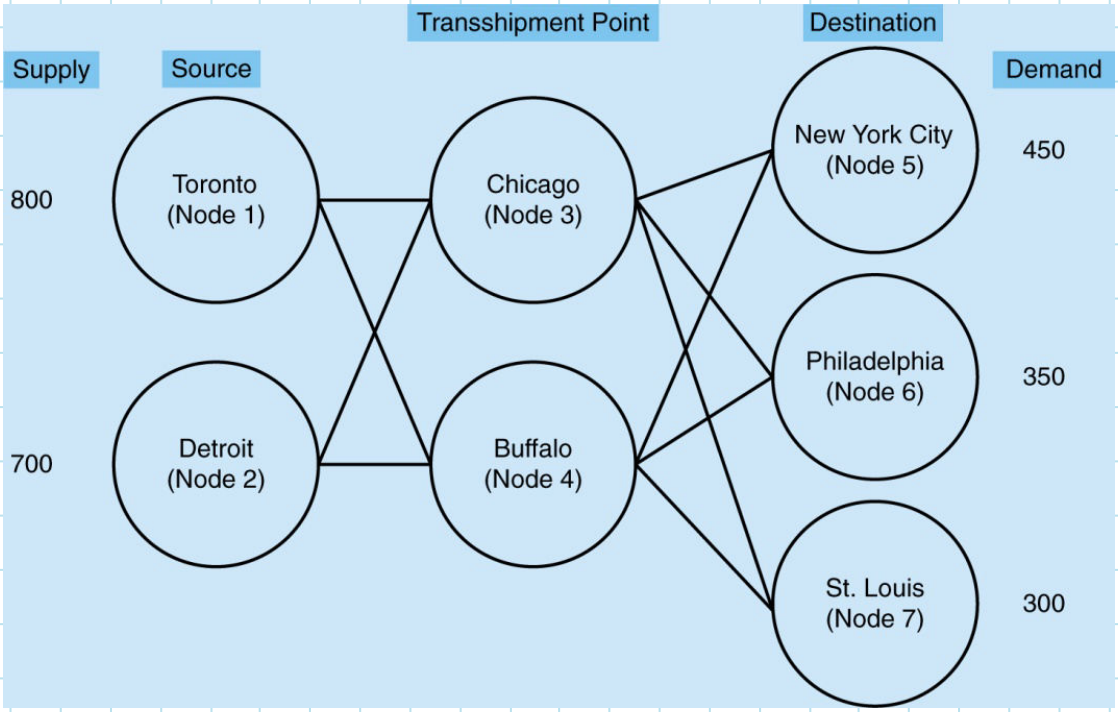

Figure 9.3

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Transshipment Applications

Frosty Machines Transshipment Data

	TO					
FROM	CHICAGO	BUFFALO	NEW YORK CITY	PHILADELPHIA	ST LOUIS	SUPPLY
Toronto	$4	$7	—	—	—	800
Detroit	$5	$7	—	—	—	700
Chicago	—	—	$6	$4	$5	—
Buffalo	—	—	$2	$3	$4	—
Demand	—	—	450	350	300	

Table 9.1

Frosty would like to minimize the transportation costs associated with shipping snow blowers to meet the demands at the supply centers given the supplies available.

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Transshipment Applications

A description of the problem would be to minimize cost subject to:

1. **The number of units shipped from Toronto is not more than 800.**
2. **The number of units shipped from Detroit is not more than 700.**
3. **The number of units shipped to New York is 450.**
4. **The number of units shipped to Philadelphia is 350.**
5. **The number of units shipped to St Louis is 300.**
6. **The number of units shipped out of Chicago is equal to the number of units shipped into Chicago.**
7. **The number of units shipped out of Buffalo is equal to the number of units shipped into Buffalo.**

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Transshipment Applications

The decision variables should represent the number of units shipped from each source to the transshipment points and from there to the final destinations.

X_{13} = the number of units shipped from Toronto to Chicago
X_{14} = the number of units shipped from Toronto to Buffalo
X_{23} = the number of units shipped from Detroit to Chicago
X_{24} = the number of units shipped from Detroit to Buffalo
X_{35} = the number of units shipped from Chicago to New York
X_{36} = the number of units shipped from Chicago to Philadelphia
X_{37} = the number of units shipped from Chicago to St Louis
X_{45} = the number of units shipped from Buffalo to New York
X_{46} = the number of units shipped from Buffalo to Philadelphia
X_{47} = the number of units shipped from Buffalo to St Louis

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Transshipment Applications

The linear program is:

Minimize cost = $4X_{13} + 7X_{14} + 5X_{23} + 7X_{24} + 6X_{35} + 4X_{36} + 5X_{37} + 2X_{45} + 3X_{46} + 4X_{47}$

subject to

$X_{13} + X_{14} \leq 800$ (supply at Toronto)
$X_{23} + X_{24} \leq 700$ (supply at Detroit)
$X_{35} + X_{45} = 450$ (demand at New York)
$X_{36} + X_{46} = 350$ (demand at Philadelphia)
$X_{37} + X_{47} = 300$ (demand at St Louis)
$X_{13} + X_{23} = X_{35} + X_{36} + X_{37}$ (shipping through Chicago)
$X_{14} + X_{24} = X_{45} + X_{46} + X_{47}$ (shipping through Buffalo)
$X_{ij} \geq 0$ for all *i* and *j* (nonnegativity)

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Solution to Frosty Machines Transshipment Problem

	A	B	C	D	E	F	G	H
1	Frosty Machines Transshipment Problem							
2								
3				Shipping Cost Per Unit				
4	From\To	Chicago	Buffalo	NYC	Phil.	St.Louis		
5	Toronto	4	7					
6	Detroit	5	7					
7	Chicago			6	4	5		
8	Buffalo			2	3	4		
9								
10			Solution - Number of units shipped					
11		Chicago	Buffalo	NYC	Phil.	St.Louis	Total shipped	Supply
12	Toronto	650	150				800	800
13	Detroit	0	300				300	700
14	Chicago			0	350	300	650	
15	Buffalo			450	0	0	450	
16	Total received	650	450	450	350	300		
17	Demand			450	350	300		
18								
19	Total cost =	9550						

	G
11	Total shipped
12	=SUM(B12:C12)

	G
14	=SUM(D14:F14)

	B
16	=SUM(B12:B13)

	D
16	=SUM(D14:D15)

Program 9.3

	B
19	=SUMPRODUCT(B5:F8,B12:F15)

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The Transportation Algorithm

- This is an iterative procedure in which a solution to a transportation problem is found and evaluated using a special procedure to determine whether the solution is optimal.
  - When the solution is optimal, the process stops.
  - If not, then a new solution is generated.

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Transportation Table for Executive Furniture Corporation

Des Moines capacity constraint

FROM \ TO	WAREHOUSE AT ALBUQUERQUE	WAREHOUSE AT BOSTON	WAREHOUSE AT CLEVELAND	FACTORY CAPACITY
DES MOINES FACTORY	$5	$4	$3	100
EVANSVILLE FACTORY	$8	$4	$3	300
FORT LAUDERDALE FACTORY	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.2

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Cleveland warehouse demand

Total supply and demand

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

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Developing an Initial Solution: Northwest Corner Rule

- Once we have arranged the data in a table, we must establish an initial feasible solution.
- One systematic approach is known as the *northwest corner rule.*
- Start in the upper left-hand cell and allocate units to shipping routes as follows:
  1. Exhaust the supply (factory capacity) of each row before moving down to the next row.
  2. Exhaust the demand (warehouse) requirements of each column before moving to the right to the next column.
  3. Check that all supply and demand requirements are met.
- This problem takes five steps to make the initial shipping assignments.

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Developing an Initial Solution: Northwest Corner Rule

1. Beginning in the upper left hand corner, we assign 100 units from Des Moines to Albuquerque. This exhaust the supply from Des Moines but leaves Albuquerque 200 desks short. We move to the second row in the same column.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 $5	$4	$3	100
EVANSVILLE (*E*)	$8	$4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

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Developing an Initial Solution: Northwest Corner Rule

2. Assign 200 units from Evansville to Albuquerque. This meets Albuquerque's demand. Evansville has 100 units remaining so we move to the right to the next column of the second row.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 $5	$4	$3	100
EVANSVILLE (*E*)	200 $8	$4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

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Developing an Initial Solution: Northwest Corner Rule

3. Assign 100 units from Evansville to Boston. The Evansville supply has now been exhausted but Boston is still 100 units short. We move down vertically to the next row in the Boston column.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 $5	$4	$3	100
EVANSVILLE (*E*)	200 $8	100 $4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

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Developing an Initial Solution: Northwest Corner Rule

4. Assign 100 units from Fort Lauderdale to Boston. This fulfills Boston's demand and Fort Lauderdale still has 200 units available.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 $5	$4	$3	100
EVANSVILLE (*E*)	200 $8	100 $4	$3	300
FORT LAUDERDALE (*F*)	$9	100 $7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

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Developing an Initial Solution: Northwest Corner Rule

5. Assign 200 units from Fort Lauderdale to Cleveland. This exhausts Fort Lauderdale's supply and Cleveland's demand. The initial shipment schedule is now complete.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 $5	$4	$3	100
EVANSVILLE (*E*)	200 $8	100 $4	$3	300
FORT LAUDERDALE (*F*)	$9	100 $7	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.3

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Developing an Initial Solution: Northwest Corner Rule

The cost of this shipping assignment:

ROUTE FROM	ROUTE TO	UNITS SHIPPED	x	PER UNIT COST ($)	=	TOTAL COST ($)
D	*A*	100		5		500
E	*A*	200		8		1,600
E	*B*	100		4		400
F	*B*	100		7		700
F	*C*	200		5		1,000
						4,200

This solution is feasible but we need to check to see if it is optimal.

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Stepping-Stone Method: Finding a Least Cost Solution

- The ***stepping-stone method*** is an iterative technique for moving from an initial feasible solution to an optimal feasible solution.
- There are two distinct parts to the process:
  - Testing the current solution to determine if improvement is possible.
  - Making changes to the current solution to obtain an improved solution.
- This process continues until the optimal solution is reached.

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Stepping-Stone Method: Finding a Least Cost Solution

- There is one very important rule: ***The number of occupied routes (or squares) must always be equal to one less than the sum of the number of rows plus the number of columns***
  - In the Executive Furniture problem this means the initial solution must have 3 + 3 – 1 = 5 squares used.

$$\text{Occupied shipping routes (squares)} = \text{Number of rows} + \text{Number of columns} - 1$$

- When the number of occupied rows is less than this, the solution is called ***degenerate.***

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Testing the Solution for Possible Improvement

- The stepping-stone method works by testing each unused square in the transportation table to see what would happen to total shipping costs if one unit of the product were tentatively shipped on an unused route.
- There are five steps in the process.

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Five Steps to Test Unused Squares with the Stepping-Stone Method

1. Select an unused square to evaluate.
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used with only horizontal or vertical moves allowed.
3. Beginning with a plus (+) sign at the unused square, place alternate minus (–) signs and plus signs on each corner square of the closed path just traced.

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Five Steps to Test Unused Squares with the Stepping-Stone Method

4. Calculate an *improvement index* by adding together the unit cost figures found in each square containing a plus sign and then subtracting the unit costs in each square containing a minus sign.
5. Repeat steps 1 to 4 until an improvement index has been calculated for all unused squares. If all indices computed are greater than or equal to zero, an optimal solution has been reached. If not, it is possible to improve the current solution and decrease total shipping costs.

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Five Steps to Test Unused Squares with the Stepping-Stone Method

For the Executive Furniture Corporation data:

Steps 1 and 2. Beginning with Des Moines–Boston route we trace a closed path using only currently occupied squares, alternately placing plus and minus signs in the corners of the path.

- In a *closed path*, only squares currently used for shipping can be used in turning corners.
- *Only one* closed route is possible for each square we wish to test.

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Five Steps to Test Unused Squares with the Stepping-Stone Method

Step 3. Test the cost-effectiveness of the Des Moines–Boston shipping route by pretending that we are shipping one desk from Des Moines to Boston. Put a plus in that box.

- But if we ship one ***more*** unit out of Des Moines we will be sending out 101 units.
- Since the Des Moines factory capacity is only 100, we must ship ***fewer*** desks from Des Moines to Albuquerque so place a minus sign in that box.
- But that leaves Albuquerque one unit short so increase the shipment from Evansville to Albuquerque by one unit and so on until the entire closed path is completed.

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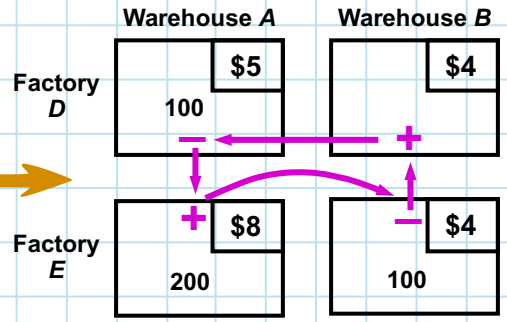
Five Steps to Test Unused Squares with the Stepping-Stone Method

Evaluating the unused Des Moines–Boston shipping route



TO / FROM	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 $5	$4	$3	100
EVANSVILLE	200 $8	100 $4	$3	300
FORT LAUDERDALE	$9	100 $7	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.3

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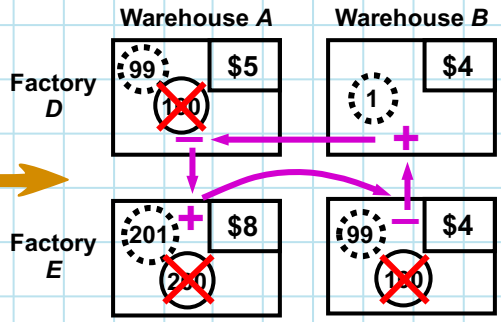
Five Steps to Test Unused Squares with the Stepping-Stone Method

Evaluating the unused Des Moines–Boston shipping route



FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 $5	$4	$3	100
EVANSVILLE	200 $8	100 $4	$3	300
FORT LAUDERDALE	$9	100 $7	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.4

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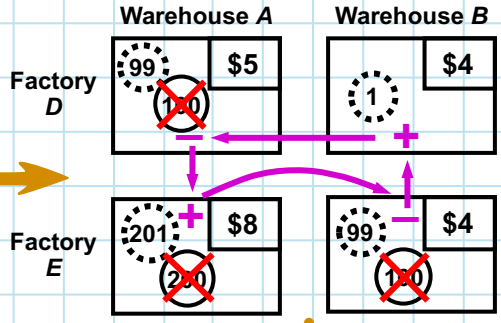
Five Steps to Test Unused Squares with the Stepping-Stone Method

Evaluating the unused Des Moines–Boston shipping route



FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	
DES MOINES	100 $5	$4	$3	
EVANSVILLE	200 $8	100 $4	$3	
FORT LAUDERDALE	$9	100 $7	200 $5	
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.4

Result of Proposed Shift in Allocation

= 1 x $4
– 1 x $5
+ 1 x $8
– 1 x $4 = +$3

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Five Steps to Test Unused Squares with the Stepping-Stone Method

Step 4. Now compute an ***improvement index*** (I_{ij}) for the Des Moines–Boston route.

Add the costs in the squares with plus signs and subtract the costs in the squares with minus signs:

Des Moines–Boston index $= I_{DB} = +\$4 - \$5 + \$5 - \$4 = +\$3$

This means for every desk shipped via the Des Moines–Boston route, total transportation cost will ***increase*** by $3 over their current level.

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Five Steps to Test Unused Squares with the Stepping-Stone Method

Step 5. Now examine the Des Moines–Cleveland unused route which is slightly more difficult to draw.

- Again, only turn corners at squares that represent existing routes.
- Pass through the Evansville–Cleveland square but we can not turn there or put a + or – sign.
- The closed path we will use is:

$$+ DC - DA + EA - EB + FB - FC$$

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Five Steps to Test Unused Squares with the Stepping-Stone Method

Evaluating the Des Moines–Cleveland Shipping Route

FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 $5	$4	Start $3	100
EVANSVILLE	200 $8	100 $4	$3	300
FORT LAUDERDALE	$9	100 $7	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.5

Des Moines–Cleveland improvement index $= I_{DC} = +\$3 - \$5 + \$8 - \$4 + \$7 - \$5 = +\$4$

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Five Steps to Test Unused Squares with the Stepping-Stone Method

Opening the Des Moines–Cleveland route will not lower our total shipping costs.

Evaluating the other two routes we find:

Evansville-Cleveland index $= I_{EC} = +\$3 - \$4 + \$7 - \$5 = +\$1$

The closed path is

$+ EC - EB + FB - FC$

Fort Lauderdale–Albuquerque index $= I_{FA} = +\$9 - \$7 + \$4 - \$8 = -\$2$

The closed path is

$+ FA - FB + EB - EA$

Opening the Fort Lauderdale-Albuquerque route *will* lower our total transportation costs.

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Obtaining an Improved Solution

- In the Executive Furniture problem there is only one unused route with a negative index (Fort Lauderdale-Albuquerque).
  - If there was more than one route with a negative index, we would choose the one with the largest improvement
- We now want to ship the maximum allowable number of units on the new route
- The quantity to ship is found by referring to the closed path of plus and minus signs for the new route and selecting the *smallest number* found in those squares containing minus signs.

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Obtaining an Improved Solution

- To obtain a new solution, that number is added to all squares on the closed path with plus signs and subtracted from all squares the closed path with minus signs.
- All other squares are unchanged.
- In this case, the maximum number that can be shipped is 100 desks as this is the smallest value in a box with a negative sign (*FB* route).
- We add 100 units to the *FA* and *EB* routes and subtract 100 from *FB* and *EA* routes.
- This leaves balanced rows and columns and an improved solution.

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Obtaining an Improved Solution

Stepping-Stone Path Used to Evaluate Route *F-A*

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 $5	$4	$3	100
E	200 $8	100 $4	$3	300
F	$9	100 $7	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.6

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Obtaining an Improved Solution

Second Solution to the Executive Furniture Problem

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 $5	$4	$3	100
E	100 $8	200 $4	$3	300
F	100 $9	$7	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.7

Total shipping costs have been reduced by (100 units) x ($2 saved per unit) and now equals $4,000.

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Obtaining an Improved Solution

- This second solution may or may not be optimal.
- To determine whether further improvement is possible, we return to the first five steps to test each square that is *now* unused.
- The four new improvement indices are:

D to *B* = I_{DB} = + \$4 – \$5 + \$8 – \$4 = + \$3
(closed path: + *DB* – *DA* + *EA* – *EB*)

D to *C* = I_{DC} = + \$3 – \$5 + \$9 – \$5 = + \$2
(closed path: + *DC* – *DA* + *FA* – *FC*)

E to *C* = I_{EC} = + \$3 – \$8 + \$9 – \$5 = – \$1
(closed path: + *EC* – *EA* + *FA* – *FC*)

F to *B* = I_{FB} = + \$7 – \$4 + \$8 – \$9 = + \$2
(closed path: + *FB* – *EB* + *EA* – *FA*)

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Obtaining an Improved Solution

Path to Evaluate the *E-C* Route

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 \$5	\$4	\$3	100
E	100 \$8	200 \$4	Start \$3	300
F	100 \$9	\$7	200 \$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.8

An improvement can be made by shipping the maximum allowable number of units from *E* to *C*.

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Obtaining an Improved Solution

Total cost of third solution:

ROUTE FROM	ROUTE TO	DESKS SHIPPED	x	PER UNIT COST ($)	=	TOTAL COST ($)
D	*A*	100		5		500
E	*B*	200		4		800
E	*C*	100		3		300
F	*A*	200		9		1,800
F	*C*	100		5		500
						3,900

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Obtaining an Improved Solution

Third and optimal solution:

FROM \ TO	*A*	*B*	*C*	FACTORY CAPACITY
D	100 $5	$4	$3	100
E	$8	200 $4	100 $3	300
F	200 $9	$7	100 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.9

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Obtaining an Improved Solution

This solution is optimal as the improvement indices that can be computed are all greater than or equal to zero.

D to *B* = I_{DB} = + \$4 – \$5 + \$9 – \$5 + \$3 – \$4 = + \$2
(closed path: + *DB* – *DA* + *FA* – *FC* + *EC* – *EB*)

D to *C* = I_{DC} = + \$3 – \$5 + \$9 – \$5 = + \$2
(closed path: + *DC* – *DA* + *FA* – *FC*)

E to *A* = I_{EA} = + \$8 – \$9 + \$5 – \$3 = + \$1
(closed path: + *EA* – *FA* + *FC* – *EC*)

F to *B* = I_{FB} = + \$7 – \$5 + \$3 – \$4 = + \$1
(closed path: + *FB* – *FC* + *EC* – *EB*)

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Summary of Steps in Transportation Algorithm (Minimization)

1. Set up a balanced transportation table.
2. Develop initial solution using the northwest corner method method.
3. Calculate an improvement index for each empty cell using the stepping-stone method. If improvement indices are all nonnegative, stop as the optimal solution has been found. If any index is negative, continue to step 4.
4. Select the cell with the improvement index indicating the greatest decrease in cost. Fill this cell using the stepping-stone path and go to step 3.

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Unbalanced Transportation Problems

- In real-life problems, total demand is frequently not equal to total supply.
- These *unbalanced problems* can be handled easily by introducing *dummy sources* or *dummy destinations.*
- If total supply is greater than total demand, a dummy destination (warehouse), with demand exactly equal to the surplus, is created.
- If total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand over supply.

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Special Situations with the Transportation Algorithm

- Unbalanced Transportation Problems
  - In either case, shipping cost coefficients of zero are assigned to each dummy location or route as no goods will actually be shipped.
  - Any units assigned to a dummy destination represent excess capacity.
  - Any units assigned to a dummy source represent unmet demand.

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Demand Less Than Supply

- Suppose that the Des Moines factory increases its rate of production from 100 to 250 desks.
- The firm is now able to supply a total of 850 desks each period.
- Warehouse requirements remain the same (700) so the row and column totals do not balance.
- We add a dummy column that will represent a fake warehouse requiring 150 desks.
- This is somewhat analogous to adding a slack variable.
- We use the stepping-stone method to find the optimal solution.

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Demand Less Than Supply

Initial Solution to an Unbalanced Problem Where Demand is Less Than Supply

FROM \ TO	A	B	C	DUMMY WAREHOUSE	TOTAL AVAILABLE
D	250 $5	$4	$3	0	250
E	50 $8	200 $4	50 $3	0	300
F	$9	$7	150 $5	150 0	300
WAREHOUSE REQUIREMENTS	300	200	200	150	850

Total cost = 250($5) + 50($8) + 200($4) + 50($3) + 150($5) + 150(0) = $3,350

Table 9.10

New Des Moines capacity

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Demand Greater than Supply

- The second type of unbalanced condition occurs when total demand is greater than total supply.
- In this case we need to add a dummy row representing a fake factory.
- The new factory will have a supply exactly equal to the difference between total demand and total real supply.
- The shipping costs from the dummy factory to each destination will be zero.

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Demand Greater than Supply

Unbalanced Transportation Table for Happy Sound Stereo Company

FROM \ TO	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	$6	$4	$9	200
PLANT *X*	$10	$5	$8	175
PLANT *Y*	$12	$7	$6	75
WAREHOUSE DEMAND	250	100	150	500 / 450

Totals do not balance

Table 9.11

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Demand Greater than Supply

Initial Solution to an Unbalanced Problem in Which Demand is Greater Than Supply

FROM \ TO	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	200 $6	$4	$9	200
PLANT *X*	50 $10	100 $5	25 $8	175
PLANT *Y*	$12	$7	75 $6	75
PLANT *Y*	0	0	50 0	50
WAREHOUSE DEMAND	250	100	150	500

Total cost of initial solution = 200($6) + 50($10) + 100($5) + 25($8) + 75($6) + $50(0) = $2,850

Table 9.12

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Degeneracy in Transportation Problems

- *Degeneracy* occurs when the number of occupied squares or routes in a transportation table solution is less than the number of rows plus the number of columns minus 1.
- Such a situation may arise in the initial solution or in any subsequent solution.
- Degeneracy requires a special procedure to correct the problem since there are not enough occupied squares to trace a closed path for each unused route and it would be impossible to apply the stepping-stone method.

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Degeneracy in Transportation Problems

- To handle degenerate problems, create an artificially occupied cell.
- That is, place a zero (representing a fake shipment) in one of the unused squares and then treat that square as if it were occupied.
- The square chosen must be in such a position as to allow all stepping-stone paths to be closed.
- There is usually a good deal of flexibility in selecting the unused square that will receive the zero.

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Degeneracy in an Initial Solution

- The Martin Shipping Company example illustrates degeneracy in an initial solution.
- It has three warehouses which supply three major retail customers.
- Applying the northwest corner rule the initial solution has only four occupied squares
- To correct this problem, place a zero in an unused square, typically one adjacent to the last filled cell.

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Degeneracy in an Initial Solution

Initial Solution of a Degenerate Problem

FROM \ TO	CUSTOMER 1	CUSTOMER 2	CUSTOMER 3	WAREHOUSE SUPPLY
WAREHOUSE 1	100 $8	0 $2	$6	100
WAREHOUSE 2	0 $10	100 $9	20 $9	120
WAREHOUSE 3	$7	$10	80 $7	80
CUSTOMER DEMAND	100	100	100	300

Table 9.13

Possible choices of cells to address the degenerate solution

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Degeneracy During Later Solution Stages

- A transportation problem can become degenerate after the initial solution stage if the filling of an empty square results in two or more cells becoming empty simultaneously.
- This problem can occur when two or more cells with minus signs tie for the lowest quantity.
- To correct this problem, place a zero in one of the previously filled cells so that only one cell becomes empty.

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Degeneracy During Later Solution Stages

Bagwell Paint Example

- After one iteration, the cost analysis at Bagwell Paint produced a transportation table that was not degenerate but was not optimal.
- The improvement indices are:

factory *A* – warehouse 2 index = +2
factory *A* – warehouse 3 index = +1
factory *B* – warehouse 3 index = –15
factory *C* – warehouse 2 index = +11

Only route with a negative index

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Degeneracy During Later Solution Stages

Bagwell Paint Transportation Table

FROM \ TO	WAREHOUSE 1	WAREHOUSE 2	WAREHOUSE 3	FACTORY CAPACITY
FACTORY *A*	70 $8	$5	$16	70
FACTORY *B*	50 $15	80 $10	$7	130
FACTORY *C*	30 $3	$9	50 $10	80
WAREHOUSE REQUIREMENT	150	80	50	280

Table 9.14

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Degeneracy During Later Solution Stages

Tracing a Closed Path for the Factory B – Warehouse 3 Route

FROM \ TO	WAREHOUSE 1	WAREHOUSE 3
FACTORY *B*	50 $15	$7
FACTORY *C*	30 $3	50 $10

Table 9.15

- This would cause two cells to drop to zero.
- We need to place an artificial zero in one of these cells to avoid degeneracy.

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More Than One Optimal Solution

- It is possible for a transportation problem to have multiple optimal solutions.
- This happens when one or more of the improvement indices is zero in the optimal solution.
  - This means that it is possible to design alternative shipping routes with the same total shipping cost.
  - The alternate optimal solution can be found by shipping the most to this unused square using a stepping-stone path.
- In the real world, alternate optimal solutions provide management with greater flexibility in selecting and using resources.

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Maximization Transportation Problems

- If the objective in a transportation problem is to maximize profit, a minor change is required in the transportation algorithm.
- Now the optimal solution is reached when all the improvement indices are negative or zero.
- The cell with the largest positive improvement index is selected to be filled using a stepping-stone path.
- This new solution is evaluated and the process continues until there are no positive improvement indices.

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Unacceptable Or Prohibited Routes

- At times there are transportation problems in which one of the sources is unable to ship to one or more of the destinations.
  - The problem is said to have an *unacceptable* or *prohibited route.*
- In a minimization problem, such a prohibited route is assigned a very high cost to prevent this route from ever being used in the optimal solution.
- In a maximization problem, the very high cost used in minimization problems is given a negative sign, turning it into a very bad profit.

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Facility Location Analysis

- The transportation method is especially useful in helping a firm to decide where to locate a new factory or warehouse.
- Each alternative location should be analyzed within the framework of one *overall* distribution system.
- The new location that yields the minimum cost for the *entire system* is the one that should be chosen.

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Locating a New Factory for Hardgrave Machine Company

- Hardgrave Machine produces computer components at three plants and ships to four warehouses.
- The plants have not been able to keep up with demand so the firm wants to build a new plant.
- Two sites are being considered, Seattle and Birmingham.
- Data has been collected for each possible location. Which new location will yield the lowest cost for the firm in combination with the existing plants and warehouses?

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Locating a New Factory for Hardgrave Machine Company

Hardgrave's Demand and Supply Data

WAREHOUSE	MONTHLY DEMAND (UNITS)	PRODUCTION PLANT	MONTHLY SUPPLY	COST TO PRODUCE ONE UNIT ($)
Detroit	10,000	Cincinnati	15,000	48
Dallas	12,000	Salt Lake	6,000	50
New York	15,000	Pittsburgh	14,000	52
Los Angeles	9,000		35,000	
	46,000			

Supply needed from new plant = 46,000 – 35,000 = 11,000 units per month

ESTIMATED PRODUCTION COST PER UNIT AT PROPOSED PLANTS	
Seattle	$53
Birmingham	$49

Table 9.16

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Locating a New Factory for Hardgrave Machine Company

Hardgrave's Shipping Costs

FROM \ TO	DETROIT	DALLAS	NEW YORK	LOS ANGELES
CINCINNATI	$25	$55	$40	$60
SALT LAKE	35	30	50	40
PITTSBURGH	36	45	26	66
SEATTLE	60	38	65	27
BIRMINGHAM	35	30	41	50

Table 9.17

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Locating a New Factory for Hardgrave Machine Company

Birmingham Plant Optimal Solution: Total Hardgrave Cost is $3,741,000

FROM \ TO	DETROIT	DALLAS	NEW YORK	LOS ANGELES	FACTORY CAPACITY
CINCINNATI	10,000 / 73	/ 103	1,000 / 88	4,000 / 108	15,000
SALT LAKE	/ 85	1,000 / 80	/ 100	5,000 / 90	6,000
PITTSBURGH	/ 88	/ 97	14,000 / 78	/ 118	14,000
BIRMINGHAM	/ 84	11,000 / 79	/ 90	/ 99	11,000
WAREHOUSE REQUIREMENT	10,000	12,000	15,000	9,000	46,000

Table 9.18

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Locating a New Factory for Hardgrave Machine Company

Seattle Plant Optimal Solution: Total Hardgrave Cost is $3,704,000.

FROM \ TO	DETROIT	DALLAS	NEW YORK	LOS ANGELES	FACTORY CAPACITY
CINCINNATI	10,000 / 73	4,000 / 103	1,000 / 88	/ 108	15,000
SALT LAKE	/ 85	6,000 / 80	/ 100	/ 90	6,000
PITTSBURGH	/ 88	/ 97	14,000 / 78	/ 118	14,000
SEATTLE	/ 113	2,000 / 91	/ 118	9,000 / 80	11,000
WAREHOUSE REQUIREMENT	10,000	12,000	15,000	9,000	46,000

Table 9.19

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Locating a New Factory for Hardgrave Machine Company

- By comparing the total system costs of the two alternatives, Hardgrave can select the lowest cost option:
  - The Birmingham location yields a total system cost of $3,741,000.
  - The Seattle location yields a total system cost of $3,704,000.
- With the lower total system cost, the Seattle location is favored.
- Excel QM can also be used as a solution tool.

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Excel QM Solution for Facility Location Example

	A	B	C	D	E	F	G	H
1	Birmingham							
2								
3	Transportation							
4								
5								
6								
7								
8	Data							
9	COSTS	Dest 1	Dest 2	Dest 3	Dest 4	Supply		
10	Origin 1	73	103	88	108	15000		
11	Origin 2	85	80	100	90	6000		
12	Origin 3	88	97	78	118	14000		
13	Origin 4	84	79	90	99	11000		
14	Demand	10000	12000	15000	9000	46000 \ 46000		
15								
16	Shipments							
17	Shipments	Dest 1	Dest 2	Dest 3	Dest 4	Row Total		
18	Origin 1	10000		1000	4000	15000		
19	Origin 2		1000		5000	6000		
20	Origin 3			14000		14000		
21	Origin 4		11000			11000		
22	Column Total	10000	12000	15000	9000	46000 \ 46000		
23								
24	Total Cost	3741000						

From the Data tab, select Solver and click Solve.

Enter the transportation data in the shaded area. Then go to the DATA Tab on the ribbon, click on Solver in the Data Analysis Group and then click SOLVE.
If SOLVER is not on the Data Tab then please see the Help file (Solver) for instructions.

Enter the costs, supplies, and demands in this table.

Solver puts the solution here.

Program 9.4

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The Assignment Algorithm

- The second special-purpose LP algorithm is the assignment method.
- Each assignment problem has associated with it a table, or matrix.
- Generally, the rows contain the objects or people we wish to assign, and the columns comprise the tasks or things to which we want them assigned.
- The numbers in the table are the costs associated with each particular assignment.
- An assignment problem can be viewed as a transportation problem in which the capacity from each source is 1 and the demand at each destination is 1.

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Assignment Model Approach

- The Fix-It Shop has three rush projects to repair.
- The shop has three repair persons with different talents and abilities.
- The owner has estimates of wage costs for each worker for each project.
- The owner's objective is to assign the three project to the workers in a way that will result in the lowest cost to the shop.
- Each project will be assigned exclusively to one worker.

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Assignment Model Approach

Estimated Project Repair Costs for the Fix-It Shop Assignment Problem

	PROJECT		
PERSON	1	2	3
Adams	$11	$14	$6
Brown	8	10	11
Cooper	9	12	7

Table 9.20

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Assignment Model Approach

Summary of Fix-It Shop Assignment Alternatives and Costs

PRODUCT ASSIGNMENT				
1	2	3	LABOR COSTS ($)	TOTAL COSTS ($)
Adams	Brown	Cooper	11 + 10 + 7	28
Adams	Cooper	Brown	11 + 12 + 11	34
Brown	Adams	Cooper	8 + 14 + 7	29
Brown	Cooper	Adams	8 + 12 + 6	26
Cooper	Adams	Brown	9 + 14 + 11	34
Cooper	Brown	Adams	9 + 10 + 6	25

Table 9.21

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The Hungarian Method (Flood's Technique)

- The *Hungarian method* is an efficient method of finding the optimal solution to an assignment problem without having to make direct comparisons of every option.
- It operates on the principle of *matrix reduction.*
- By subtracting and adding appropriate numbers in the cost table or matrix, we can reduce the problem to a matrix of *opportunity costs.*
- Opportunity costs show the relative penalty associated with assigning any person to a project as opposed to making the *best* assignment.
- We want to make assignment so that the opportunity cost for each assignment is zero.

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Three Steps of the Assignment Method

1. *Find the opportunity cost table by*:
   (a) Subtracting the smallest number in each row of the original cost table or matrix from every number in that row.
   (b) Then subtracting the smallest number in each column of the table obtained in part (a) from every number in that column.
2. *Test the table resulting from step 1 to see whether an optimal assignment can be made* by drawing the minimum number of vertical and horizontal straight lines necessary to cover all the zeros in the table. If the number of lines is less than the number of rows or columns, proceed to step 3.

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Three Steps of the Assignment Method

3. ***Revise the opportunity cost table*** **by *subtracting* the smallest number not covered by a line from all numbers not covered by a straight line. This same number is also added to every number lying at the intersection of any two lines. Return to step 2 and continue the cycle until an optimal assignment is possible.**

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Steps in the Assignment Method

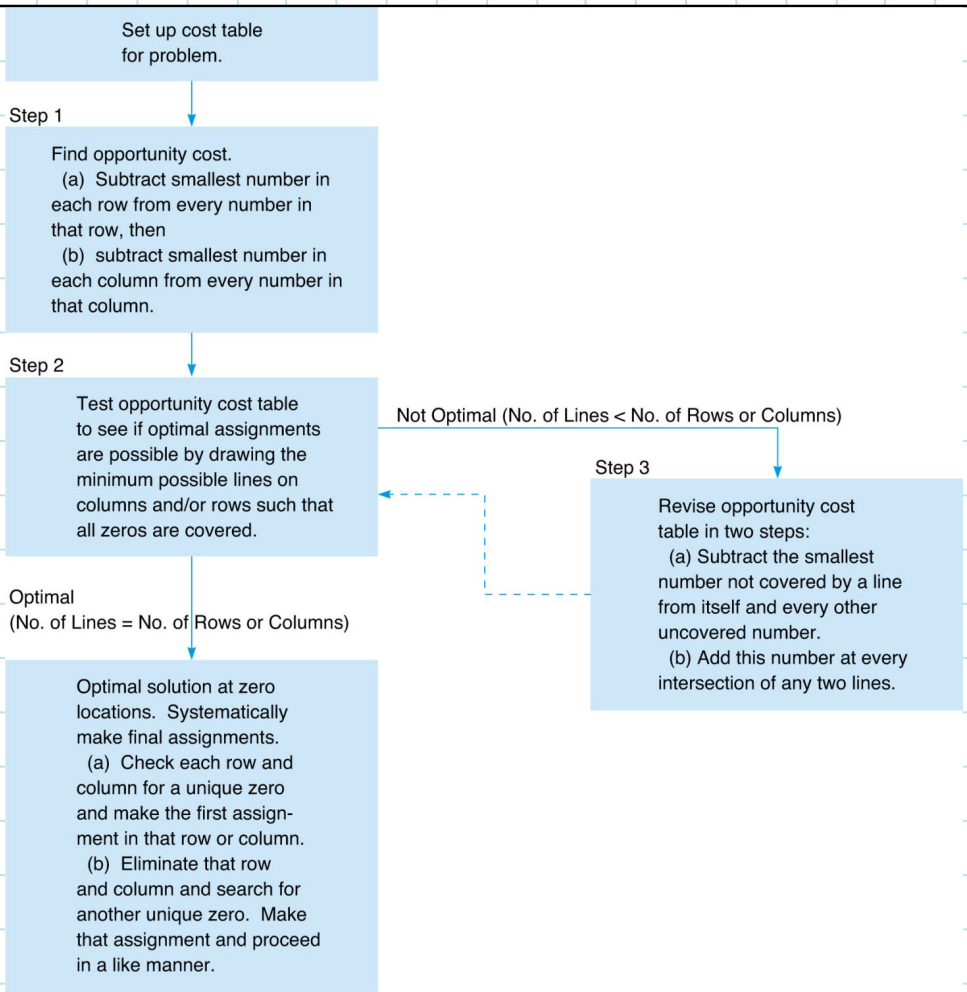

Figure 9.4

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The Hungarian Method (Flood's Technique)

- **Step 1: Find the opportunity cost table.**
  - We can compute *row* opportunity costs and *column* opportunity costs.
  - What we need is the *total* opportunity cost.
  - We derive this by taking the row opportunity costs and subtract the smallest number in that column from each number in that column.

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The Hungarian Method (Flood's Technique)

Cost of Each Person-Project Assignment for the Fix-it Shop Problem

PERSON	PROJECT 1	PROJECT 2	PROJECT 3
Adams	$11	$14	$6
Brown	8	10	11
Cooper	9	12	7

Row Opportunity Cost Table for the Fix-it Shop Step 1, Part (a)

PERSON	PROJECT 1	PROJECT 2	PROJECT 3
Adams	$5	$8	$0
Brown	0	2	3
Cooper	2	5	0

Tables 9.22-9.23

The opportunity cost of assigning Cooper to project 2 is $12 – $7 = $5.

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The Hungarian Method (Flood's Technique)

Derive the total opportunity costs by taking the costs in Table 9.23 and subtract the smallest number in each column from each number in that column.

Total Opportunity Cost Table for the Fix-it Shop Step 1, Part (b)

	PROJECT		
PERSON	1	2	3
Adams	$5	$6	$0
Brown	0	0	3
Cooper	2	3	0

Table 9.24

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The Hungarian Method (Flood's Technique)

- Step 2: Test for the optimal assignment.
  - We want to assign workers to projects in such a way that the total labor costs are at a minimum.
  - We would like to have a total assigned opportunity cost of zero.
  - The test to determine if we have reached an optimal solution is simple.
  - We find the *minimum* number of straight lines necessary to cover all the zeros in the table.
  - If the number of lines equals the number of rows or columns, an optimal solution has been reached.

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The Hungarian Method (Flood's Technique)

Test for Optimal Solution to Fix-it Shop Problem

PERSON	PROJECT 1	PROJECT 2	PROJECT 3
Adams	$5	$6	$0
Brown	0	0	0
Cooper	2	3	0

Covering line 1

Covering line 2

Table 9.25

This requires only two lines to cover the zeros so the solution is not optimal.

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The Hungarian Method (Flood's Technique)

- **Step 3: Revise the opportunity-cost table.**
  - **We *subtract* the smallest number not covered by a line from all numbers not covered by a straight line.**
  - **The same number is added to every number lying at the intersection of any two lines.**
  - **We then return to step 2 to test this new table.**

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The Hungarian Method (Flood's Technique)

Revised Opportunity Cost Table for the Fix-it Shop Problem

PERSON	PROJECT 1	PROJECT 2	PROJECT 3
Adams	$3	$4	$0
Brown	0	0	5
Cooper	0	1	0

Table 9.26

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The Hungarian Method (Flood's Technique)

Optimality Test on the Revised Fix-it Shop Opportunity Cost Table

PERSON	PROJECT 1	PROJECT 2	PROJECT 3
Adams	$3	$4	$0
Brown	0	0	5
Cooper	0	1	0

Table 9.27

Covering line 1

Covering line 2

Covering line 3

This requires three lines to cover the zeros so the solution is optimal.

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Making the Final Assignment

- The optimal assignment is Adams to project 3, Brown to project 2, and Cooper to project 1.
- For larger problems one approach to making the final assignment is to select a row or column that contains only one zero.
  - Make the assignment to that cell and rule out its row and column.
  - Follow this same approach for all the remaining cells.

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Making the Final Assignment

Total labor costs of this assignment are:

ASSIGNMENT	COST ($)
Adams to project 3	6
Brown to project 2	10
Cooper to project 1	9
Total cost	25

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Making the Final Assignment

Making the Final Fix-it Shop Assignments

(A) FIRST ASSIGNMENT				(B) SECOND ASSIGNMENT				(C) THIRD ASSIGNMENT			
	1	2	3		1	2	3		1	2	3
Adams	3	4	0	Adams	3	4	0	Adams	3	4	0
Brown	0	0	5	Brown	0	0	5	Brown	0	0	5
Cooper	0	1	0	Cooper	0	1	0	Cooper	0	1	0

Table 9.28

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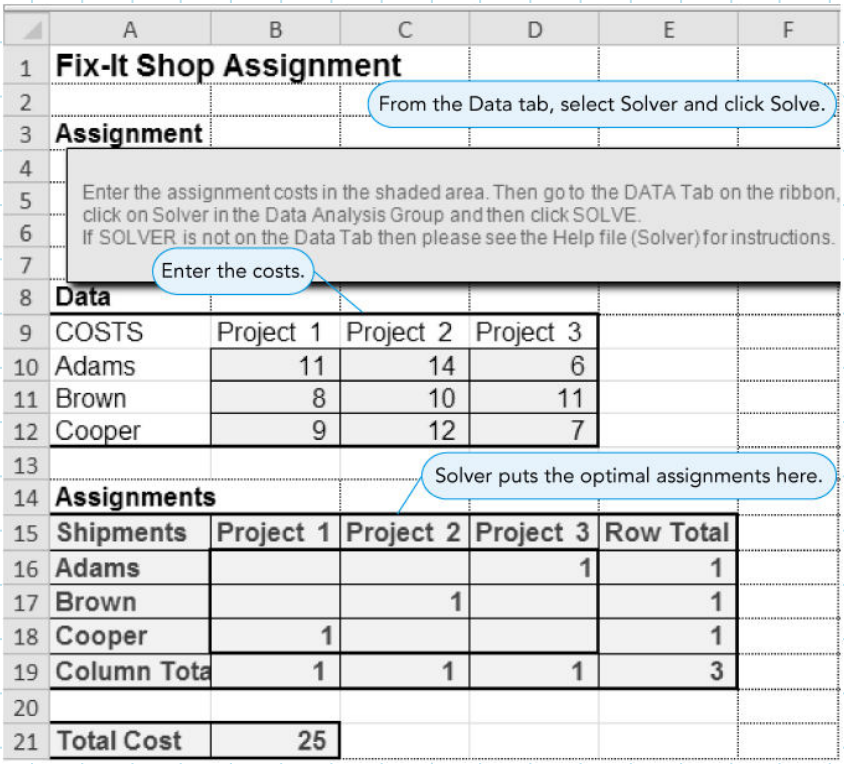
Excel QM Solution for Fix-It Shop Assignment Problem



Fix-It Shop Assignment

From the Data tab, select Solver and click Solve.

Assignment

Enter the assignment costs in the shaded area. Then go to the DATA Tab on the ribbon, click on Solver in the Data Analysis Group and then click SOLVE.
If SOLVER is not on the Data Tab then please see the Help file (Solver) for instructions.

Enter the costs.

Data

COSTS	Project 1	Project 2	Project 3
Adams	11	14	6
Brown	8	10	11
Cooper	9	12	7

Solver puts the optimal assignments here.

Assignments

Shipments	Project 1	Project 2	Project 3	Row Total
Adams			1	1
Brown		1		1
Cooper	1			1
Column Tota	1	1	1	3

Total Cost	25

Program 9.5

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Unbalanced Assignment Problems

- Often the number of people or objects to be assigned does not equal the number of tasks or clients or machines listed in the columns, and the problem is *unbalanced.*
- When this occurs, and there are more rows than columns, simply add a *dummy column* or task.
- If the number of tasks exceeds the number of people available, we add a *dummy row*.
- Since the dummy task or person is nonexistent, we enter zeros in its row or column as the cost or time estimate.

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Unbalanced Assignment Problems

- Suppose the Fix-It Shop has another worker available.
- The shop owner still has the same basic problem of assigning workers to projects, but the problem now needs a dummy column to balance the four workers and three projects.

	PROJECT			
PERSON	1	2	3	DUMMY
Adams	$11	$14	$6	$0
Brown	8	10	11	0
Cooper	9	12	7	0
Davis	10	13	8	0

Table 9.29

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Maximization Assignment Problems

- Some assignment problems are phrased in terms of maximizing the payoff, profit, or effectiveness.
- It is easy to obtain an equivalent minimization problem by converting all numbers in the table to opportunity costs.
- This is brought about by subtracting every number in the original payoff table from the largest single number in that table.
- Transformed entries represent opportunity costs.
- Once the optimal assignment has been found, the total payoff is found by adding the original payoffs of those cells that are in the optimal assignment.

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Maximization Assignment Problems

- The British navy wishes to assign four ships to patrol four sectors of the North Sea.
- Ships are rated for their probable efficiency in each sector.
- The commander wants to determine patrol assignments producing the greatest overall efficiencies.

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Maximization Assignment Problems

Efficiencies of British Ships in Patrol Sectors

	SECTOR			
SHIP	*A*	*B*	*C*	*D*
1	20	60	50	55
2	60	30	80	75
3	80	100	90	80
4	65	80	75	70

Table 9.30

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Maximization Assignment Problems

Opportunity Costs of British Ships

	SECTOR			
SHIP	*A*	*B*	*C*	*D*
1	80	40	50	45
2	40	70	20	25
3	20	0	10	20
4	35	20	25	30

Table 9.31

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Maximization Assignment Problems

- Convert the maximization efficiency table into a minimizing opportunity cost table by subtracting each rating from 100, the largest rating in the whole table.
- The smallest number in each row is subtracted from every number in that row and the smallest number in each column is subtracted from every number in that column.
- The minimum number of lines needed to cover the zeros in the table is four, so this represents an optimal solution.

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Maximization Assignment Problems

Row Opportunity Costs for the British Navy Problem

	SECTOR			
SHIP	*A*	*B*	*C*	*D*
1	40	0	10	5
2	20	50	0	5
3	20	0	10	20
4	15	0	5	10

Table 9.32

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Maximization Assignment Problems

Total Opportunity Costs for the British Navy Problem

	SECTOR			
SHIP	*A*	*B*	*C*	*D*
1	25	0	10	0
2	5	50	0	0
3	5	0	10	15
4	0	0	5	5

Table 9.33

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Maximization Assignment Problems

The overall efficiency

ASSIGNMENT	EFFICIENCY
Ship 1 to sector *D*	55
Ship 2 to sector *C*	80
Ship 3 to sector *B*	100
Ship 4 to sector *A*	65
Total efficiency	300

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Chapter 6

Inventory Control Models

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. Understand the importance of inventory control and ABC analysis.
2. Use the economic order quantity (EOQ) to determine how much to order.
3. Compute the reorder point (ROP) in determining when to order more inventory.
4. Handle inventory problems that allow quantity discounts or non-instantaneous receipt.

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Learning Objectives

After completing this chapter, students will be able to:

5. Understand the use of safety stock.
6. Describe the use of material requirements planning in solving dependent-demand inventory problems.
7. Discuss just-in-time inventory concepts to reduce inventory levels and costs.
8. Discuss enterprise resource planning systems.

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Chapter Outline

6.1 Introduction
6.2 Importance of Inventory Control
6.3 Inventory Decisions
6.4 Economic Order Quantity: Determining How Much to Order
6.5 Reorder Point: Determining When to Order
6.6 EOQ Without the Instantaneous Receipt Assumption
6.7 Quantity Discount Models
6.8 Use of Safety Stock

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Chapter Outline

6.9 Single-Period Inventory Models
6.10 ABC Analysis
6.11 Dependent Demand: The Case for Material Requirements Planning
6.12 Just-in-Time Inventory Control
6.13 Enterprise Resource Planning

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Introduction

- Inventory is an expensive and important asset to many companies.
- Inventory is any stored resource used to satisfy a current or future need.
- Common examples are raw materials, work-in-process, and finished goods.
- Most companies try to balance high and low inventory levels with cost minimization as a goal.
  - Lower inventory levels can reduce costs.
  - Low inventory levels may result in stockouts and dissatisfied customers.

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Introduction

- All organizations have some type of inventory control system.
- Inventory planning helps determine what goods and/or services need to be produced.
- Inventory planning helps determine whether the organization produces the goods or services or whether they are purchased from another organization.
- Inventory planning also involves demand forecasting.

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Introduction

Inventory planning and control

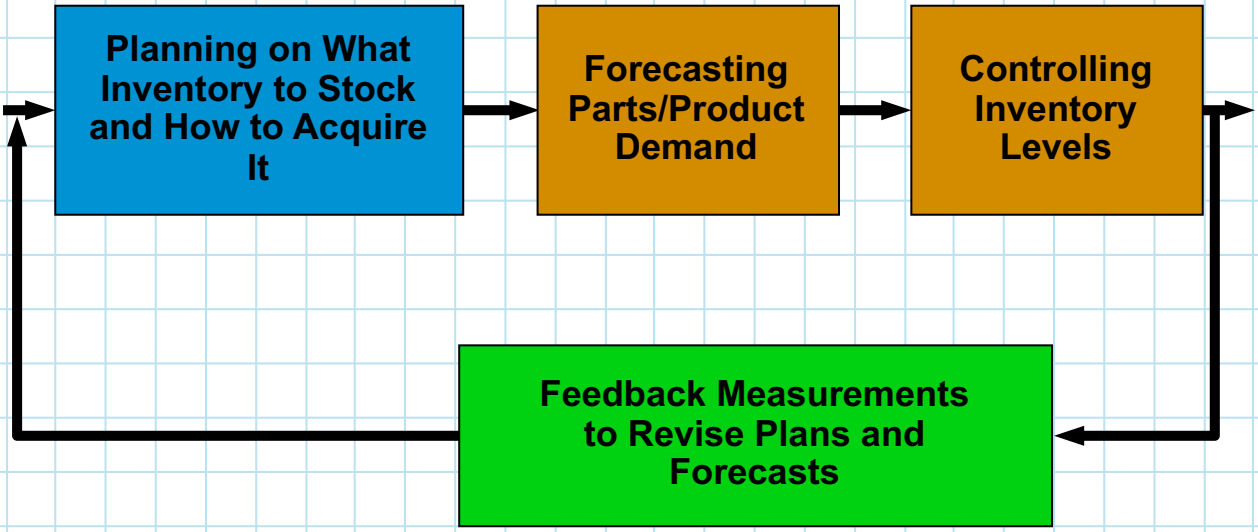

Figure 6.1

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Importance of Inventory Control

- Five uses of inventory:
  - The decoupling function
  - Storing resources
  - Irregular supply and demand
  - Quantity discounts
  - Avoiding stockouts and shortages
- Decouple manufacturing processes.
  - Inventory is used as a buffer between stages in a manufacturing process.
  - This reduces delays and improves efficiency.

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Importance of Inventory Control

- Storing resources.
  - Seasonal products may be stored to satisfy off-season demand.
  - Materials can be stored as raw materials, work-in-process, or finished goods.
  - Labor can be stored as a component of partially completed subassemblies.
- Compensate for irregular supply and demand.
  - Demand and supply may not be constant over time.
  - Inventory can be used to buffer the variability.

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Importance of Inventory Control

- Take advantage of quantity discounts.
  - Lower prices may be available for larger orders.
  - Extra costs associated with holding more inventory must be balanced against lower purchase price.
- Avoid stockouts and shortages.
  - Stockouts may result in lost sales.
  - Dissatisfied customers may choose to buy from another supplier.

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Inventory Decisions

- There are two fundamental decisions in controlling inventory:
  - How much to order.
  - When to order.
- The major objective is to minimize total inventory costs.
- Common inventory costs are:
  - Cost of the items (purchase or material cost).
  - Cost of ordering.
  - Cost of carrying, or holding, inventory.
  - Cost of stockouts.

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Inventory Cost Factors

ORDERING COST FACTORS	CARRYING COST FACTORS
Developing and sending purchase orders	Cost of capital
Processing and inspecting incoming inventory	Taxes
Bill paying	Insurance
Inventory inquiries	Spoilage
Utilities, phone bills, and so on, for the purchasing department	Theft
Salaries and wages for the purchasing department employees	Obsolescence
Supplies, such as forms and paper, for the purchasing department	Salaries and wages for warehouse employees
	Utilities and building costs for the warehouse
	Supplies, such as forms and paper, for the warehouse

Table 6.1

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Inventory Cost Factors

- Ordering costs are generally independent of order quantity.
  - Many involve personnel time.
  - The amount of work is the same no matter the size of the order.
- Carrying costs generally varies with the amount of inventory, or the order size.
  - The labor, space, and other costs increase as the order size increases.
- The actual cost of items purchased can vary if there are quantity discounts available.

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Economic Order Quantity

- The *economic order quantity* (*EOQ*) model is one of the oldest and most commonly known inventory control techniques.
- It is easy to use but has a number of important assumptions.
- Objective is to minimize total cost of inventory.

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Economic Order Quantity

Assumptions:

1. Demand is known and constant.
2. Lead time is known and constant.
3. Receipt of inventory is instantaneous.
4. Purchase cost per unit is constant throughout the year.
5. The only variable costs are the cost of placing an order, *ordering cost*, and the cost of holding or storing inventory over time, *holding* or *carrying cost*, and these are constant throughout the year.
6. Orders are placed so that stockouts or shortages are avoided completely.

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Inventory Usage Over Time

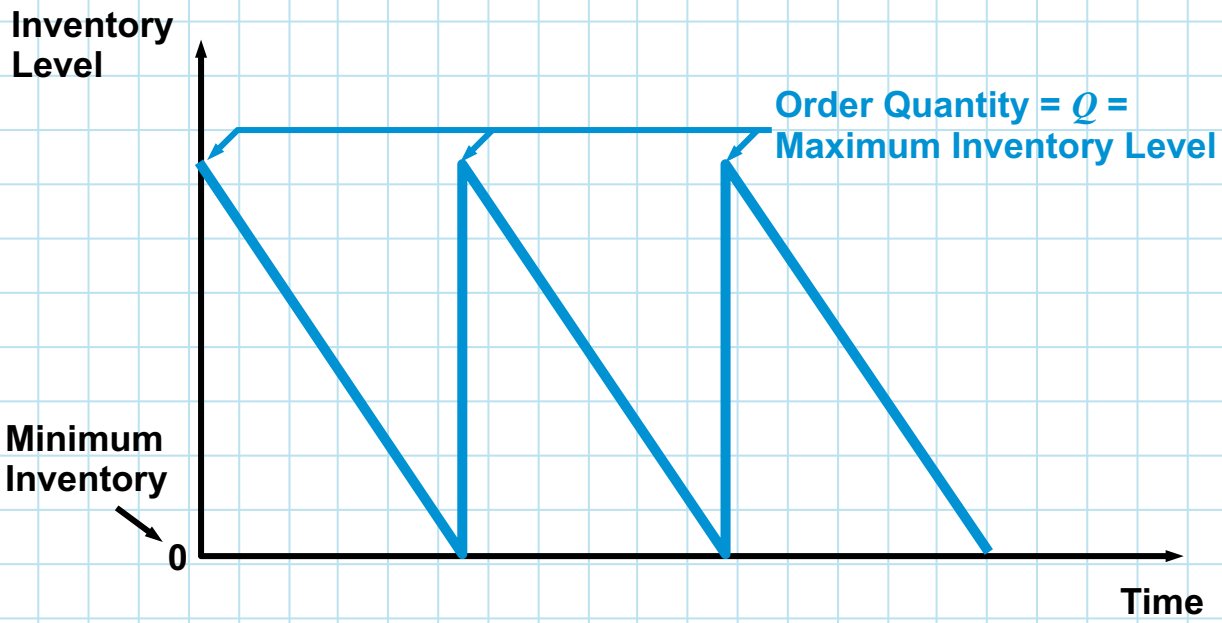

Figure 6.2

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Inventory Costs in the EOQ Situation

Computing Average Inventory

$$\text{Average inventory level} = \frac{Q}{2}$$

DAY	INVENTORY LEVEL		
	BEGINNING	ENDING	AVERAGE
April 1 (order received)	10	8	9
April 2	8	6	7
April 3	6	4	5
April 4	4	2	3
April 5	2	0	1

Maximum level April 1 = 10 units
Total of daily averages = 9 + 7 + 5 + 3 + 1 = 25
Number of days = 5
Average inventory level = 25/5 = 5 units

Table 6.2

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Inventory Costs in the EOQ Situation

Mathematical equations can be developed using:

Q = number of pieces to order
EOQ = Q^* = optimal number of pieces to order
D = annual demand in units for the inventory item
C_o = ordering cost of each order
C_h = holding or carrying cost per unit per year

$$\text{Annual ordering cost} = \left(\begin{array}{c}\text{Number of}\\ \text{orders placed}\\ \text{per year}\end{array}\right) \times \left(\begin{array}{c}\text{Ordering}\\ \text{cost per}\\ \text{order}\end{array}\right)$$

$$= \frac{D}{Q} C_o$$

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Inventory Costs in the *EOQ* Situation

Mathematical equations can be developed using:

Q = number of pieces to order
EOQ = Q^* = optimal number of pieces to order
D = annual demand in units for the inventory item
C_o = ordering cost of each order
C_h = holding or carrying cost per unit per year

$$\text{Annual holding cost} = \begin{pmatrix}\text{Average}\\ \text{inventory}\end{pmatrix} \times \begin{pmatrix}\text{Carrying}\\ \text{cost per unit}\\ \text{per year}\end{pmatrix}$$

$$= \frac{Q}{2} C_h$$

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Inventory Costs in the *EOQ* Situation

Total Cost as a Function of Order Quantity

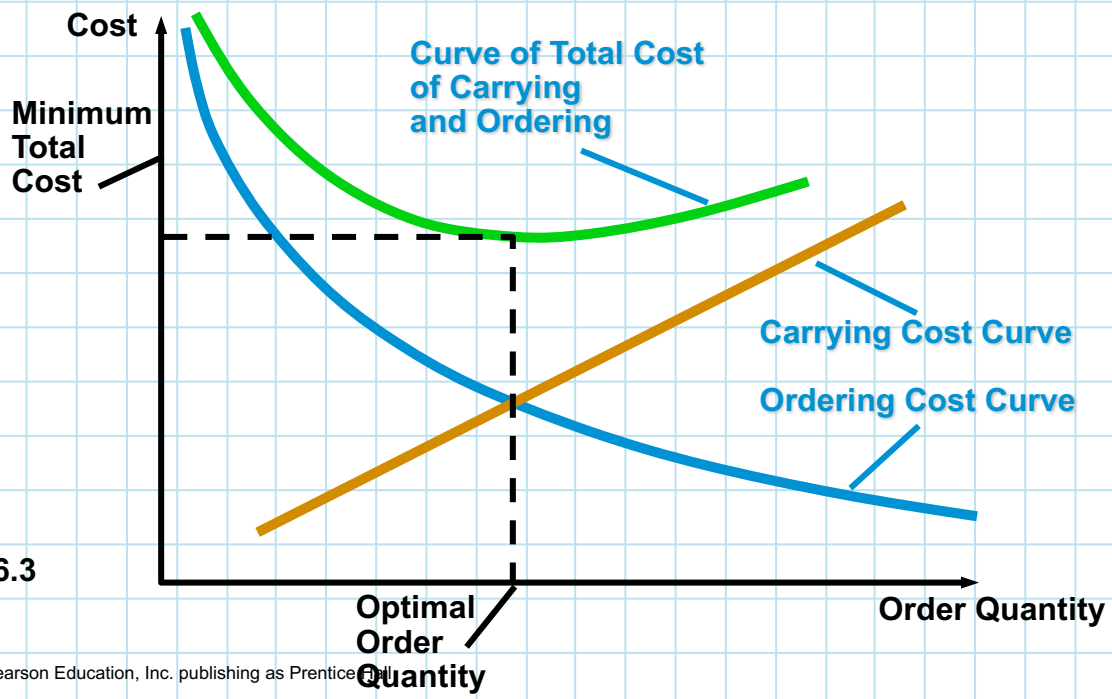

Figure 6.3

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Finding the EOQ

According to the graph, when the EOQ assumptions are met, total cost is minimized when annual ordering cost equals annual holding cost.

$$\frac{D}{Q}C_o = \frac{Q}{2}C_h$$

Solving for Q

$$2DC_o = Q^2 C_h$$

$$\frac{2DC_o}{C_h} = Q^2$$

$$\sqrt{\frac{2DC_o}{C_h}} = Q = \text{EOQ} = Q^*$$

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Economic Order Quantity (EOQ) Model

Summary of equations:

$$\text{Annual ordering cost} = \frac{D}{Q}C_o$$

$$\text{Annual holding cost} = \frac{Q}{2}C_h$$

$$\text{EOQ} = Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

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Sumco Pump Company

- Sumco Pump Company sells pump housings to other companies.
- The firm would like to reduce inventory costs by finding optimal order quantity.
  - Annual demand = 1,000 units
  - Ordering cost = $10 per order
  - Average carrying cost per unit per year = $0.50

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2(1{,}000)(10)}{0.50}} = \sqrt{40{,}000} = 200 \text{ units}$$

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Sumco Pump Company

Total annual cost = Order cost + Holding cost

$$TC = \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

$$= \frac{1{,}000}{200}(10) + \frac{200}{2}(0.5)$$

$$= \$50 + \$50 = \$100$$

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Sumco Pump Company

Input Data and Excel QM formulas for the Sumco Pump Company Example

	A	B	C	D
1	Sumco Pump Company			
2				
3	Inventory	Economic Order Quantity Model		
4	Enter the data in the shaded area			
5				
6				
7	Data			
8	Demand rate, D	1000		
9	Setup/order cost, S	10		
10	Holding cost, H	0.5	(fixed amount)	
11	Unit Price, P			
12				
13	Results			
14	Optimal Order Quantity, Q*	=SQRT(2*B8*B9/B10)		
15	Maximum Inventory	=B14		
16	Average Inventory	=B14/2		
17	Number of Orders	=B8/B14		
18				
19	Holding cost	=B16*B10		
20	Setup cost	=B17*B9		
21				
22	Unit costs	=B11*B8		
23	Total cost, T_c	=B19+B20+B22		
24				

Enter demand rate, setup/ordering cost, holding cost, and unit price.

If unit price is available, it is entered here.

On input screen, you may specify whether the holding cost is fixed amount or a percentage of the unit (purchase) cost.

Total unit (purchase) cost is given here.

Program 6.1A

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Sumco Pump Company

Excel QM Solution for the Sumco Pump Company Example

	A	B	C	D	E	F
1	Sumco Pump Company					
2						
3	Inventory	Economic Order Quantity Model				
4	Enter the data in the shaded area					
5						
6						
7	Data					
8	Demand rate, D	1000				
9	Setup/order cost, S	10				
10	Holding cost, H	0.5	(fixed amount)			
11	Unit Price, P					
12						
13	Results					
14	Optimal Order Quantity, Q*	200				
15	Maximum Inventory	200				
16	Average Inventory	100				
17	Number of Orders	5				
18						
19	Holding cost	$50.00				
20	Setup cost	$50.00				
21						
22	Unit costs	$0.00				
23	Total cost, T_c	$100.00				
24						

Total cost includes holding cost, ordering/setup cost, and unit/purchase cost if the unit cost is input.

Program 6.1B

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Purchase Cost of Inventory Items

- Total inventory cost can be written to include the cost of purchased items.
- Given the EOQ assumptions, the annual purchase cost is constant at $D \times C$ no matter the order policy, where
  - C is the purchase cost per unit.
  - D is the annual demand in units.
- At times it may be useful to know the average dollar level of inventory:

$$\text{Average dollar level} = \frac{(CQ)}{2}$$

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Purchase Cost of Inventory Items

- Inventory carrying cost is often expressed as an annual percentage of the unit cost or price of the inventory.
- This requires a new variable.

$$I = \begin{pmatrix} \text{Annual inventory holding charge as} \\ \text{a percentage of unit price or cost} \end{pmatrix}$$

- The cost of storing inventory for one year is then

$$C_h = IC$$

thus, $$Q^* = \sqrt{\frac{2DC_o}{IC}}$$

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Sensitivity Analysis with the *EOQ* Model

- The EOQ model assumes all values are know and fixed over time.
- Generally, however, some values are estimated or may change.
- Determining the effects of these changes is called *sensitivity analysis.*
- Because of the square root in the formula, changes in the inputs result in relatively small changes in the order quantity.

$$\text{EOQ} = \sqrt{\frac{2DC_o}{C_h}}$$

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Sensitivity Analysis with the *EOQ* Model

- In the Sumco Pump example:

$$\text{EOQ} = \sqrt{\frac{2(1{,}000)(10)}{0.50}} = 200 \text{ units}$$

- If the ordering cost were increased four times from \$10 to \$40, the order quantity would only double

$$\text{EOQ} = \sqrt{\frac{2(1{,}000)(40)}{0.50}} = 400 \text{ units}$$

- In general, the EOQ changes by the square root of the change to any of the inputs.

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Reorder Point: Determining When To Order

- Once the order quantity is determined, the next decision is *when to order.*
- The time between placing an order and its receipt is called the *lead time* (L) or *delivery time.*
- When to order is generally expressed as a *reorder point* (ROP).

$$\text{ROP} = \begin{pmatrix}\text{Demand}\\ \text{per day}\end{pmatrix} \times \begin{pmatrix}\text{Lead time for a}\\ \text{new order in days}\end{pmatrix}$$

$$= d \times L$$

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Procomp's Computer Chips

- Demand for the computer chip is 8,000 per year.
- Daily demand is 40 units.
- Delivery takes three working days.

$$\text{ROP} = d \times L = 40 \text{ units per day} \times 3 \text{ days}$$
$$= 120 \text{ units}$$

- An order based on the EOQ calculation is placed when the inventory reaches 120 units.
- The order arrives 3 days later just as the inventory is depleted.

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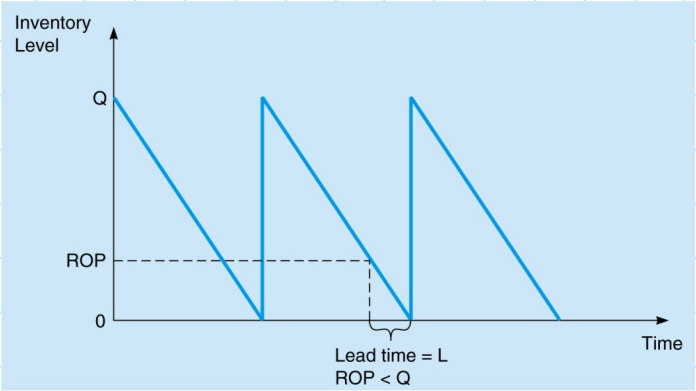
Reorder Point Graphs



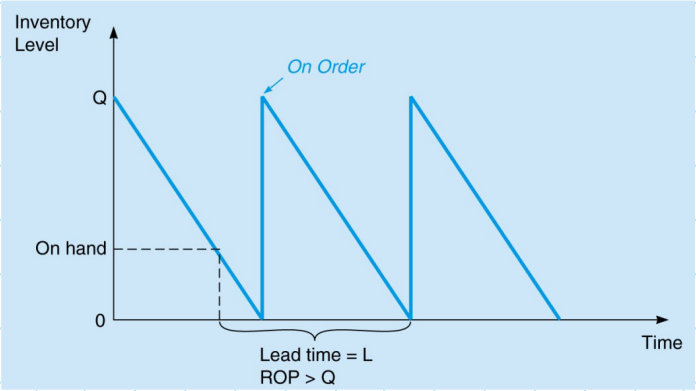


Figure 6.4

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EOQ Without The Instantaneous Receipt Assumption

- **When inventory accumulates over time, the *instantaneous receipt* assumption does not apply.**
- **Daily demand rate must be taken into account.**
- **The revised model is often called the *production run model.***

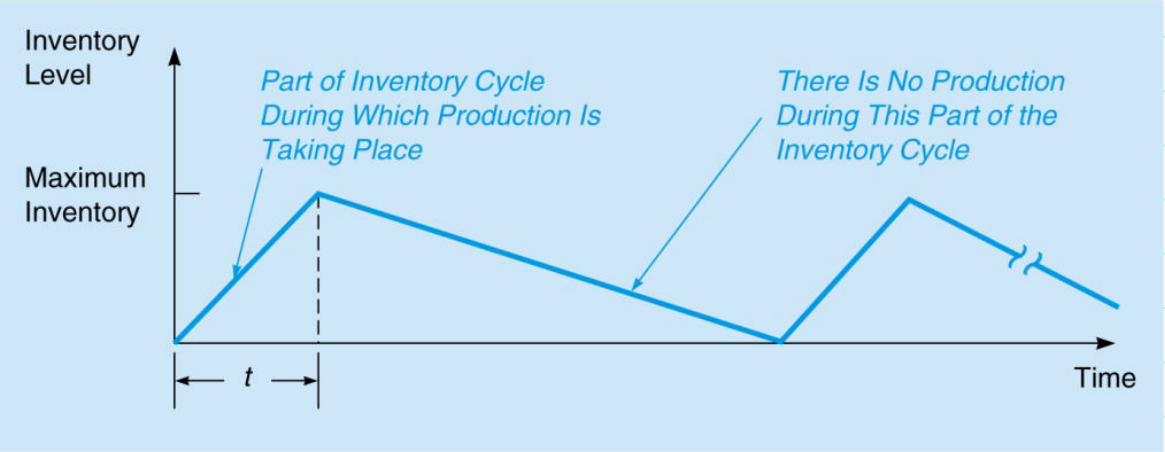

Figure 6.5

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Annual Carrying Cost for Production Run Model

- In production runs, *setup cost* replaces ordering cost.
- The model uses the following variables:

Q = number of pieces per order, or production run

C_s = setup cost

C_h = holding or carrying cost per unit per year

p = daily production rate

d = daily demand rate

t = length of production run in days

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Annual Carrying Cost for Production Run Model

Maximum inventory level

= (Total produced during the production run) – (Total used during the production run)

= (Daily production rate)(Number of days production) – (Daily demand)(Number of days production)

= $(pt) - (dt)$

since Total produced $= Q = pt$

we know $t = \dfrac{Q}{p}$

Maximum inventory level $= pt - dt = p\dfrac{Q}{p} - d\dfrac{Q}{p} = Q\left(1 - \dfrac{d}{p}\right)$

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Annual Carrying Cost for Production Run Model

Since the average inventory is one-half the maximum:

$$\text{Average inventory} = \frac{Q}{2}\left(1 - \frac{d}{p}\right)$$

and

$$\text{Annual holding cost} = \frac{Q}{2}\left(1 - \frac{d}{p}\right)C_h$$

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Annual Setup Cost for Production Run Model

Setup cost replaces ordering cost when a product is produced over time.

$$\text{Annual setup cost} = \frac{D}{Q}C_s$$

replaces

$$\text{Annual ordering cost} = \frac{D}{Q}C_o$$

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Determining the Optimal Production Quantity

By setting setup costs equal to holding costs, we can solve for the optimal order quantity

Annual holding cost = Annual setup cost

$$\frac{Q}{2}\left(1-\frac{d}{p}\right)C_h = \frac{D}{Q}C_s$$

Solving for Q, we get

$$Q^* = \sqrt{\frac{2DC_s}{C_h\left(1-\frac{d}{p}\right)}}$$

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Production Run Model

Summary of equations

$$\text{Annual holding cost} = \frac{Q}{2}\left(1-\frac{d}{p}\right)C_h$$

$$\text{Annual setup cost} = \frac{D}{Q}C_s$$

$$\text{Optimal production quantity } Q^* = \sqrt{\frac{2DC_s}{C_h\left(1-\frac{d}{p}\right)}}$$

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Brown Manufacturing

Brown Manufacturing produces commercial refrigeration units in batches.

Annual demand $= D =$ 10,000 units

Setup cost $= C_s =$ \$100

Carrying cost $= C_h =$ \$0.50 per unit per year

Daily production rate $= p =$ 80 units daily

Daily demand rate $= d =$ 60 units daily

1. How many units should Brown produce in each batch?
2. How long should the production part of the cycle last?

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Brown Manufacturing Example

1. $$Q^* = \sqrt{\frac{2DC_s}{C_h\left(1-\frac{d}{p}\right)}}$$

$$Q^* = \sqrt{\frac{2\times 10{,}000\times 100}{0.5\left(1-\frac{60}{80}\right)}}$$

$$= \sqrt{\frac{2{,}000{,}000}{0.5\left(1/4\right)}} = \sqrt{16{,}000{,}000}$$

$$= 4{,}000 \text{ units}$$

2. $$\text{Production cycle} = \frac{Q}{p}$$

$$= \frac{4{,}000}{80} = 50 \text{ days}$$

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Brown Manufacturing

Excel QM Formulas and Input Data for the Brown Manufacturing Problem

	A	B	C
1	**Brown Manufacturing**		
2			
3	**Inventory**	Production Order Quantity Model	
4	Enter the data in the shaded area. You may have to do some work to enter the daily		
5			
6	Data		
7	Demand rate, D	10000	
8	Setup cost, S	100	
9	Holding cost, H	0.5	(fixed amount)
10	Daily production rate, p	80	
11	Daily demand rate, d	60	
12	Unit price, P	0	
13			
14	**Results**		
15	Optimal production quantity, Q*	=SQRT(2*B7*B8/B9)*SQRT(B10/(B10-B11))	
16	Maximum Inventory	=B15*(B10-B11)/B10	
17	Average Inventory	=B16/2	
18	Number of Setups	=B7/B15	
19			
20	Holding cost	=B17*B9	
21	Setup cost	=B18*B8	
22			
23	Unit costs	=B12*B7	
24			
25	Total cost, T_c	=B20+B21+B23	

Enter the demand rate, setup cost, and holding cost. Notice that the holding cost is a fixed dollar amount rather than a percentage of the unit price.

Enter daily production rate and daily demand rate.

Calculate the optimal production quantity.

Calculate the maximum inventory.

Calculate the average number of setups.

Calculate the annual holding costs based on average inventory and the annual setup cost based on the number of setups.

Program 6.2A

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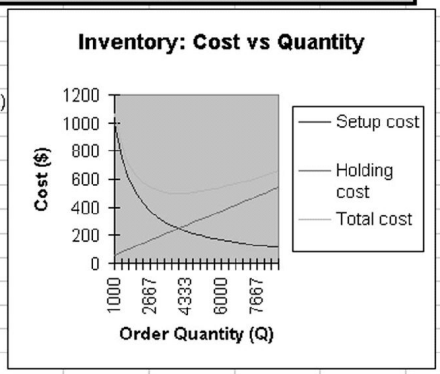
Brown Manufacturing

The Solution Results for the Brown Manufacturing Problem Using Excel QM

	A	B	C
1	**Brown Manufacturing**		
2			
3	**Inventory**	Production Order Quantity Model	
4	Enter the data in the shaded area. You may have to do some work to enter the daily production rate.		
5			
6	Data		
7	Demand rate, D	10000	
8	Setup cost, S	100	
9	Holding cost, H	0.5	(fixed amount)
10	Daily production rate, p	80	
11	Daily demand rate, d	60	
12	Unit price, P	0	
13			
14	**Results**		
15	Optimal production quantity, Q*	4000	
16	Maximum Inventory	1000	
17	Average Inventory	500	
18	Number of Setups	2.5	
19			
20	Holding cost	250	
21	Setup cost	250	
22			
23	Unit costs	0	
24			
25	Total cost, T_c	500	



Program 6.2B

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Quantity Discount Models

- Quantity discounts are commonly available.
- The basic EOQ model is adjusted by adding in the purchase or materials cost.

Total cost = Material cost + Ordering cost + Holding cost

$$\text{Total cost} = DC + \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

where

D = annual demand in units

C_o = ordering cost of each order

C = cost per unit

C_h = holding or carrying cost per unit per year

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Quantity Discount Models

Because unit cost is now variable,

Holding cost = $C_h = IC$

I = holding cost as a percentage of the unit cost (C)

$$\text{Total cost} = DC + \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

where

D = annual demand in units

C_o = ordering cost of each order

C = cost per unit

C_h = holding or carrying cost per unit per year

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Quantity Discount Models

- A typical quantity discount schedule can look like the table below.
- However, buying at the lowest unit cost is not always the best choice.

DISCOUNT NUMBER	DISCOUNT QUANTITY	DISCOUNT (%)	DISCOUNT COST ($)
1	0 to 999	0	5.00
2	1,000 to 1,999	4	4.80
3	2,000 and over	5	4.75

Table 6.3

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Quantity Discount Models

Total cost curve for the quantity discount model

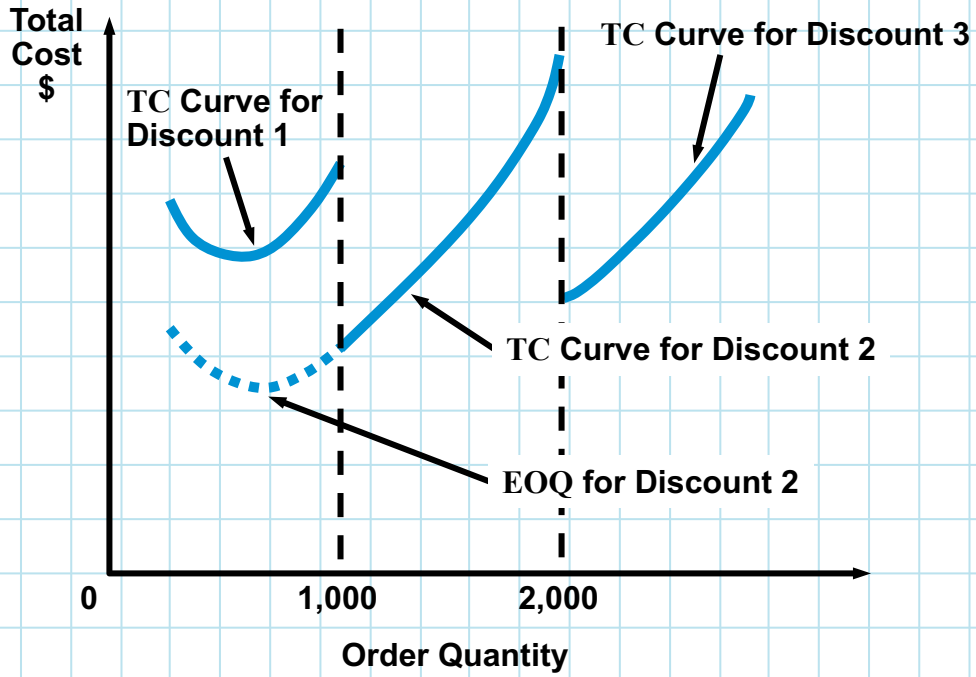

Figure 6.6

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Brass Department Store

- Brass Department Store stocks toy race cars.
- Their supplier has given them the quantity discount schedule shown in Table 6.3.
  - Annual demand is 5,000 cars, ordering cost is $49, and holding cost is 20% of the cost of the car
- The first step is to compute EOQ values for each discount.

$$EOQ_1 = \sqrt{\frac{(2)(5{,}000)(49)}{(0.2)(5.00)}} = 700 \text{ cars per order}$$

$$EOQ_2 = \sqrt{\frac{(2)(5{,}000)(49)}{(0.2)(4.80)}} = 714 \text{ cars per order}$$

$$EOQ_3 = \sqrt{\frac{(2)(5{,}000)(49)}{(0.2)(4.75)}} = 718 \text{ cars per order}$$

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Brass Department Store Example

- The second step is adjust quantities below the allowable discount range.
- The EOQ for discount 1 is allowable.
- The EOQs for discounts 2 and 3 are outside the allowable range and have to be adjusted to the smallest quantity possible to purchase and receive the discount:

$$Q_1 = 700$$
$$Q_2 = 1{,}000$$
$$Q_3 = 2{,}000$$

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Brass Department Store

The third step is to compute the total cost for each quantity.

DISCOUNT NUMBER	UNIT PRICE (C)	ORDER QUANTITY (Q)	ANNUAL MATERIAL COST ($) = DC	ANNUAL ORDERING COST ($) = $(D/Q)C_o$	ANNUAL CARRYING COST ($) = $(Q/2)C_h$	TOTAL ($)
1	$5.00	700	25,000	350.00	350.00	25,700.00
2	4.80	1,000	24,000	245.00	480.00	24,725.00
3	4.75	2,000	23,750	122.50	950.00	24,822.50

The final step is to choose the alternative with the lowest total cost.

Table 6.4

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Brass Department Store

Excel QM's Formulas and the Input Data for the Brass Department Store Quantity Discount Problem

	A	B	C	D	E	F
1	**Brass Department Store**					
2						
3	**Inventory**	Quantity Discount Model				
4						
5	Data					
6	Demand rate, D	5000				
7	Setup cost, S	49				
8	Holding cost %, I	20%				
9						
10		Range 1	Range 2	Range 3		
11	Minimum quantity	0	1000	2000		
12	Unit Price, P	5	4.8	4.75		
13						
14	**Results**					
15		=B10	=C10	=D10		
16	Q* (Square root formula)	=SQRT(2*$B$6*$B$7/($B$8*B12))	=SQRT(2*$B$6*$B	=SQRT(2*$B$6*$B$7/($B$8*D12))		
17	Order Quantity	=IF(B16>=B11,B16,B11)	=IF(C16>=C11,C1	=IF(D16>=D11,D16,D11)	=	
18						
19	Holding cost	=B17*$B$8*B12/2	=C17*$B$8*C12/2	=D17*$B$8*D12/2		
20	Setup cost	=$B$7*$B$6/B17	=$B$7*$B$6/C17	=$B$7*$B$6/D17		
21						
22	Unit costs	=B12*$B$6	=C12*$B$6	=D12*$B$6		
23						
24	Total cost, T_c	=B19+B20+B22	=C19+C20+C22	=D19+D20+D22	minimum	=MIN(B24:D24)
25	Optimal Order Quantity	=IF(B24=$F$24,B17,"")	=IF(C24=$F$24,C1	=IF(D24=$F$24,D17,"")		
26						
27						
28						

Enter demand rate, setup cost, and holding cost.

Enter the quantity discount schedule of quantities and unit prices for each price break.

Compute the order quantities for each price break and adjust them upward if necessary.

Compute holding, setup, and unit cost for each price break.

Compute the total cost for each price break.

Determine the optimal order quantity by finding the order quantity that minimizes total costs.

Program 6.3A

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Brass Department Store

Excel QM's Solution to the Brass Department Store Problem

Program 6.3B

	A	B	C	D	E	F	G	H
1	**Brass Department Store**							
2								
3	**Inventory**	Quantity Discount Model						
4								
5	Data							
6	Demand rate, D	5000						
7	Setup cost, S	49						
8	Holding cost %, I	20%						
9								
10		Range 1	Range 2	Range 3				
11	Minimum quantity	0	1000	2000				
12	Unit Price, P	5	4.8	4.75				
13								
14	**Results**							
15		Range 1	Range 2	Range 3				
16	Q* (Square root formula)	700	714.434508	718.184846				
17	Order Quantity	700	1000	2000		=		
18								
19	Holding cost	$350.00	$480.00	$950.00				
20	Setup cost	$350.00	$245.00	$122.50				
21								
22	Unit costs	$25,000.00	$24,000.00	$23,750.00				
23								
24	Total cost, T_c	$25,700.00	$24,725.00	$24,822.50	minimum	$24,725.00		
25	Optimal Order Quantity		1000					

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Use of Safety Stock

- If demand or the lead time are uncertain, the exact ROP will not be known with certainty.
- To prevent *stockouts*, it is necessary to carry extra inventory called *safety stock.*
- Safety stock can prevent stockouts when demand is unusually high.
- Safety stock can be implemented by adjusting the ROP.

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Use of Safety Stock

- The basic ROP equation is

$$\text{ROP} = d \times L$$

$d =$ daily demand (or average daily demand)

$L =$ order lead time or the number of working days it takes to deliver an order (or average lead time)

- A safety stock variable is added to the equation to accommodate uncertain demand during lead time

$$\text{ROP} = d \times L + SS$$

where

$SS =$ safety stock

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Use of Safety Stock

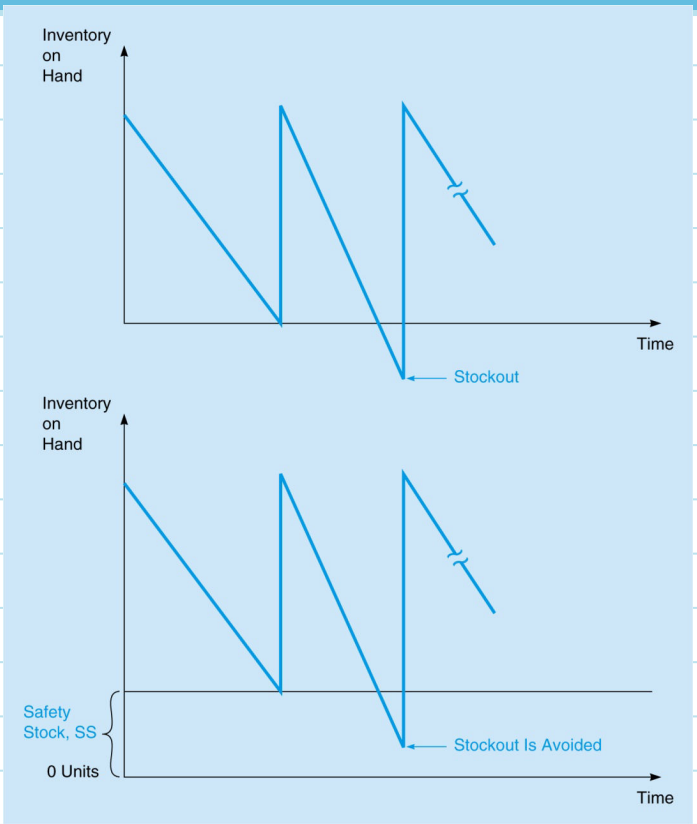

Figure 6.7

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ROP with Known Stockout Costs

- With a fixed EOQ and an ROP for placing orders, stockouts can only occur during lead time.
- Our objective is to find the safety stock quantity that will minimize the total of stockout cost and holding cost.
- We need to know the stockout cost per unit and the probability distribution of demand during lead time.
- Estimating stockout costs can be difficult as there may be direct and indirect costs.

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Safety Stock with Unknown Stockout Costs

- There are many situations when stockout costs are unknown.
- An alternative approach to determining safety stock levels is to use a *service level*.
- A service level is the percent of time you will not be out of stock of a particular item.

Service level = 1 – Probability of a stockout

or

Probability of a stockout = 1 – Service level

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Safety Stock with the Normal Distribution

ROP = (Average Demand During Lead Time) + $Z\sigma_{dLT}$

Safety Stock = $Z\sigma_{dLT}$

Where:

Z = number of standard deviations for a given service level

σ_{dLT} = standard deviation of demand during lead time

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Hinsdale Company

- Inventory demand during lead time is normally distributed.
- Mean demand during lead time is 350 units with a standard deviation of 10.
- The company wants stockouts to occur only 5% of the time.

μ = Mean demand = 350

σ_{dLT} = Standard deviation = 10

X = Mean demand + Safety stock

SS = Safety stock = $X - \mu = Z\sigma$

$$Z = \frac{X - \mu}{\sigma}$$

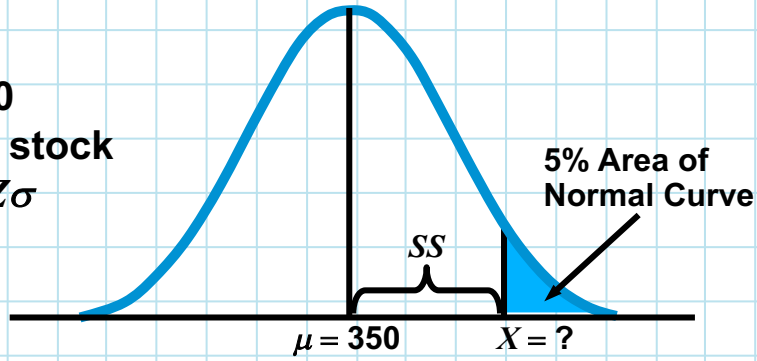

Figure 6.8

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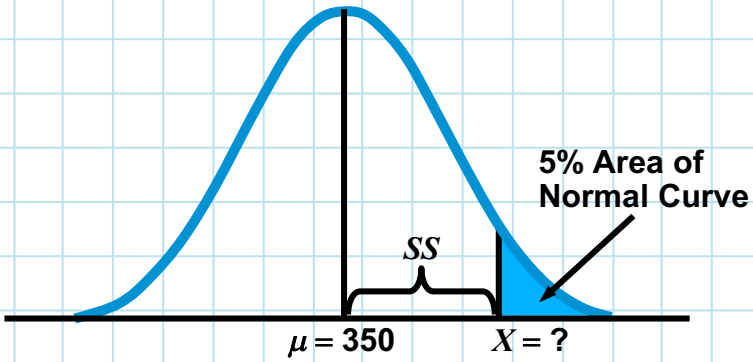
Hinsdale Company Example

- From Appendix A we find $Z = 1.65 = \frac{X - \mu}{\sigma} = \frac{SS}{\sigma}$
- Solving for safety stock:

$SS = 1.65(10) = 16.5$ units, or 17 units

X = ROP = 350+16.5 = 366.5 units

Figure 6.9



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Hinsdale Company

- Different safety stock levels will be generated for different service levels.
- However, the relationship is not linear.
  - You should be aware of what a service level is costing in terms of carrying the safety stock in inventory.
- The relationship between Z and safety stock can be developed as follows:

1. We know that $Z = \frac{X - \mu}{\sigma}$
2. We also know that $SS = X - \mu$
3. Thus $Z = \frac{SS}{\sigma}$
4. So we have
   $SS = Z\sigma$
   $= Z(10)$

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Hinsdale Company

Safety Stock at different service levels

SERVICE LEVEL (%)	Z VALUE FROM NORMAL CURVE TABLE	SAFETY STOCK (UNITS)
90	1.28	12.8
91	1.34	13.4
92	1.41	14.1
93	1.48	14.8
94	1.55	15.5
95	1.65	16.5
96	1.75	17.5
97	1.88	18.8
98	2.05	20.5
99	2.33	23.3
99.99	3.72	37.2

Table 6.5

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Hinsdale Company

Service level versus annual carrying costs

This graph was developed for a specific case, but the general shape of the curve is the same for all service-level problems.

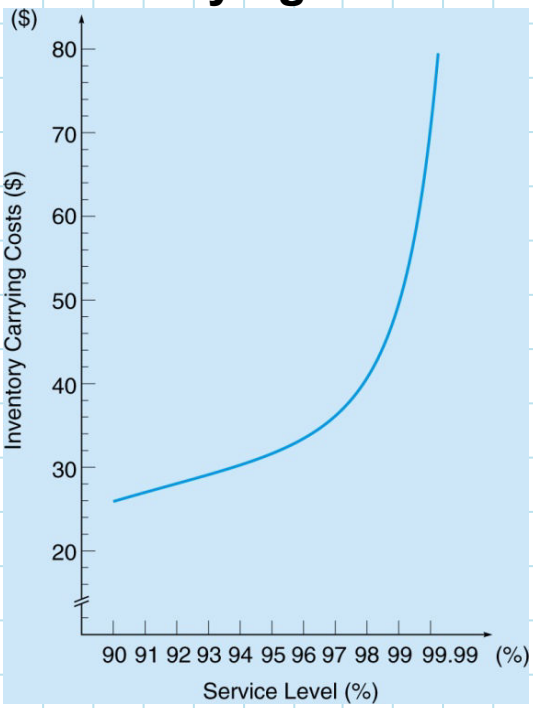

Figure 6.9

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Calculating Lead Time Demand and Standard Deviation

- There are three situations to consider:
  - Demand is variable but lead time is constant.
  - Demand is constant but lead time is variable.
  - Both demand and lead time are variable.

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Calculating Lead Time Demand and Standard Deviation

Demand is variable but lead time is constant:

$$ROP = \bar{d}L + Z(\sigma_d\sqrt{L})$$

Where:

$$\bar{d} = average\ daily\ demand$$

$$\sigma_d = standard\ deviation\ of\ daily\ demand$$

$$L = lead\ time\ in\ days$$

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Calculating Lead Time Demand and Standard Deviation

Demand is constant but lead time is variable:

$$ROP = d\overline{L} + Z(d\sigma_L)$$

Where:

$$\overline{L} = average\ lead\ time$$

$$\sigma_L = standard\ deviation\ of\ lead\ time$$

$$d = daily\ demand$$

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Calculating Lead Time Demand and Standard Deviation

Both demand and lead time are variable.

$$ROP = \overline{dL} + Z(\sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2})$$

Notice that this is the most general case and that the other two cases can be derived from this formula.

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Hinsdale Company

Suppose for product SKU F5402, daily demand is normally distributed, with a mean of 15 units and a standard deviation of 3. Lead time is exactly 4 days. To maintain a 97% service level, what is the ROP, and how much safety stock should be carried?

$$ROP = \bar{d}L + Z(\sigma_d\sqrt{L})$$

ROP = 15(4)+1.88(3*2)
= 60 + 11.28
=71.28

So the average demand during lead time is 60 units, and safety stock is 11.28 units.

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Hinsdale Company

Suppose for product SKU B7319, daily demand is constant at 25 units per day, but lead time is normally distributed, with a mean of 6 days and a standard deviation of 3. To maintain a 98% service level, what is the ROP?

$$ROP = d\bar{L} + Z(d\sigma_L)$$

ROP = 25(6)+2.05(25*3)
= 150+153.75
=303.75

So the average demand during lead time is 150 units, and safety stock is 153.75 units.

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Hinsdale Company

Suppose for product SKU F9004, daily demand is normally distributed, with a mean of 20 units and a standard deviation of 4. Lead time is normally distributed, with a mean of 5 days and a standard deviation of 2 days. To maintain a 94% service level, what is the ROP?

$$ROP = \overline{dL} + Z(\sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2})$$

$$ROP = 20(5) + 1.55(\sqrt{5(16) + 400(4)})$$

ROP = 100+1.55(40.99)
= 163.53

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Calculating Annual Holding Cost with Safety Stock

- **Under standard assumptions of EOQ, average inventory is just *Q*/2.**
- **So annual holding cost is: (*Q*/2)*C_h.**
- **This is not the case with safety stock because safety stock is not meant to be drawn down.**

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Calculating Annual Holding Cost with Safety Stock

Total annual holding cost = holding cost of regular inventory + holding cost of safety stock

$$THC = \frac{Q}{2}C_h + (SS)C_h$$

Where:

THC	= total annual holding cost
Q	= order quantity
C_h	= holding cost per unit per year
SS	= safety stock

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Excel QM Formulas and Input Data for the Hinsdale Safety Stock Problems

The average demand and standard deviation during the lead time are entered here, if available.

If daily demand is normally distributed but lead time is constant, the data are entered here.

If lead time is normally distributed, data are entered here. If daily demand is constant, enter 0 for the standard deviation.

The standard deviation of demand during the lead time is calculated here.

	A		
1	Hinsdale Company Safety Stock		
2			
3	Inventory		Safety stock - Normal distribution
4	Select a model and then enter the data in the shaded area. The model on		
5			
6	Model: Demand during leadtime and its standard deviat		Model: Daily demand and its standard deviation are gi
7			
8	Data		Data
9	Average demand during lead time, μ	350	Average daily demand 15
10	Standard deviation of σ_{dLT}	10	Standard deviation of daily demand, σ_d 3
11	Service level (% of demand met)	0.95	Lead time days 4
12			Service level (% of demand met) 0.97
13			
14	Results		Results
15	Z-value	=NORMSINV(B11)	Z-value =NORMSINV(H12)
16	Safety stock	=B15*B10	Average demand during lead time =H9*H11
17			Standard deviation of demand during lead time, σ_{dLT} =H10*SQRT(H11)
18			=H15*H17
19			=H18+H16
20			
21			
22	Models: Either daily demand, lead time or both are varia		
23			
24	Data		
25	Average daily demand	25	
26	Standard deviation of daily demand	0	Enter 0 if demand is constant
27	Average lead time (in days)	6	
28	Standard deviation of lead time, σ_{LT}	3	Enter 0 if lead time is constant
29	Service level (% of demand met)	0.98	
30			
31	Results		
32	Z-value	=NORMSINV(B29)	
33	Average demand during lead time	=B27*B25	
34	Standard deviation of demand during lead time, σ_{dLT}	=SQRT(B27*B26^2+B25^2*B28^2)	
35	Safety stock	=B34*B32	
36	Reorder point	=B33+B35	

Program 6.4A

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Excel QM Solution to the Hinsdale Safety Stock Problem

Hinsdale Company Safety Stock

Inventory — **Safety stock - Normal distribution**

Select a model and then enter the data in the shaded area. The model on the bottom left represents the 3 models described in the textbook under Other Probabilistic Models

Model: Demand during leadtime and its standard deviation given

Data	
Average demand during lead time, μ	350
Standard deviation of σ_{dLT}	10
Service level (% of demand met)	95.00%
Results	
Z-value	1.64
Safety stock	16.45

Solution to the first Hinsdale example, where standard deviation of demand during lead time was given.

Model: Daily demand and its standard deviation are given

Data	
Average daily demand	15
Standard deviation of daily demand, σ_d	3
Lead time days	4
Service level (% of demand met)	97.00%
Results	
Z-value	1.88
Average demand during lead time	60
Standard deviation of demand during lead time, σ_{dLT}	6.00
Safety stock	11.28
Reorder Point	71.28

Solution to the second Hinsdale, example, where daily demand was normally distributed.

Models: Either daily demand, lead time or both are variable

Data		
Average daily demand	25	
Standard deviation of daily demand	0	**Enter 0 if demand is constant**
Average lead time (in days)	6	
Standard deviation of lead time, σ_{LT}	3	**Enter 0 if lead time is constant**
Service level (% of demand met)	98.00%	
Results		
Z-value	2.05	
Average demand during lead time	150	
Standard deviation of demand during lead time, σ_{dLT}	75.00	
Safety stock	154.03	
Reorder point	304.03	

Solution to the third Hinsdale example, where daily demand was constant but lead time was normally distributed.

Program 6.4B

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Single-Period Inventory Models

- Some products have no future value beyond the current period.
- These situations are called *news vendor* problems or *single-period inventory models.*
- Analysis uses marginal profit (MP) and marginal loss (ML) and is called marginal analysis.
- With a manageable number of states of nature and alternatives, discrete distributions can be used.
- When there are a large number of alternatives or states of nature, the normal distribution may be used.

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Marginal Analysis with Discrete Distributions

We stock an additional unit only if the expected marginal profit for that unit exceeds the expected marginal loss.

$P =$ probability that demand will be greater than or equal to a given supply (or the probability of selling at least one additional unit).

$1 - P =$ probability that demand will be less than supply (or the probability that one additional unit will not sell).

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Marginal Analysis with Discrete Distributions

- The expected marginal profit is $P(\text{MP})$.
- The expected marginal loss is $(1 - P)(\text{ML})$.
- The optimal decision rule is to stock the additional unit if:

$$P(\text{MP}) \geq (1 - P)\text{ML}$$

- With some basic manipulation:

$$P(\text{MP}) \geq \text{ML} - P(\text{ML})$$
$$P(\text{MP}) + P(\text{ML}) \geq \text{ML}$$
$$P(\text{MP} + \text{ML}) \geq \text{ML}$$

or $$P \geq \frac{\text{ML}}{\text{ML} + \text{MP}}$$

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Steps of Marginal Analysis with Discrete Distributions

1. Determine the value of $\frac{ML}{ML + MP}$ for the problem.
2. Construct a probability table and add a cumulative probability column.
3. Keep ordering inventory as long as the probability (P) of selling at least one additional unit is greater than $\frac{ML}{ML + MP}$

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Café du Donut

- The café buys donuts each day for $4 per carton of 2 dozen donuts.
- Any cartons not sold are thrown away at the end of the day.
- If a carton is sold, the total revenue is $6.
- The marginal profit per carton is.

$$MP = \text{Marginal profit} = \$6 - \$4 = \$2$$

- The marginal loss is $4 per carton since cartons can not be returned or salvaged.

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Café du Donut's Probability Distribution

DAILY SALES (CARTONS OF DOUGHNUTS)	PROBABILITY (P) THAT DEMAND WILL BE AT THIS LEVEL
4	0.05
5	0.15
6	0.15
7	0.20
8	0.25
9	0.10
10	0.10
Total	1.00

Table 6.6

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Café du Donut Example

Step 1. Determine the value of $\frac{\text{ML}}{\text{ML}+\text{MP}}$ for the decision rule.

$$P \geq \frac{\text{ML}}{\text{ML}+\text{MP}} = \frac{\$4}{\$4+\$2} = \frac{4}{6} = 0.67$$

$$P \geq 0.67$$

Step 2. Add a new column to the table to reflect the probability that doughnut sales will be at each level or greater.

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Marginal Analysis for Café du Donut

DAILY SALES (CARTONS OF DOUGHNUTS)	PROBABILITY (P) THAT DEMAND WILL BE AT THIS LEVEL	PROBABILITY (P) THAT DEMAND WILL BE AT THIS LEVEL OR GREATER
4	0.05	1.00 ≥ 0.66
5	0.15	0.95 ≥ 0.66
6	0.15	0.80 ≥ 0.66
7	0.20	0.65
8	0.25	0.45
9	0.10	0.20
10	0.10	0.10
Total	1.00	

Table 6.7

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Café du Donut Example

Step 3. Keep ordering additional cartons as long as the probability of selling at least one additional carton is greater than P, which is the indifference probability.

P at 6 cartons = 0.80 > 0.67

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Marginal Analysis with the Normal Distribution

- We first need to find four values:
  1. The average or mean sales for the product, μ
  2. The standard deviation of sales, σ
  3. The marginal profit for the product, MP.
  4. The marginal loss for the product, ML.
- We let X^* = optimal stocking level.

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Steps of Marginal Analysis with the Normal Distribution

1. Determine the value of $\dfrac{\mathrm{ML}}{\mathrm{ML}+\mathrm{MP}}$ for the problem.
2. Locate P on the normal distribution (Appendix A) and find the associated Z-value.
3. Find X^* using the relationship:

$$Z = \frac{X^* - \mu}{\sigma}$$

to solve for the resulting stocking policy:

$$X^* = \mu + Z\sigma$$

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Joe's Stocking Decision for the Chicago Tribune

- Demand for the *Chicago Tribune* at Joe's Newsstand averages 60 papers a day with a standard deviation of 10.
- The marginal loss is 20 cents and the marginal profit is 30 cents.

Step 1. Joe should stock the *Tribune* as long as the probability of selling the last unit is at least ML/(ML + MP):

$$\frac{\text{ML}}{\text{ML}+\text{MP}} = \frac{20 \text{ cents}}{20 \text{ cents} + 30 \text{ cents}} = \frac{20}{50} = 0.40$$

Let P = 0.40.

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Joe's Stocking Decision for the Chicago Tribune

Step 2. Using the normal distribution in Figure 6.10, we find the appropriate Z value

Z = 0.25 standard deviations from the mean

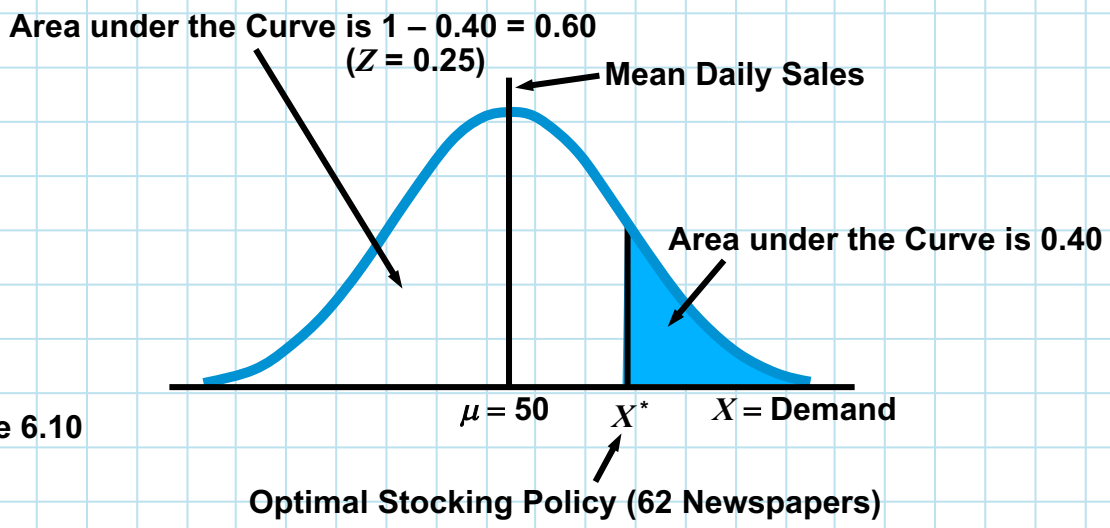

Figure 6.10

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Joe's Stocking Decision for the Chicago Tribune

Step 3. In this problem, $\mu = 60$ and $\sigma = 10$, so

$$0.25 = \frac{X^* - 60}{10}$$

or

$$X^* = 60 + 0.25(10) = 62.5, \text{ or 62 newspapers}$$

Joe should order 62 newspapers since the probability of selling 63 newspapers is slightly less than 0.40

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Joe's Stocking Decision for the Chicago Tribune

- The procedure is the same when $P > 0.50$.
- Joe also stocks the *Chicago Sun-Times.*
- Marginal loss is 40 cents and marginal profit is 10 cents.
- Daily sales average 100 copies with a standard deviation of 10 papers.

$$\frac{\text{ML}}{\text{ML} + \text{MP}} = \frac{40 \text{ cents}}{40 \text{ cents} + 10 \text{ cents}} = \frac{40}{50} = 0.80$$

- From Appendix A:

$Z = -0.84$ standard deviations from the mean

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Joe's Stocking Decision for the Chicago Tribune

With $\mu = 100$ and $\sigma = 10$

$$-0.84 = \frac{X^* - 100}{10}$$

or

$$X^* = 100 - 0.84(10)$$
$$= 91.6, \text{ or } 91 \text{ newspapers}$$

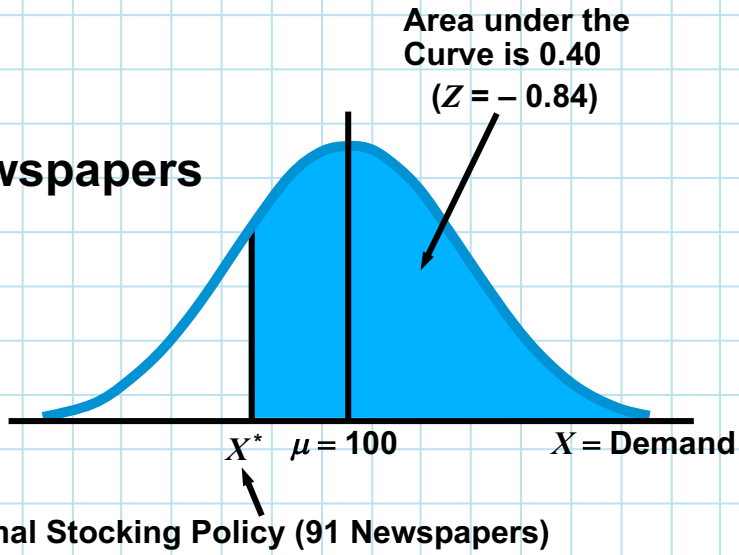

Figure 6.11

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ABC Analysis

- The purpose of ABC analysis is to divide the inventory into three groups based on the overall inventory value of the items.
- Group A items account for the major portion of inventory costs.
  - Typically about 70% of the dollar value but only 10% of the quantity of items.
  - Forecasting and inventory management must be done carefully.
- Group B items are more moderately priced.
  - May represent 20% of the cost and 20% of the quantity.
- Group C items are very low cost but high volume.
  - It is not cost effective to spend a lot of time managing these items.

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Summary of ABC Analysis

INVENTORY GROUP	DOLLAR USAGE (%)	INVENTORY ITEMS (%)	ARE QUANTITATIVE CONTROL TECHNIQUES USED?
A	70	10	Yes
B	20	20	In some cases
C	10	70	No

Table 6.8

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Dependent Demand: The Case for Material Requirements Planning

- All the inventory models discussed so far have assumed demand for one item is independent of the demand for any other item.
- However, in many situations items demand is dependent on demand for one or more other items.
- In these situations, *Material Requirements Planning (MRP)* can be employed effectively.

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Dependent Demand: The Case for Material Requirements Planning

- Some of the benefits of MRP are:
  1. Increased customer service levels.
  2. Reduced inventory costs.
  3. Better inventory planning and scheduling.
  4. Higher total sales.
  5. Faster response to market changes and shifts.
  6. Reduced inventory levels without reduced customer service.
- Most MRP systems are computerized, but the basic analysis is straightforward.

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Material Structure Tree

- The first step is to develop a *bill of materials* (*BOM*).
- The BOM identifies components, descriptions, and the number required for production of one unit of the final product.
- From the BOM we can develop a material structure tree.
- We use the following data:
  - Demand for product A is 50 units.
  - Each A requires 2 units of B and 3 units of C.
  - Each B requires 2 units of D and 3 units of E.
  - Each C requires 1 unit of E and 2 units of F.

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Material Structure Tree

Material structure tree for Item A

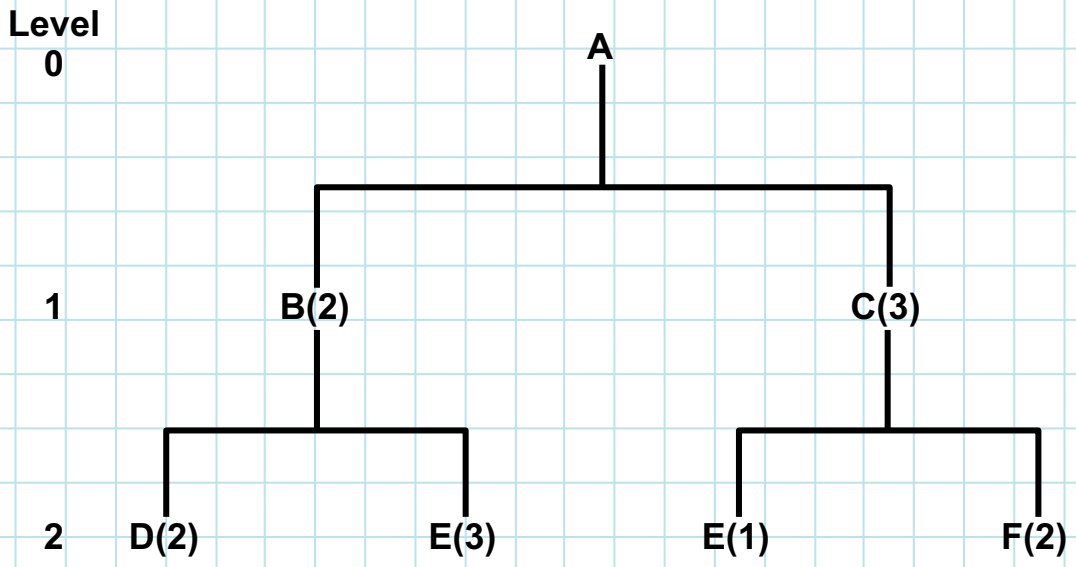

Figure 6.12

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Material Structure Tree

- It is clear from the three that the demand for B, C, D, E, and F is completely dependent on the demand for A.
- The material structure tree has three levels: 0, 1, and 2.
- Items above a level are called *parents.*
- Items below any level are called *components.*
- The number in parenthesis beside each item shows how many are required to make the item above it.

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Material Structure Tree

We can use the material structure tree and the demand for Item A to compute demands for the other items.

Part B: $2 \times$ number of A's $= 2 \times 50 = 100$.
Part C: $3 \times$ number of A's $= 3 \times 50 = 150$.
Part D: $2 \times$ number of B's $= 2 \times 100 = 200$.
Part E: $3 \times$ number of B's $+ 1 \times$ number of C's
$= 3 \times 100 + 1 \times 150 = 450$.
Part F: $2 \times$ number of C's $= 2 \times 150 = 300$.

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Gross and Net Material Requirements Plan

- Once the materials structure tree is done, we construct a gross material requirements plan.
- This is a time schedule that shows:
  - when an item must be ordered when there is no inventory on hand, or
  - when the production of an item must be started in order to satisfy the demand for the finished product at a particular date.
- We need lead times for each of the items.

Item A – 1 week	Item D – 1 week
Item B – 2 weeks	Item E – 2 weeks
Item C – 1 week	Item F – 3 weeks

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Gross Material Requirements Plan for 50 Units of A

		Week 1	2	3	4	5	6	
A	Required Date						50	Lead Time = 1 Week
	Order Release					50		
B	Required Date					100		Lead Time = 2 Weeks
	Order Release			100				
C	Required Date					150		Lead Time = 1 Week
	Order Release				150			
D	Required Date			200				Lead Time = 1 Week
	Order Release		200					
E	Required Date			300	150			Lead Time = 2 Weeks
	Order Release	300	150					
F	Required Date				300			Lead Time = 3 Weeks
	Order Release	300						

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Figure 6.13

Net Material Requirements Plan

A net material requirements plan can be constructed from the gross materials requirements plan and the following on-hand inventory information:

ITEM	ON-HAND INVENTORY
A	10
B	15
C	20
D	10
E	10
F	5

Table 6.9

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Net Material Requirements Plan

- Using this data we can construct a plan that includes:
  - Gross requirements.
  - On-hand inventory.
  - Net requirements.
  - Planned-order receipts.
  - Planned-order releases.
- The net requirements plan is constructed like the gross requirements plan.

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Net Material Requirements Plan for 50 Units of A

Item		Week 1	2	3	4	5	6	Lead Time
A	Gross						50	1
	On-Hand 10						10	
	Net						40	
	Order Receipt						40	
	Order Release					40		
B	Gross					80[A]		2
	On-Hand 15					15		
	Net					65		
	Order Receipt					65		
	Order Release			65				

Figure 6.14(a)

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Net Material Requirements Plan for 50 Units of A

Item		Week 1	2	3	4	5	6	Lead Time
C	Gross					120[A]		1
	On-Hand 20					10		
	Net					100		
	Order Receipt					100		
	Order Release				100			
D	Gross			130[B]				1
	On-Hand 10			10				
	Net			120				
	Order Receipt			120				
	Order Release		120					

Figure 6.14(b)

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Net Material Requirements Plan for 50 Units of A

Item		Week 1	2	3	4	5	6	Lead Time
E	Gross			195[B]	100[C]			2
	On-Hand 10			10	0			
	Net			185	100			
	Order Receipt			185	100			
	Order Release	185	100					
F	Gross				200[C]			3
	On-Hand 5				5			
	Net				195			
	Order Receipt				195			
	Order Release	195						

Figure 6.14(c)

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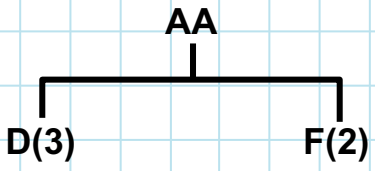
Two or More End Products

- Most manufacturing companies have more than one end item.
- In this example, the second product is AA and it has the following material structure tree:



- If we require 10 units of AA, the gross requirements for parts D and F can be computed:

Part D: $3 \times$ number of AA's $= 3 \times 10 = 30$

Part F: $2 \times$ number of AA's $= 2 \times 10 = 20$

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Two or More End Products

- The lead time for AA is one week.
- The gross requirement for AA is 10 units in week 6 and there are no units on hand.
- This new product can be added to the MRP process.
- The addition of AA will only change the MRP schedules for the parts contained in AA.

- MRP can also schedule spare parts and components.
- These have to be included as gross requirements.

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Net Material Requirements Plan, Including AA

Item	Inventory	Week 1	2	3	4	5	6	Lead Time
AA	Gross						10	1 Week
	On-Hand: 0						0	
	Net						10	
	Order Receipt						10	
	Order Release					10		
A	Gross						50	1 Week
	On-Hand: 10						10	
	Net						40	
	Order Receipt						40	
	Order Release					40		
B	Gross					80 [A]		2 Weeks
	On-Hand: 15					15		
	Net					65		
	Order Receipt					65		
	Order Release			65				
C	Gross					120 [A]		1 Week
	On-Hand: 20					20		
	Net					100		
	Order Receipt					100		
	Order Release				100			
D	Gross			130 [B]		30 [AA]		1 Week
	On-Hand: 10			10		0		
	Net			120		30		
	Order Receipt			120		30		
	Order Release		120		30			
E	Gross			195 [B]	100 [C]			2 Weeks
	On-Hand: 10			10	0			
	Net			185	100			
	Order Receipt			185	100			
	Order Release	185	100					
F	Gross				200 [C]	20 [AA]		3 Weeks
	On-Hand: 5				5	0		
	Net				195	20		
	Order Receipt				195	20		
	Order Release	195	20					

Figure 6.15

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Just-in-Time (JIT) Inventory Control

- To achieve greater efficiency in the production process, organizations have tried to have less in-process inventory on hand.
- This is known as *JIT inventory.*
- The inventory arrives just in time to be used during the manufacturing process.
- One technique of implementing JIT is a manual procedure called *kanban.*

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Just-in-Time Inventory Control

- Kanban in Japanese means "card."
- With a dual-card kanban system, there is a conveyance kanban, or C-kanban, and a production kanban, or P-kanban.
- Kanban systems are quite simple, but they require considerable discipline.
- As there is little inventory to cover variability, the schedule must be followed exactly.

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4 Steps of Kanban

1. A user takes a container of parts or inventory along with its C-kanban to his or her work area.

   When there are no more parts or the container is empty, the user returns the container along with the C-kanban to the producer area.
2. At the producer area, there is a full container of parts along with a P-kanban.

   The user detaches the P-kanban from the full container and takes the container and the C-kanban back to his or her area for immediate use.

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4 Steps of Kanban

3. The detached P-kanban goes back to the producer area along with the empty container

   The P-kanban is a signal that new parts are to be manufactured or that new parts are to be placed in the container and is attached to the container when it is filled .

4. This process repeats itself during the typical workday.

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The Kanban System

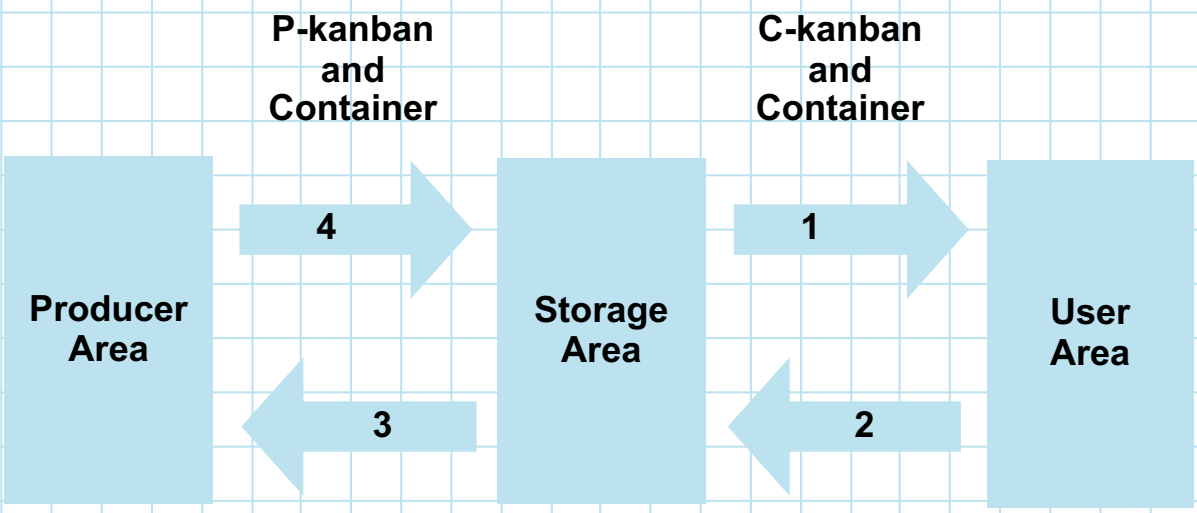

Figure 6.16

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Enterprise Resource Planning

- MRP has evolved to include not only the materials required in production, but also the labor hours, material cost, and other resources related to production.
- In this approach the term MRP II is often used and the word *resource* replaces the word *requirements.*
- As this concept evolved and sophisticated software was developed, these systems became known as *enterprise resource planning* (*ERP*) systems.

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Enterprise Resource Planning

- The objective of an ERP System is to reduce costs by integrating all of the operations of a firm.
- Starts with the supplier of materials needed and flows through the organization to include invoicing the customer of the final product.
- Data are entered only once into a database where it can be quickly and easily accessed by anyone in the organization.
- Benefits include:
  - Reduced transaction costs.
  - Increased speed and accuracy of information.

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Enterprise Resource Planning

- Drawbacks to ERP:
  - The software is expensive to buy and costly to customize.
    - Small systems can cost hundreds of thousands of dollars.
    - Large systems can cost hundreds of millions.
  - The implementation of an ERP system may require a company to change its normal operations.
  - Employees are often resistant to change.
  - Training employees on the use of the new software can be expensive.

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Chapter 12

Project Management

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. Understand how to plan, monitor, and control projects with the use of PERT and CPM.
2. Determine earliest start, earliest finish, latest start, latest finish, and slack times for each activity, along with the total project completion time.
3. Reduce total project time at the least total cost by crashing the network using manual or linear programming techniques.
4. Understand the important role of software in project management.

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Chapter Outline

12.1 Introduction
12.2 PERT/CPM
12.3 PERT/Cost
12.4 Project Crashing
12.5 Other Topics in Project Management

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Introduction

- Managing large-scale, complicated projects effectively is a difficult problem and the stakes are high.
- The first step in planning and scheduling a project is to develop the *work breakdown structure.*
- Time, cost, resource requirements, predecessors, and people required are identified for each activity.
- A schedule for the project then can be developed.

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Introduction

- The *program evaluation and review technique* (*PERT*) and the *critical path method* (*CPM*) are two popular quantitative analysis techniques to help plan, schedule, monitor, and control projects.
- Originally the approaches differed in how they estimated activity times.
  - PERT used three time estimates to develop a probabilistic estimate of completion time.
  - CPM was a more deterministic technique.
- They have become so similar they are commonly considered one technique, PERT/CPM.

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Six Steps of PERT/CPM

1. Define the project and all of its significant activities or tasks.
2. Develop the relationships among the activities and decide which activities must precede others.
3. Draw the network connecting all of the activities.
4. Assign time and/or cost estimates to each activity.
5. Compute the longest time path through the network; this is called the *critical path.*
6. Use the network to help plan, schedule, monitor, and control the project.

The critical path is important since any delay in these activities can delay the completion of the project.

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PERT/CPM

Given the large number of tasks in a project, it is easy to see why the following questions are important:

1. **When will the entire project be completed?**
2. **What are the *critical* activities or tasks in the project, that is, the ones that will delay the entire project if they are late?**
3. **Which are the *non-critical* activities, that is, the ones that can run late without delaying the entire project's completion?**
4. **If there are three time estimates, what is the probability that the project will be completed by a specific date?**

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PERT/CPM

5. **At any particular date, is the project on schedule, behind schedule, or ahead of schedule?**
6. **On any given date, is the money spent equal to, less than, or greater than the budgeted amount?**
7. **Are there enough resources available to finish the project on time?**

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General Foundry Example of PERT/CPM

- General Foundry, Inc. has long been trying to avoid the expense of installing air pollution control equipment.
- The local environmental protection group has recently given the foundry 16 weeks to install a complex air filter system on its main smokestack.
- General Foundry was warned that it will be forced to close unless the device is installed in the allotted period.
- They want to make sure that installation of the filtering system progresses smoothly and on time.

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General Foundry Example of PERT/CPM

Activities and immediate predecessors for General Foundry

ACTIVITY	DESCRIPTION	IMMEDIATE PREDECESSORS
A	Build internal components	—
B	Modify roof and floor	—
C	Construct collection stack	*A*
D	Pour concrete and install frame	*B*
E	Build high-temperature burner	*C*
F	Install control system	*C*
G	Install air pollution device	*D, E*
H	Inspect and test	*F, G*

Table 12.1

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Drawing the PERT/CPM Network

- There are two common techniques for drawing PERT networks.
- *Activity-on-node* (*AON*) where the nodes represent activities.
- *Activity-on-arc* (*AOA*) where the arcs are used to represent the activities.
- The AON approach is easier and more commonly found in software packages.
- One node represents the start of the project, one node for the end of the project, and nodes for each of the activities.
- The arcs are used to show the predecessors for each activity.

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General Foundry Example of PERT/CPM

Network for General Foundry

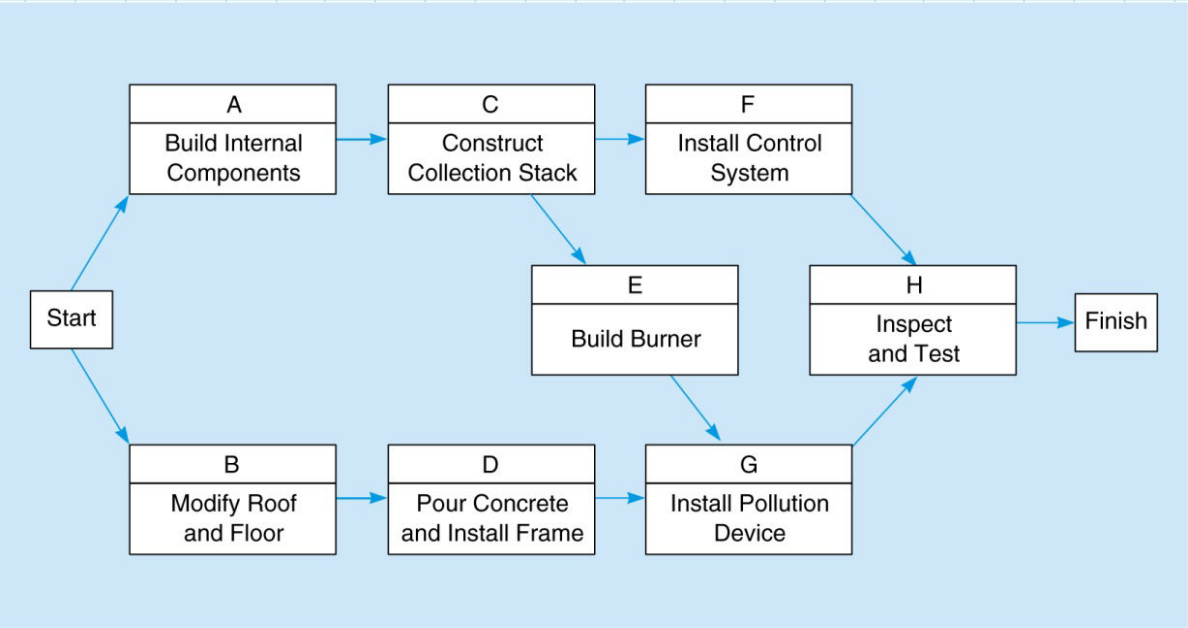

Figure 12.1

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Activity Times

- In some situations, activity times are known with certainty.
- The CPM assigns just one time estimate to each activity and this is used to find the critical path.
- In many projects there is uncertainty about activity times.
- PERT employs a probability distribution based on three time estimates for each activity, and a weighted average of these estimates is used for the time estimate and this is used to determine the critical path.
  - PERT often assumes time estimates follow a ***beta probability distribution***.

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Activity Times

The time estimates in PERT are:

Optimistic time (a) = time an activity will take if everything goes as well as possible. There should be only a small probability (say, $^1/_{100}$) of this occurring.

Pessimistic time (b) = time an activity would take assuming very unfavorable conditions. There should also be only a small probability that the activity will really take this long.

Most likely time (m) = most realistic time estimate to complete the activity

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Activity Times

Beta Probability Distribution with Three Time Estimates

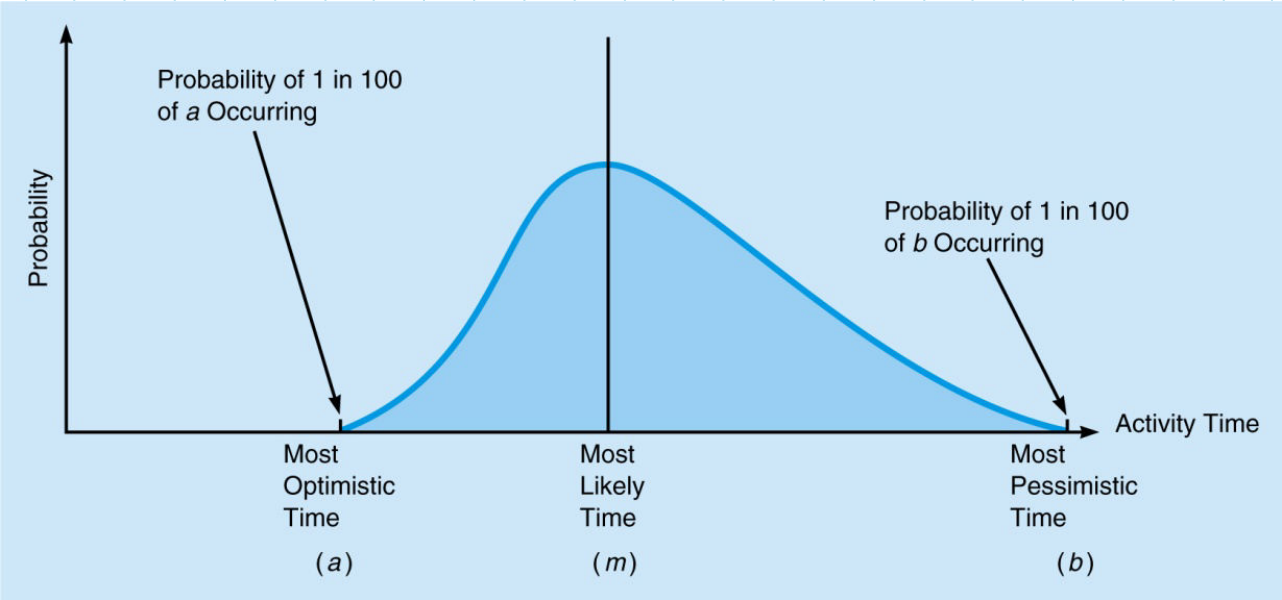

Figure 12.2

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Activity Times

To find the *expected activity time* (*t*), the beta distribution weights the estimates as follows:

$$t = \frac{a + 4m + b}{6}$$

To compute the dispersion or *variance of activity completion time*, we use the formula:

$$\text{Variance} = \left(\frac{b - a}{6}\right)^2$$

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Activity Times

Time estimates (weeks) for General Foundry

ACTIVITY	OPTIMISTIC, a	MOST PROBABLE, m	PESSIMISTIC, b	EXPECTED TIME, $t = [(a + 4m + b)/6]$	VARIANCE, $[(b - a)/6]^2$
A	1	2	3	2	4/36
B	2	3	4	3	4/36
C	1	2	3	2	4/36
D	2	4	6	4	16/36
E	1	4	7	4	36/36
F	1	2	9	3	64/36
G	3	4	11	5	64/36
H	1	2	3	2	4/36
				25	

Table 12.2

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How to Find the Critical Path

- We accept the expected completion time for each task as the actual time for now.
- The total of 25 weeks in Table 12.2 does not take into account the obvious fact that some of the tasks could be taking place at the same time.
- To find out how long the project will take we perform the critical path analysis for the network.
- The *critical path* is the longest path through the network.

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How to Find the Critical Path

General Foundry's Network With Expected Activity Times

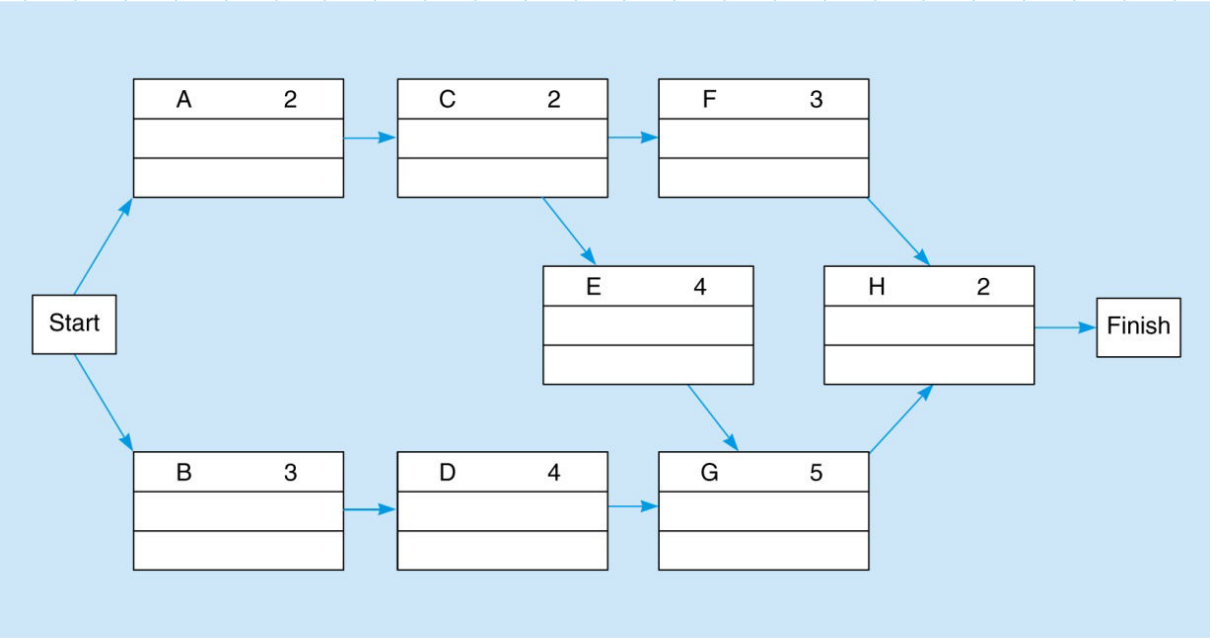

Figure 12.3

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How to Find the Critical Path

To find the critical path, we need to determine the following quantities for each activity in the network.

1. ***Earliest start time*** **(*ES*): the earliest time an activity can begin without violation of immediate predecessor requirements.**
2. ***Earliest finish time*** **(*EF*): the earliest time at which an activity can end.**
3. ***Latest start time*** **(*LS*): the latest time an activity can begin without delaying the entire project.**
4. ***Latest finish time*** **(*LF*): the latest time an activity can end without delaying the entire project.**

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How to Find the Critical Path

In the nodes, the activity time and the early and late start and finish times are represented in the following manner.

ACTIVITY	t
ES	EF
LS	LF

Earliest times are computed as:

Earliest finish time = Earliest start time + Expected activity time

$$EF = ES + t$$

Earliest start = Largest of the earliest finish times of immediate predecessors

ES = Largest EF of immediate predecessors

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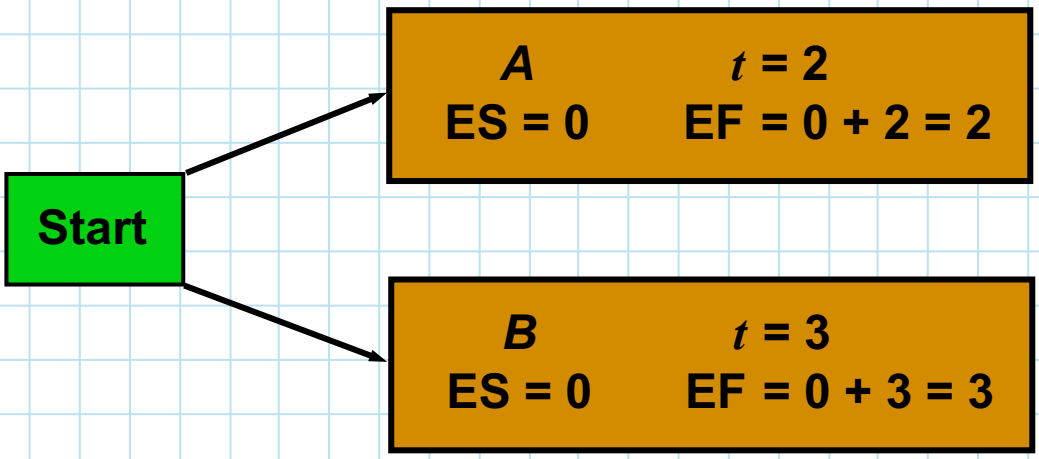
How to Find the Critical Path

- At the start of the project we set the time to zero.
- Thus ES = 0 for both *A* and *B.*



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How to Find the Critical Path

General Foundry's Earliest Start (ES) and Earliest Finish (EF) times

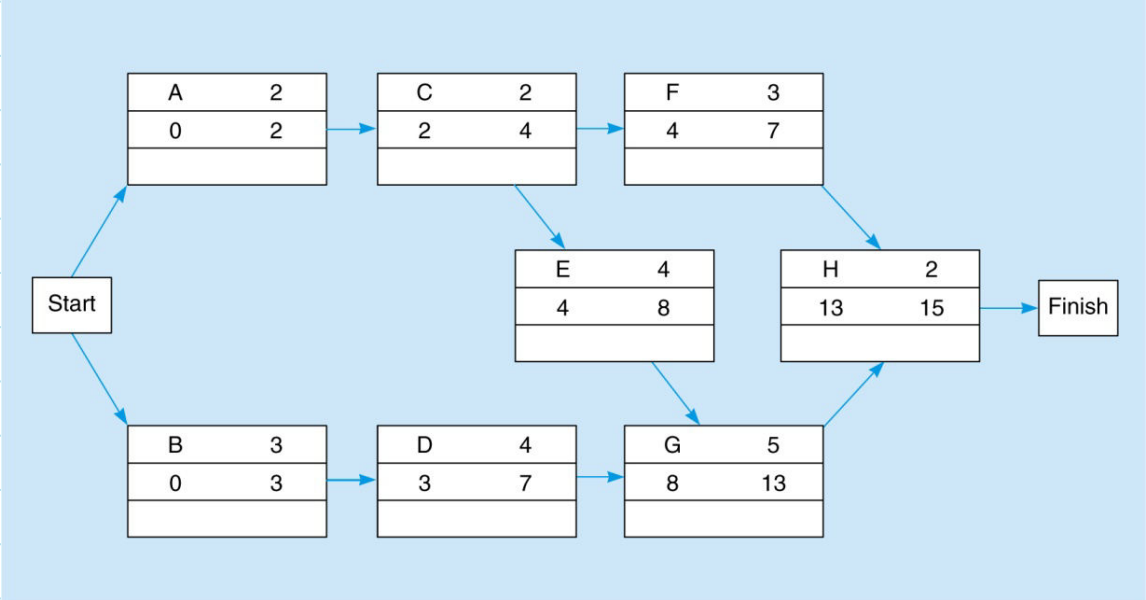

Figure 12.4

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How to Find the Critical Path

Latest times are computed as

Latest start time = Latest finish time – Expected activity time

$$LS = LF - t$$

Latest finish time = Smallest of latest start times for following activities

LF = Smallest LS of following activities

For activity *H*:

$$LS = LF - t = 15 - 2 = 13 \text{ weeks}$$

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How to Find the Critical Path

General Foundry's Latest Start (LS) and Latest Finish (LF) times

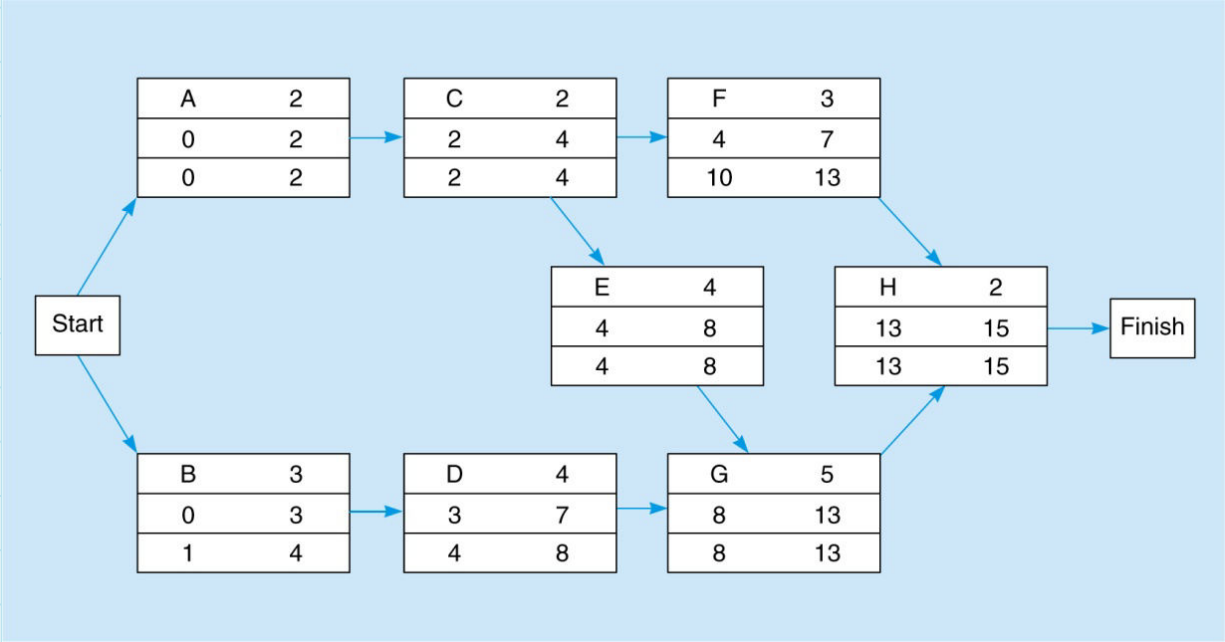

Figure 12.5

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How to Find the Critical Path

- **Once ES, LS, EF, and LF have been determined, it is a simple matter to find the amount of *slack time* that each activity has:**

Slack = LS – ES, or Slack = LF – EF

- **From Table 12.3 we see activities *A*, *C*, *E*, *G*, and *H* have no slack time.**
- **These are called *critical activities* and they are said to be on the *critical path.***
- **The total project completion time is 15 weeks.**
- **Industrial managers call this a boundary timetable.**

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How to Find the Critical Path

General Foundry's Schedule and Slack Times

ACTIVITY	EARLIEST START, ES	EARLIEST FINISH, EF	LATEST START, LS	LATEST FINISH, LF	SLACK, LS – ES	ON CRITICAL PATH?
A	0	2	0	2	0	Yes
B	0	3	1	4	1	No
C	2	4	2	4	0	Yes
D	3	7	4	8	1	No
E	4	8	4	8	0	Yes
F	4	7	10	13	6	No
G	8	13	8	13	0	Yes
H	13	15	13	15	0	Yes

Table 12.3

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How to Find the Critical Path

General Foundry's Critical Path

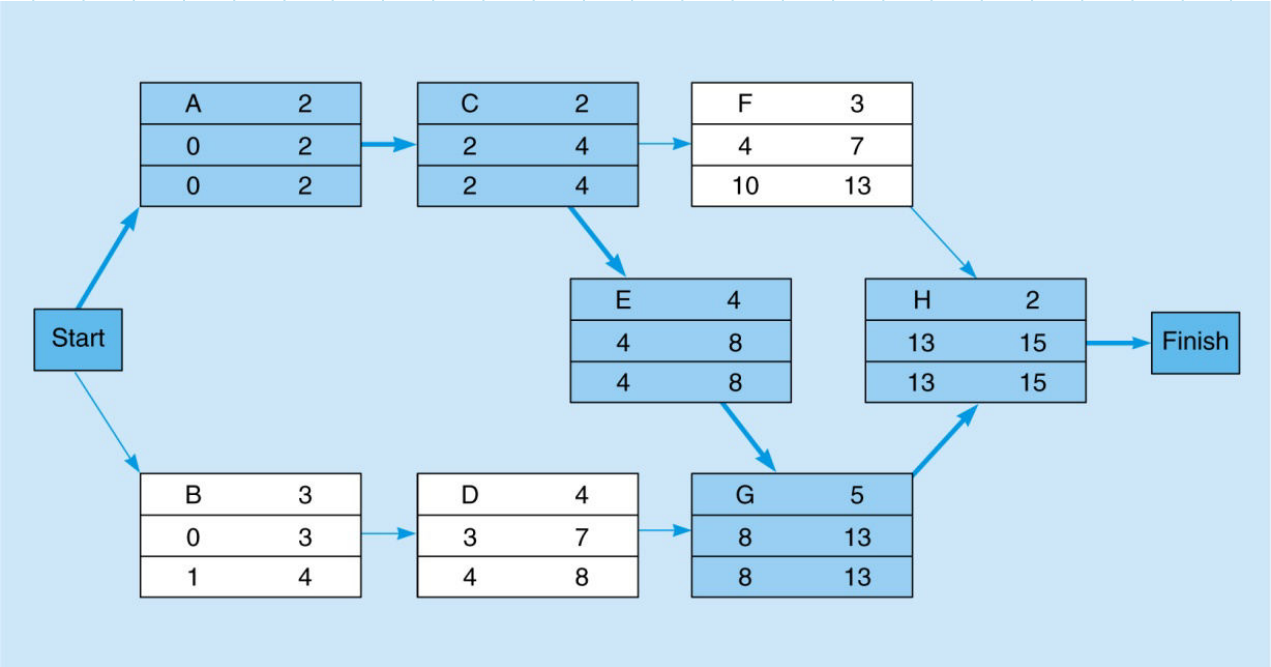

Figure 12.6

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Probability of Project Completion

- The *critical path analysis* helped determine the expected project completion time of 15 weeks.
- But variation in activities on the critical path can affect overall project completion, and this is a major concern.
- If the project is not complete in 16 weeks, the foundry will have to close.
- PERT uses the variance of critical path activities to help determine the variance of the overall project.

$$\text{Project variance} = \sum \begin{array}{l}\text{variances of activities}\\ \text{on the critical path}\end{array}$$

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Probability of Project Completion

- From Table 12.2 we know that

ACTIVITY	VARIANCE
A	4/36
C	4/36
E	36/36
G	64/36
H	4/36

- Hence, the project variance is

$$\text{Project variance} = {}^{4}/_{36} + {}^{4}/_{36} + {}^{36}/_{36} + {}^{64}/_{36} + {}^{4}/_{36} = {}^{112}/_{36} = 3.111$$

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Probability of Project Completion

- We know the standard deviation is just the square root of the variance, so:

$$\text{Project standard deviation} = \sigma_T = \sqrt{\text{Project variance}}$$
$$= \sqrt{3.11} = 1.76 \text{ weeks}$$

- We assume activity times are independent and that total project completion time is normally distributed.

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Probability Distribution for Project Completion Times

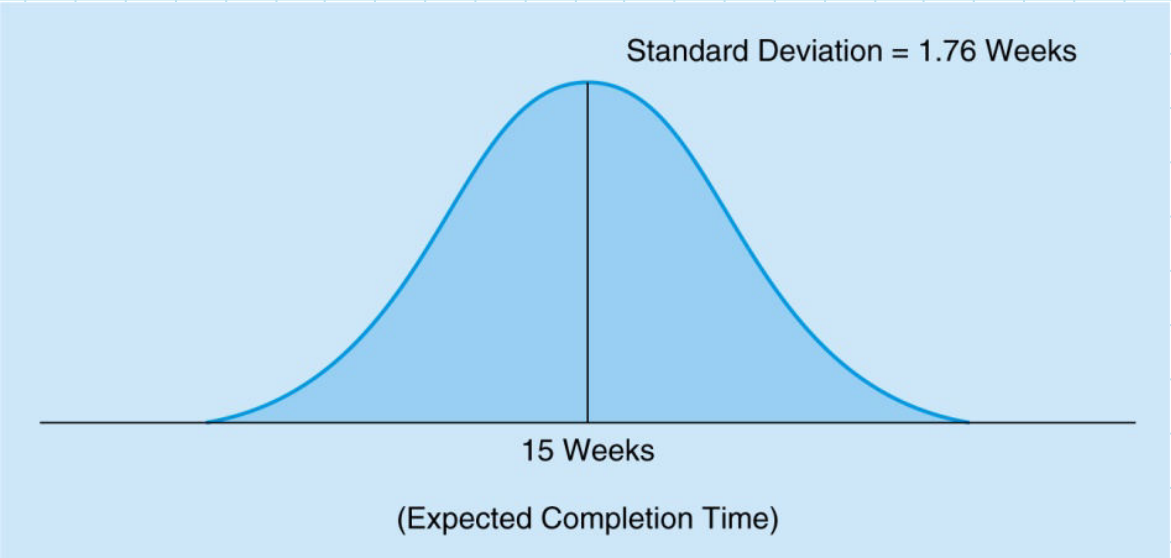

Figure 12.7

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Probability of Project Completion

The standard normal equation can be applied as follows:

$$Z = \frac{\text{Due date} - \text{Expected date of completion}}{\sigma_T}$$

$$= \frac{16 \text{ weeks} - 15 \text{ weeks}}{1.76 \text{ weeks}} = 0.57$$

- From Appendix A we find the probability of 0.71566 associated with this Z value.
- That means the probability this project can be completed in 16 weeks or less is 0.716.

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Probability of General Foundry Meeting the 16-week Deadline

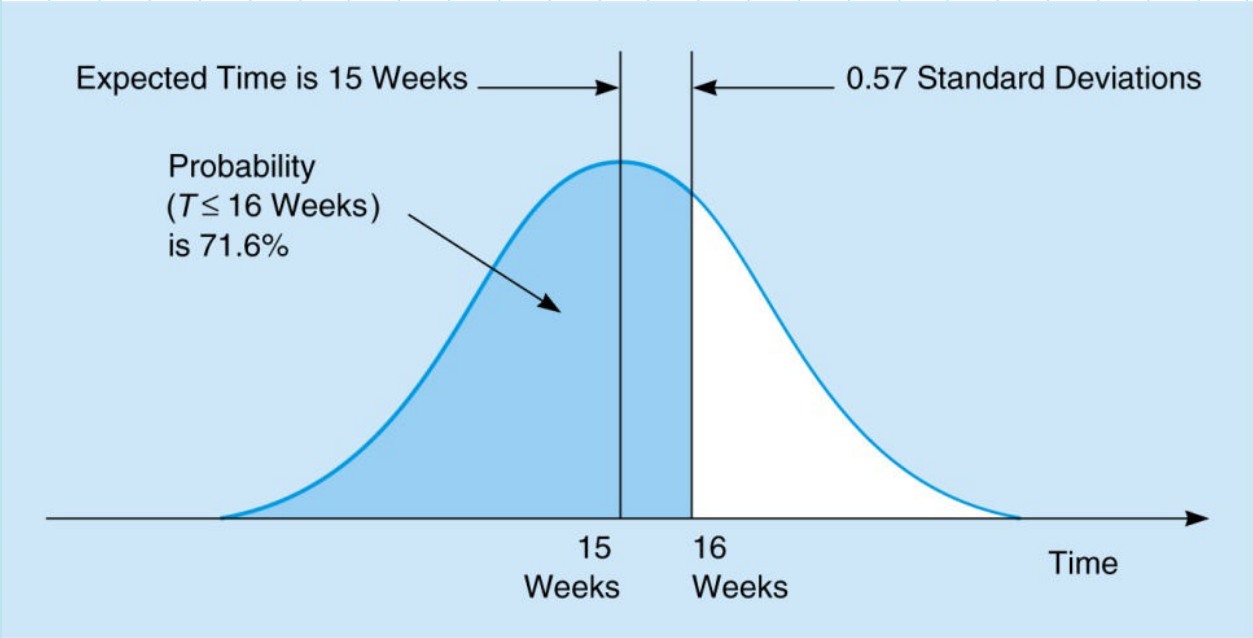

Figure 12.8

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What PERT Was Able to Provide

- PERT has been able to provide the project manager with several valuable pieces of information.
- The project's expected completion date is 15 weeks.
- There is a 71.6% chance that the equipment will be in place within the 16-week deadline.
- Five activities (*A*, *C*, *E*, *G*, *H*) are on the critical path.
- Three activities (*B*, *D*, *F*) are not critical but have some slack time built in.
- A detailed schedule of activity starting and ending dates has been made available.

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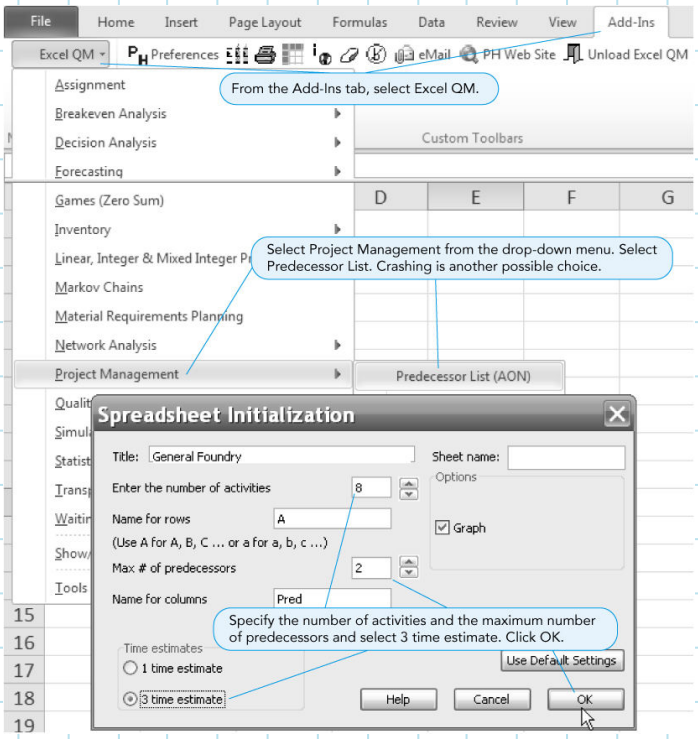
Using Excel QM in the General Foundry Example

Excel QM Initialization Screen for General Foundry Example with Three Time Estimates



Program 12.1A

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Using Excel QM in the General Foundry Example

Excel QM Input Screen and Solution for General Foundry Example with Three Time Estimates

General Foundry

Input the three time estimates for each activity.

Enter the times in the appropriate column(s). Enter the prec

Mean times and standard deviations are automatically calculated.

Data

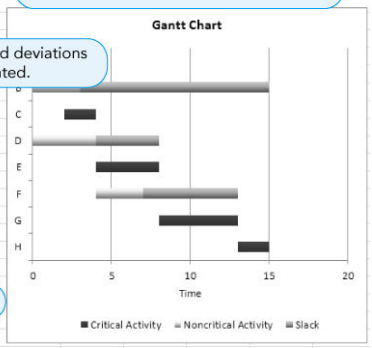
Activity	Optimistic	Likely	Pessimistic	Mean	Std dev	Variance
A	1	2	3	2	0.333333	0.111111
B	2	3	4	3	0.333333	0.111111
C	1	2	3	2	0.333333	0.111111
D	2	4	6	4	0.666667	0.444444
E	1	4	7	4	1	1
F	1	2	9	3	1.333333	1.777778
G	3	4	11	5	1.333333	1.777778
H	1	2	3	2	0.333333	0.111111

Precedences — Immediate Predecessors (1 per column)

Activity	Time	Pred 1	Pred 2
A	2		
B	3		
C	2	A	
D	4	B	
E	4	C	
F	3	C	
G	5	D	E
H	2	F	G

Enter the immediate predecessors.

The Gantt chart shows the critical path and the slack for the noncritical activities.



A table is displayed, with all the earliest, latest, and slack times.

Results

Activity	Early Start	Early Finish	Late Start	Late Finish	Slack
A	0	2	0	2	0
B	0	3	12	15	12
C	2	4	2	4	0
D	0	4	4	8	4
E	4	8	4	8	0
F	4	7	10	13	6
G	8	13	8	13	0
H	13	15	13	15	0
	Project	15			

	Variance
	0.1111111
	0.1111111
	1
	1.7777778
	0.1111111
Project	3.1111111

Program 12.1B

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Sensitivity Analysis and Project Management

- The time required to complete an activity can vary from the projected or expected time.
- If the activity is on the critical path, the completion time of the project will change.
- This will also have an impact on ES, EF, LS, and LF times for other activities.
- The exact impact depends on the relationship between the various activities.
- A *predecessor activity* is one that must be accomplished before the given activity can be started.
- A *successor activity* is one that can be started only after the given activity is finished.

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Sensitivity Analysis and Project Management

Impact of an Increase (Decrease) in an Activity Time for a Critical Path Activity

ACTIVITY TIME	SUCCESSOR ACTIVITY	PARALLEL ACTIVITY	PREDECESSOR ACTIVITY
Earliest start	Increase (decrease)	No change	No change
Earliest finish	Increase (decrease)	No change	No change
Latest start	Increase (decrease)	Increase (decrease)	No change
Latest finish	Increase (decrease)	Increase (decrease)	No change
Slack	No change	Increase (decrease)	No change

Table 12.4

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PERT/COST

- Although PERT is an excellent method of monitoring and controlling project length, it does not consider the very important factor of project cost.
- *PERT/Cost* is a modification of PERT that allows a manager to plan, schedule, monitor, and control cost as well as time.
- Using PERT/Cost to plan, schedule, monitor, and control project cost helps accomplish the sixth and final step of PERT.

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Planning and Scheduling Project Costs: Budgeting Process

- The overall approach in the budgeting process of a project is to determine how much is to be spent every week or month.
- This can be accomplished in four basic budgeting steps:

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Four Steps of the Budgeting Process

1. Identify all costs associated with each of the activities then add these costs together to get one estimated cost or budget for each activity.
2. In large projects, activities can be combined into larger work packages. A *work package* is simply a logical collection of activities.
3. Convert the budgeted cost per activity into a cost per time period by assuming that the cost of completing any activity is spent at a uniform rate over time.
4. Using the ES and LS times, find out how much money should be spent during each week or month to finish the project by the date desired.

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Budgeting for General Foundry

- The Gantt chart in Figure 12.9 illustrates this process.
- The horizontal bars shown when each activity will be performed based on its ES-EF times.
- We determine how much will be spent on each activity during each week and fill these amounts into a chart in place of the bars.
- The following two tables show the activity costs and budgeted cost for the General Foundry project.

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Budgeting for General Foundry

Gantt chart General Foundry project

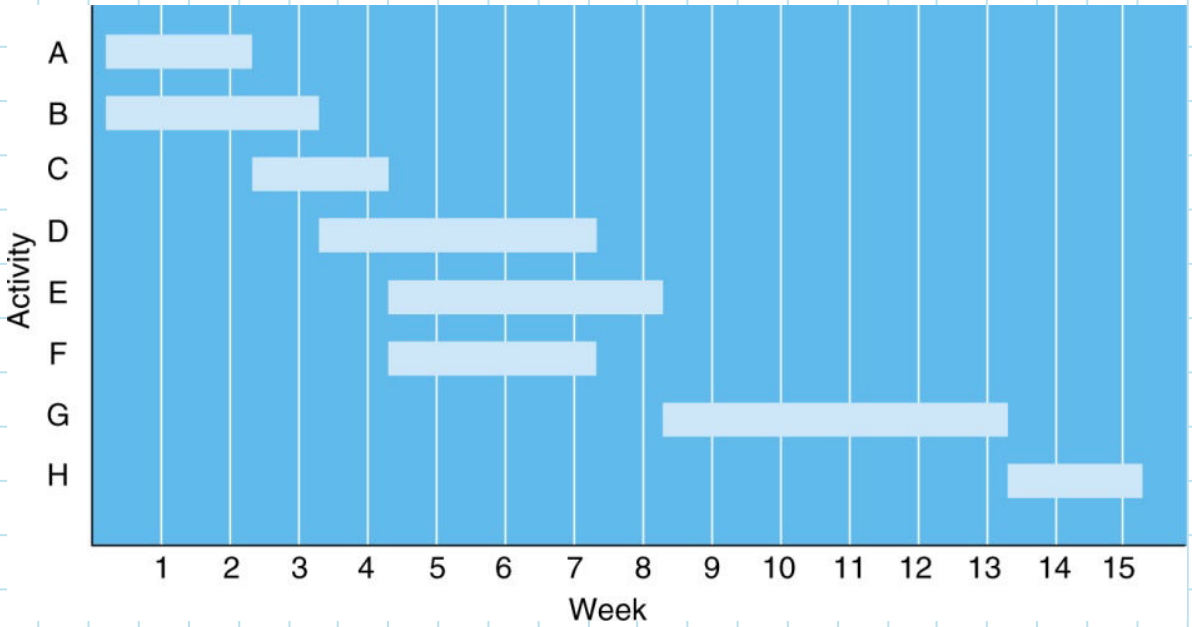

Figure 12.9

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Budgeting for General Foundry

Activity costs for General Foundry, Inc.

ACTIVITY	EARLIEST START, ES	LATEST START, LS	EXPECTED TIME, t	TOTAL BUDGETED COST ($)	BUDGETED COST PER WEEK ($)
A	0	0	2	22,000	11,000
B	0	1	3	30,000	10,000
C	2	2	2	26,000	13,000
D	3	4	4	48,000	12,000
E	4	4	4	56,000	14,000
F	4	10	3	30,000	10,000
G	8	8	5	80,000	16,000
H	13	13	2	16,000	8,000
			Total	308,000	

Table 12.5

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Budgeted Cost (Thousands of Dollars) for General Foundry, Inc., Using Earliest Start Times

ACTIVITY	WEEK 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	TOTAL
A	11	11														22
B	10	10	10													30
C			13	13												26
D				12	12	12	12									48
E					14	14	14	14								56
F					10	10	10									30
G									16	16	16	16	16			80
H														8	8	16
																308
Total per week	21	21	23	25	36	36	36	14	16	16	16	16	16	8	8	
Total to date	21	42	65	90	126	162	198	212	228	244	260	276	292	300	308	

Table 12.6

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Budgeting for General Foundry

- It is also possible to prepare a budget based on the latest starting time.
- This budget will delay the expenditure of funds until the last possible moment.
- The following table shows the latest start budget for the General Foundry project.
- The two tables form a budget range.
- Any budget can be chosen between these two values depending on when the company wants to actually spend the money.
- The budget ranges are plotted in Figure 12.10

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Budgeted Cost (Thousands of Dollars) for General Foundry, Inc., Using Latest Start Times

	WEEK															
ACTIVITY	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	TOTAL
A	11	11														22
B		10	10	10												30
C			13	13												26
D					12	12	12	12								48
E					14	14	14	14								56
F											10	10	10			30
G									16	16	16	16	16			80
H														8	8	16
																308
Total per week	11	21	23	23	26	26	26	26	16	16	26	26	26	8	8	
Total to date	11	32	55	78	104	130	156	182	198	214	240	266	292	300	308	

Table 12.7

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Budget Ranges for General Foundry

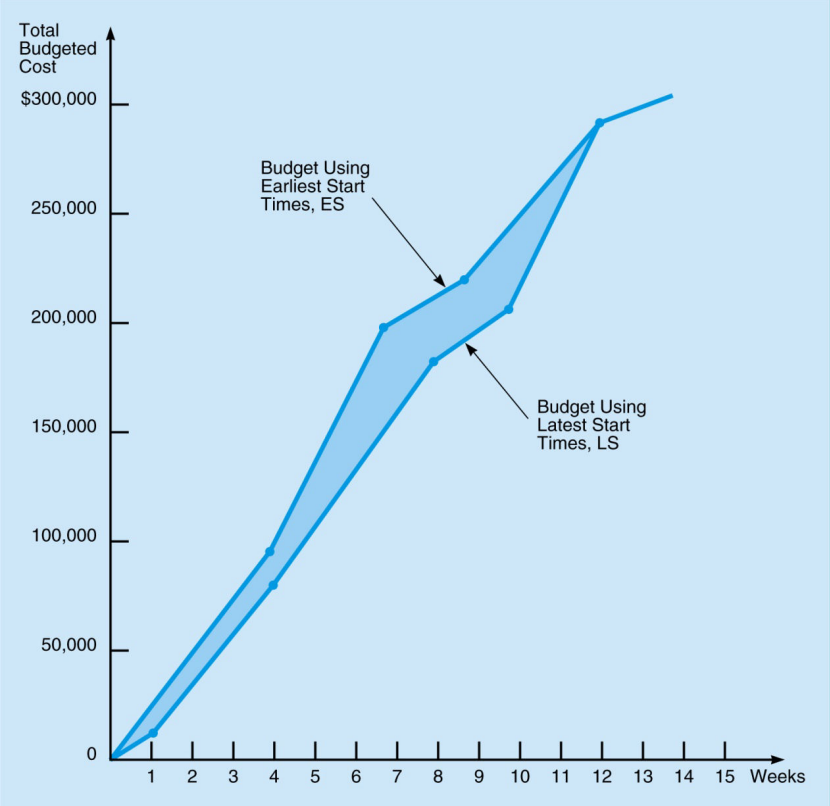

Figure 12.10

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Monitoring and Controlling Project Costs

- **Costs are monitored and controlled to ensure the project is progressing on schedule and that cost overruns are kept to a minimum.**
- **The status of the entire project should be checked periodically.**
- **The following table shows the state of the project in the sixth week.**
- **It can be used the answer questions about the schedule and costs so far.**

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Monitoring and Controlling Budgeted Cost

ACTIVITY	TOTAL BUDGETED COST ($)	PERCENT OF COMPLETION	VALUE OF WORK COMPLETED ($)	ACTUAL COST ($)	ACTIVITY DIFFERENCE ($)
A	22,000	100	22,000	20,000	–2,000
B	30,000	100	30,000	36,000	6,000
C	26,000	100	26,000	26,000	0
D	48,000	10	4,800	6,000	1,200
E	56,000	20	11,200	20,000	8,800
F	30,000	20	6,000	4,000	–2,000
G	80,000	0	0	0	0
H	16,000	0	0	0	0
		Total	100,000	112,000	12,000

Table 12.8

Overrun

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Monitoring and Controlling Project Costs

The value of work completed, or the cost to date for any activity, can be computed as follows:

Value of work completed = (Percentage of work complete) x (Total activity budget)

The activity difference is also of interest:

Activity difference = Actual cost – Value of work completed

A negative activity difference is a cost underrun and a positive activity difference is a cost overrun.

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Project Crashing

- Projects will sometimes have deadlines that are impossible to meet using normal procedures.
- By using exceptional methods it may be possible to finish the project in less time than normally required at a greater cost.
- Reducing a project's completion time is called *crashing.*

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Project Crashing

- Crashing a project starts with using the *normal time* to create the critical path.
- The *normal cost* is the cost for completing the activity using normal procedures.
- If the project will not meet the required deadline, extraordinary measures must be taken.
  - The *crash time* is the shortest possible activity time and will require additional resources.
  - The *crash cost* is the price of completing the activity in the earlier-than-normal time.

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Four Steps to Project Crashing

1. Find the normal critical path and identify the critical activities.
2. Compute the crash cost per week (or other time period) for all activities in the network using the formula:

$$\text{Crash cost/Time period} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

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Four Steps to Project Crashing

3. Select the activity on the critical path with the smallest crash cost per week and crash this activity to the maximum extent possible or to the point at which your desired deadline has been reached.
4. Check to be sure that the critical path you were crashing is still critical. If the critical path is still the longest path through the network, return to step 3. If not, find the new critical path and return to step 2.

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General Foundry

- Suppose that General Foundry has been given 14 weeks instead of 16 weeks to install the new equipment.
- The critical path for the project is 15 weeks.
- What options does the firm have?
  - The normal and crash times and costs are shown in Table 12.9.
  - Crash costs are assumed to be linear and Figure 12.11 shows the crash cost for activity *B*.
- Crashing activities *B* and *A* will shorten the completion time to 14 but it creates a second critical path.
- Any further crashing must be done to both critical paths.

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General Foundry

Normal and Crash Data for General Foundry, Inc.

	TIME (WEEKS)		COST ($)		CRASH COST PER WEEK ($)	CRITICAL PATH?
ACTIVITY	NORMAL	CRASH	NORMAL	CRASH		
A	2	1	22,000	23,000	1,000	Yes
B	3	1	30,000	34,000	2,000	No
C	2	1	26,000	27,000	1,000	Yes
D	4	3	48,000	49,000	1,000	No
E	4	2	56,000	58,000	1,000	Yes
F	3	2	30,000	30,500	500	No
G	5	2	80,000	86,000	2,000	Yes
H	2	1	16,000	19,000	3,000	Yes

Table 12.9

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General Foundry

Crash and Normal Times and Costs for Activity *B*

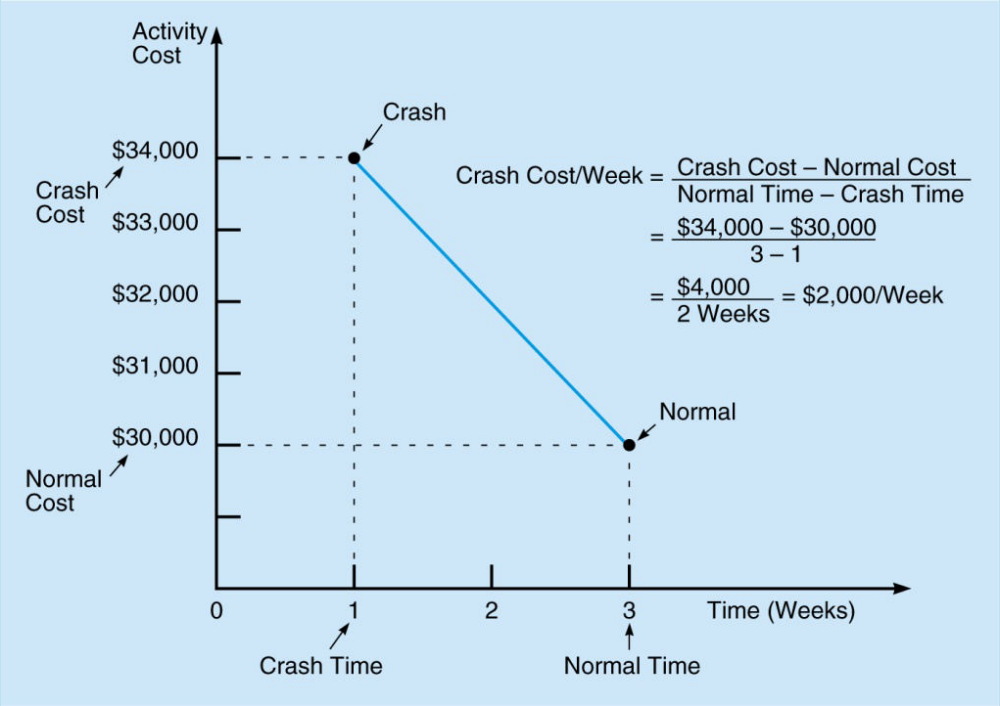

Figure 12.11

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Project Crashing with Linear Programming

- **Linear programming is another approach to finding the best project crashing schedule.**
- **The data needed are derived from the normal and crash data for General Foundry and the project network with activity times.**

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Project Crashing with Linear Programming

General Foundry's Network With Activity Times

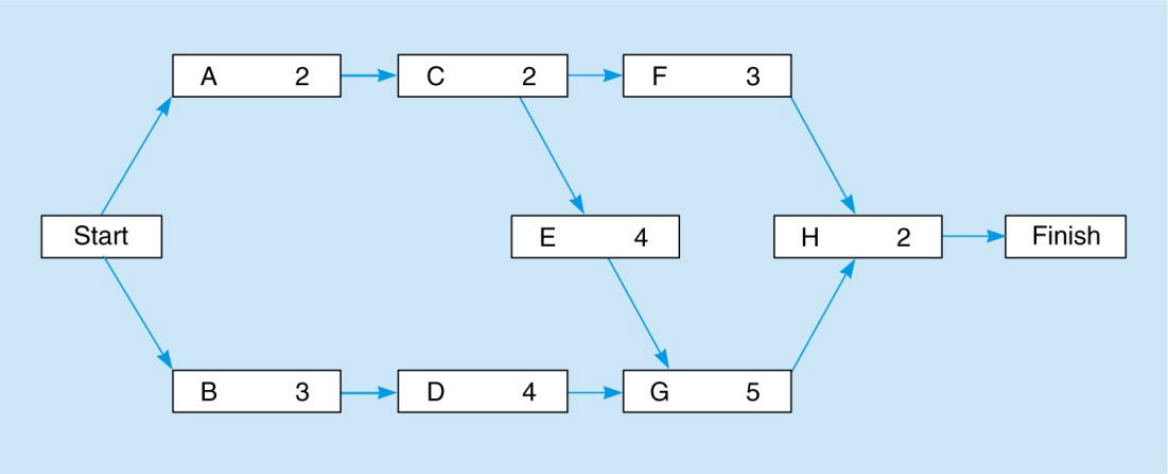

Figure 12.12

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Project Crashing with Linear Programming

The decision variables for the problem are:

X_A = EF for activity *A*
X_B = EF for activity *B*
X_C = EF for activity *C*
X_D = EF for activity *D*
X_E = EF for activity *E*
X_F = EF for activity *F*
X_G = EF for activity *G*
X_H = EF for activity *H*
X_{start} = start time for project (usually 0)
X_{finish} = earliest finish time for the project

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Project Crashing with Linear Programming

- Additional decision variables for the problem are:

  Y = the number of weeks that each activity is crashed.

  Y_A = the number of weeks activity *A* is crashed and so forth up to Y_H.

- The objective function is

$$\text{Minimize crash cost} = 1{,}000Y_A + 2{,}000Y_B + 1{,}000Y_C + 1{,}000Y_D + 1{,}000Y_E + 500Y_F + 2{,}000Y_G + 3{,}000Y_H$$

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Project Crashing with Linear Programming

- Crash time constraints ensure activities are not crashed more than is allowed.

$$Y_A \le 1$$
$$Y_B \le 2$$
$$Y_C \le 1$$
$$Y_D \le 1$$
$$Y_E \le 2$$
$$Y_F \le 1$$
$$Y_G \le 3$$
$$Y_H \le 1$$

- This completion constraint specifies that the last event must take place before the project deadline:

$$X_{\text{finish}} \le 12$$

- This constraint indicates the project is finished when activity *H* is finished:

$$X_{\text{finish}} \ge X_H$$

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Project Crashing with Linear Programming

Constraints describing the network have the form:

EF time ≥ EF time for predecessor + Activity time

$$\text{EF} \geq \text{EF}_{\text{predecessor}} + (t - Y), \text{ or}$$

$$X \geq X_{\text{predecessor}} + (t - Y)$$

For activity *A*,	$X_A \geq X_{\text{start}} + (2 - Y_A)$	or	$X_A - X_{\text{start}} + Y_A \geq 2$
For activity *B*,	$X_B \geq X_{\text{start}} + (3 - Y_B)$	or	$X_B - X_{\text{start}} + Y_B \geq 3$
For activity *C*,	$X_C \geq X_A + (2 - Y_C)$	or	$X_C - X_A + Y_C \geq 2$
For activity *D*,	$X_D \geq X_B + (4 - Y_D)$	or	$X_D - X_B + Y_D \geq 4$
For activity *E*,	$X_E \geq X_C + (4 - Y_E)$	or	$X_E - X_C + Y_E \geq 4$
For activity *F*,	$X_F \geq X_C + (3 - Y_F)$	or	$X_F - X_C + Y_F \geq 3$
For activity *G*,	$X_G \geq X_D + (5 - Y_G)$	or	$X_G - X_D + Y_G \geq 5$
For activity *G*,	$X_G \geq X_E + (5 - Y_G)$	or	$X_G - X_E + Y_G \geq 5$
For activity *H*,	$X_H \geq X_F + (2 - Y_H)$	or	$X_H - X_F + Y_H \geq 2$
For activity *H*,	$X_H \geq X_G + (2 - Y_H)$	or	$X_H - X_G + Y_H \geq 2$

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Solution to Crashing Problem Using Solver in Excel

Activity A is crashed 1 week, and activity G is crashed 2 weeks. There are other optimal solutions to this problem.

T4 = =SUMPRODUCT($B$3:$S$3,

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1	Crashing General Foundry Problem																						
2		YA	YB	YC	YD	YE	YF	YG	YH	XST	XA	XB	XC	XD	XE	XF	XG	XH	XFIN				
3	Values	1	0	0	0	0	0	2	0	0	1	3	3	7	7	10	10	12	12	Totals			
4	Minimize cost	1000	2000	1000	1000	1000	500	2000	3000											5000			
5	A crash max.	1																		1	<	1	
6	B crash max.		1																	0	<	2	
7	C crash max.			1																0	<	1	
8	D crash max.				1															0	<	1	
9	E crash max.					1														0	<	2	
10	F crash max.						1													0	<	1	
11	G crash max.							1												2	<	3	
12	H crash max.								1											0	<	1	
13	Due date																		1	12	<	12	
14	Start									1										0	=	0	
15	A constraint	1								-1	1									2	>	2	
16	B constraint		1							-1		1								3	>	3	
17	C constraint			1							-1		1							2	>	2	
18	D constraint				1							-1		1						4	>	4	
19	E constraint					1							-1		1					4	>	4	
20	F constraint						1						-1			1				7	>	3	
21	G constaint 1							1						-1			1			5	>	5	
22	G constraint 2							1							-1		1			5	>	5	
23	H constraint 1								1							-1		1		2	>	2	
24	H constraint 2								1								-1	1		2	>	2	
25	Finish constraint																	-1	1	0	>	0	

The total cost of crashing is $5,000.

The formula in cell T4 is copied down column T (from T5 to T25).

Program 12.2

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Other Topics in Project Management

- **Subprojects**
  - **For extremely large projects, an activity may be made of several smaller sub-activities which can be viewed as a smaller project or subproject of the original .**
- **Milestones**
  - **Major events in a project are often referred to as *milestones* and may be reflected in *Gantt charts* and PERT charts to highlight the importance of reaching these events.**

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Other Topics in Project Management

- **Resource Leveling**
  - ***Resource leveling* adjusts the activity start away from the early start so that resource utilization is more evenly distributed over time.**
- **Software**
  - **There are many project management software packages on the market for both personal computers and larger mainframe machines.**
  - **Most of these create PERT charts and Gantt charts and can be used to develop budget schedules, adjust future start times, and level resource utilization.**

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Chapter 14

Simulation Modeling

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. Tackle a wide variety of problems by simulation.
2. Understand the seven steps of conducting a simulation.
3. Explain the advantages and disadvantages of simulation.
4. Develop random number intervals and use them to generate outcomes.
5. Understand alternative simulation packages available.

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Chapter Outline

14.1 Introduction
14.2 Advantages and Disadvantages of Simulation
14.3 Monte Carlo Simulation
14.4 Simulation and Inventory Analysis
14.5 Simulation of a Queuing Problem
14.6 Simulation Model for a Maintenance Policy
14.7 Other Simulation Issues

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Introduction

- ***Simulation*** is one of the most widely used quantitative analysis tools.
- To ***simulate*** is to try to duplicate the features, appearance, and characteristics of a real system.
- We will build a ***mathematical model*** that comes as close as possible to representing the reality of the system.
- ***Physical*** models can also be built to test systems.

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Introduction

Using simulation, a manager should:

1. Define a problem.
2. Introduce the variables associated with the problem.
3. Construct a simulation model.
4. Set up possible courses of action for testing.
5. Run the simulation experiment.
6. Consider the results.
7. Decide what courses of action to take.

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Process of Simulation

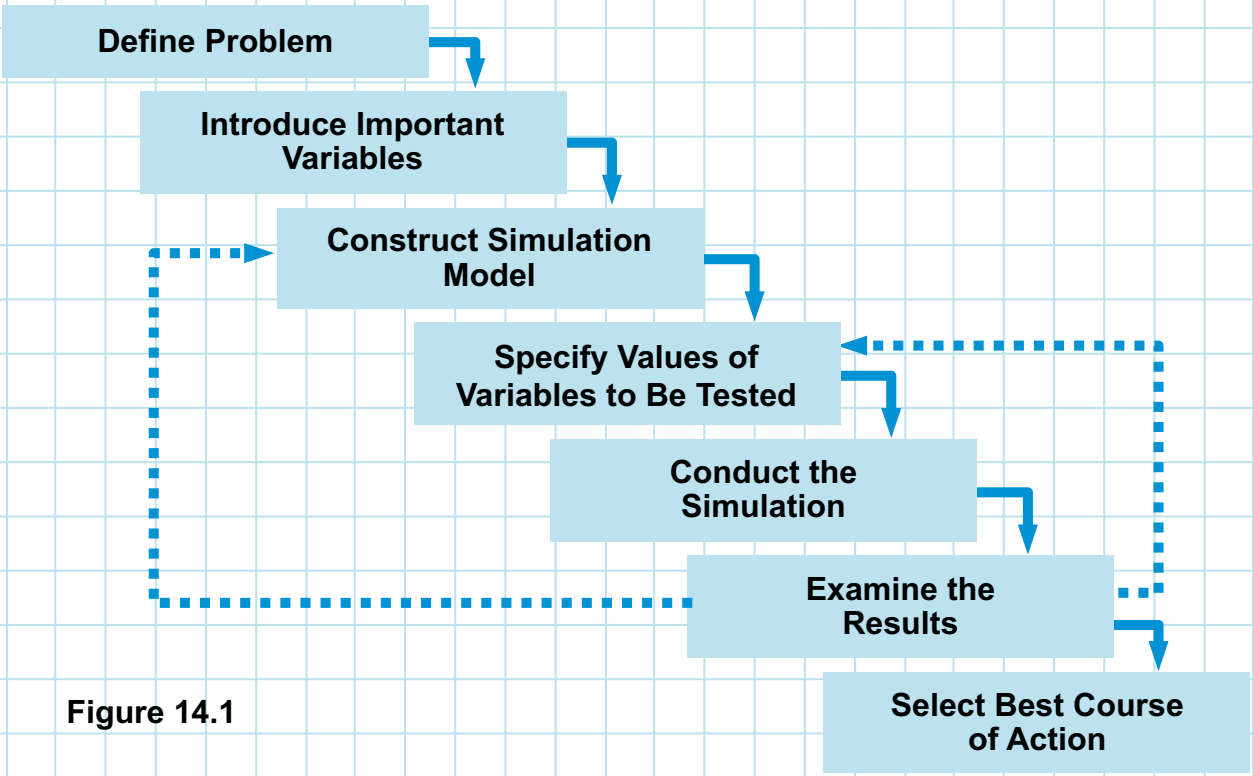

Figure 14.1

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Advantages and Disadvantages of Simulation

The main advantages of simulation are:

1. It is relatively straightforward and flexible.
2. Recent advances in computer software make simulation models very easy to develop.
3. Can be used to analyze large and complex real-world situations.
4. Allows “what-if?” type questions.
5. Does not interfere with the real-world system.
6. Enables study of interactions between components.
7. Enables time compression.
8. Enables the inclusion of real-world complications.

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Advantages and Disadvantages of Simulation

The main disadvantages of simulation are:

1. It is often expensive as it may require a long, complicated process to develop the model.
2. It does not generate optimal solutions; it is a trial-and-error approach.
3. It requires managers to generate all conditions and constraints of real-world problem.
4. Each model is unique and the solutions and inferences are not usually transferable to other problems.

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Monte Carlo Simulation

- When systems contain elements that exhibit chance in their behavior, the Monte Carlo method of simulation can be applied.
- Some examples are:
  1. Inventory demand.
  2. Lead time for inventory.
  3. Times between machine breakdowns.
  4. Times between arrivals.
  5. Service times.
  6. Times to complete project activities.
  7. Number of employees absent.

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Monte Carlo Simulation

- The basis of the Monte Carlo simulation is experimentation on the probabilistic elements through random sampling.
- It is based on the following five steps:
  1. Establishing a probability distribution for important variables.
  2. Building a cumulative probability distribution for each variable.
  3. Establishing an interval of random numbers for each variable.
  4. Generating random numbers.
  5. Actually simulating a series of trials.

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Harry's Auto Tire

- A popular radial tire accounts for a large portion of the sales at Harry's Auto Tire.
- Harry wishes to determine a policy for managing this inventory.
- He wants to simulate the daily demand for a number of days.

Step 1: Establishing probability distributions

- One way to establish a probability distribution for a given variable is to examine historical outcomes.
- Managerial estimates based on judgment and experience can also be used.

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Harry's Auto Tire

Historical Daily Demand for Radial Tires at Harry's Auto Tire and Probability Distribution

DEMAND FOR TIRES	FREQUENCY (DAYS)	PROBAILITY OF OCCURRENCE
0	10	10/200 = 0.05
1	20	20/200 = 0.10
2	40	40/200 = 0.20
3	60	60/200 = 0.30
4	40	40/200 = 0.20
5	30	30/200 = 0.15
	200	200/200 = 1.00

Table 14.1

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Harry's Auto Tire

Step 2: Building a cumulative probability distribution for each variable

- Converting from a regular probability to a cumulative distribution is an easy job.
- A cumulative probability is the probability that a variable will be less than or equal to a particular value.
- A cumulative distribution lists all of the possible values and the probabilities, as shown in Table 14.2.

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Harry's Auto Tire

Cumulative Probabilities for Radial Tires

DAILY DEMAND	PROBABILITY	CUMULATIVE PROBABILITY
0	0.05	0.05
1	0.10	0.15
2	0.20	0.35
3	0.30	0.65
4	0.20	0.85
5	0.15	1.00

Table 14.2

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Harry's Auto Tire

Step 3: Setting random number intervals

- Assign a set of numbers to represent each possible value or outcome.
  - These are *random number intervals.*
  - A *random number* is a series of digits that have been selected by a totally random process.
  - The range of the random number intervals corresponds *exactly* to the probability of the outcomes as shown in Figure 14.2.

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Harry's Auto Tire

Graphical Representation of the Cumulative Probability Distribution for Radial Tires

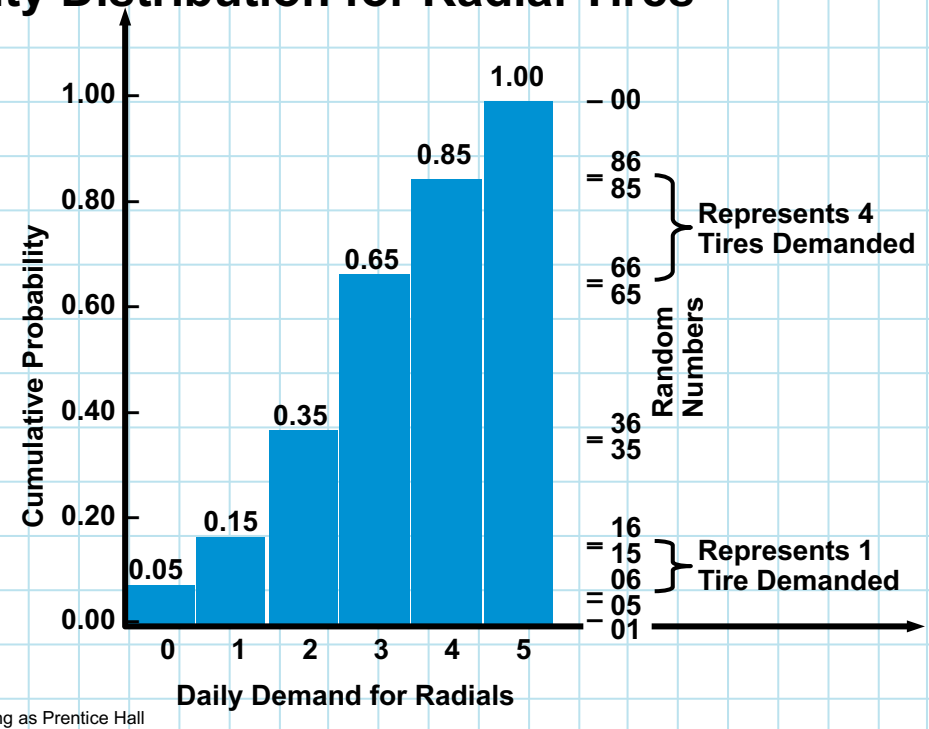

Figure 14.2

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Harry's Auto Tire

Assignment of Random Number Intervals for Harry's Auto Tire

DAILY DEMAND	PROBABILITY	CUMULATIVE PROBABILITY	INTERVAL OF RANDOM NUMBERS
0	0.05	0.05	01 to 05
1	0.10	0.15	06 to 15
2	0.20	0.35	16 to 35
3	0.30	0.65	36 to 65
4	0.20	0.85	66 to 85
5	0.15	1.00	86 to 00

Table 14.3

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Harry's Auto Tire

Step 4: Generating random numbers

- Random numbers can be generated in several ways.
- Large problems will use computer program to generate the needed random numbers.
- For small problems, random processes like roulette wheels or pulling chips from a hat may be used.
- The most common manual method is to use a random number table.
- Because *everything* is random in a random number table, we can select numbers from anywhere in the table to use in the simulation.

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Harry's Auto Tire

Table of random numbers (partial)

52	06	50	88	53	30	10	47	99	37
37	63	28	02	74	35	24	03	29	60
82	57	68	28	05	94	03	11	27	79
69	02	36	49	71	99	32	10	75	21
98	94	90	36	06	78	23	67	89	85
96	52	62	87	49	56	59	23	78	71
33	69	27	21	11	60	95	89	68	48
50	33	50	95	13	44	34	62	64	39
88	32	18	50	62	57	34	56	62	31
90	30	36	24	69	82	51	74	30	35

Table 14.4

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Harry's Auto Tire

Step 5: Simulating the experiment

- We select random numbers from Table 14.4.
- The number we select will have a corresponding range in Table 14.3.
- We use the daily demand that corresponds to the probability range aligned with the random number.

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Harry's Auto Tire

Ten-day Simulation of Demand for Radial Tires

DAY	RANDOM NUMBER	SIMULATED DAILY DEMAND
1	52	3
2	37	3
3	82	4
4	69	4
5	98	5
6	96	5
7	33	2
8	50	3
9	88	5
10	90	5
		39 = total 10-day demand
		3.9 = average daily demand for tires

Table 14.5

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Harry's Auto Tire

Note that the average demand from this simulation (3.9 tires) is different from the *expected* daily demand.

$$\text{Expected daily demand} = \sum_{i=0}^{5} (\text{Probability of } i \text{ tires})(\text{Demand of } i \text{ tires})$$

$$= (0.05)(0) + (0.10)(1) + (0.20)(2) + (0.30)(3) + (0.20)(4) + (0.15)(5)$$

$$= 2.95 \text{ tires}$$

If this simulation were repeated hundreds or thousands of times it is much more likely the average *simulated* demand would be nearly the same as the *expected* demand.

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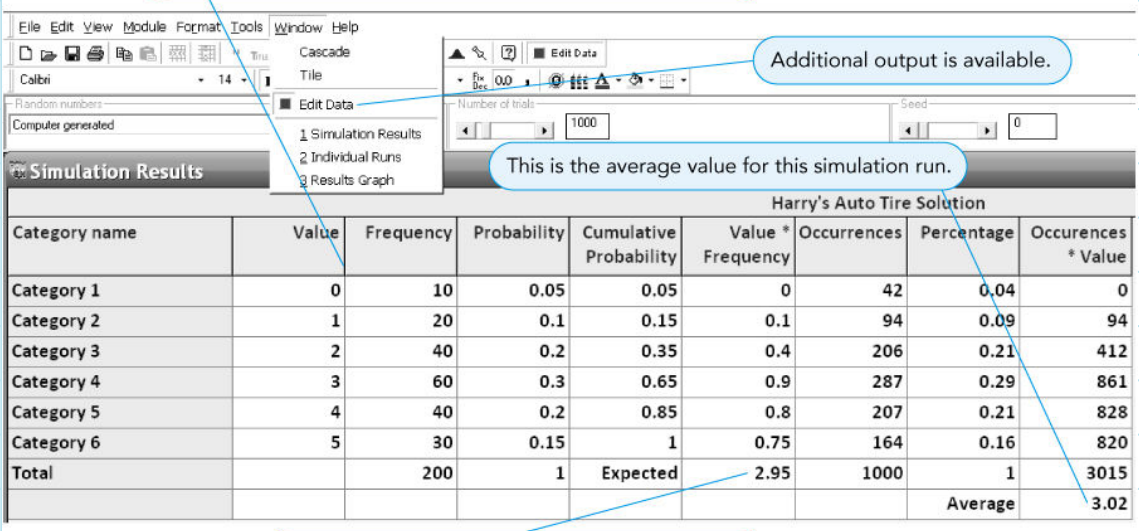
QM for Windows Output Screen for Simulation of Harry's Auto Tire Example

Once the input screen appears, enter the values and the frequencies. The probabilities will automatically be calculated.



Additional output is available.

This is the average value for this simulation run.

Category name	Value	Frequency	Probability	Cumulative Probability	Value * Frequency	Occurrences	Percentage	Occurences * Value
Category 1	0	10	0.05	0.05	0	42	0.04	0
Category 2	1	20	0.1	0.15	0.1	94	0.09	94
Category 3	2	40	0.2	0.35	0.4	206	0.21	412
Category 4	3	60	0.3	0.65	0.9	287	0.29	861
Category 5	4	40	0.2	0.85	0.8	207	0.21	828
Category 6	5	30	0.15	1	0.75	164	0.16	820
Total		200	1	Expected	2.95	1000	1	3015
							Average	3.02

The expected value is calculated mathematically.

Program 14.1

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Simulation with Excel Spreadsheets

Using Excel 2010 to Simulate Tire Demand for Harry's Auto Tire Shop

	A	B	C	D	E	F	G	H	I
1	Harry's Auto Tire Shop								
2		Probability	Probability Range (Lower)	Cumulative Probability	Tires Demand		Day	Random Number	Simulated Demand
3		0.05	0	0.05	0		1	0.628711	3
4		0.1	0.05	0.15	1		2	0.402931	3
5		0.2	0.15	0.35	2		3	0.419694	3
6		0.3	0.35	0.65	3		4	0.645743	3
7		0.2	0.65	0.85	4		5	0.446755	3
8		0.15	0.85	1	5		6	0.022622	0
9							7	0.216480	2
10							8	0.901222	5
11							9	0.794447	4
12							10	0.530363	3
13								Average =	2.9
14		Results (Frequency table)							
15		Tires Demanded	Frequency	Percentage	Cumulative %				
16		0	1	10%	10%				
17		1	0	0%	10%				
18		2	1	10%	20%				
19		3	6	60%	80%				
20		4	1	10%	90%				
21		5	1	10%	100%				
22			10	=Total					

Program 14.2

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Simulation with Excel Spreadsheets

Using Excel 2010 to Simulate Tire Demand for Harry's Auto Tire Shop

	I
13	=AVERAGE(I3:I12)

	C
22	=SUM(C16:C21)

	C	D
3	0	=B3
4	=D3	=D3+B4

	H	I
3	=RAND()	=VLOOKUP(H3,$C$3:$E$8,3,TRUE)

	C
16	=FREQUENCY($I$3:$I$12,$B$16:$B$21)

	D	E
16	=C16/$C$22	=D16
17	=C17/$C$22	=E16+D17

Program 14.2

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Simulation with Excel Spreadsheets

Generating Normal Random Numbers in Excel

	A	B	C	D	E
1	**Generating Normal Random Numbers**				
2					
3	Random number		Value	Frenquency	Percentage
4	39.54		26	0	0.0%
5	42.84		28	1	0.5%
6	36.49		30	3	1.5%
7	38.69		32	7	3.5%
8	44.15		34	14	7.0%
9	54.04		36	9	4.5%
10	32.99		38	31	15.5%
11	37.97		40	38	19.0%
12	43.47		42	31	15.5%
13	37.76		44	25	12.5%
14	38.30		46	26	13.0%
15	39.21		48	5	2.5%
16	40.08		50	6	3.0%
17	32.31		52	2	1.0%
18	39.30		54	1	0.5%
19	40.24		56	1	0.5%
20	39.35			**200**	**= Total**
201	41.15				
202	41.03		***(Rows 21-200 are hidden)***		
203	33.02				

	A
4	=NORMINV(RAND(),40,5)

	E
4	=D4/$D$20

	D
4	=FREQUENCY(A4:A203,C4:C19)

Program 14.3

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Simulation with Excel Spreadsheets

Excel QM Simulation of Harry's Auto Tire Example

Enter the values and the frequencies. The probabilities and simulation results will then appear.

Harry's Auto Tire

Enter the values and the frequencies in the top table. Press F9 to run another simulation. If you like, you may enter the random numbers in the column labeled "Random number".

Simulation

	Random Number Sorter	Data: Category name	Value	Frequency	Expected Value: Probability	Cumulative Probability	Value * Frequency	Simulation results: Value	Simulation Occurrences	Percentage	Occurences * Value
8	0	Category 1	0	10	0.05	0.05	0	0	6	0.03	0
9	5	Category 2	1	20	0.1	0.15	20	1	22	0.11	22
10	15	Category 3	2	40	0.2	0.35	80	2	34	0.17	68
11	35	Category 4	3	60	0.3	0.65	180	3	63	0.315	189
12	65	Category 5	4	40	0.2	0.85	160	4	49	0.245	196
13	85	Category 6	5	30	0.15	1	150	5	26	0.13	130
14		Total		200		Expected		Totals	200	1	605
15										Average	3.025

Program 14.4

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Simulation and Inventory Analysis

- We have seen deterministic inventory models.
- In many real-world inventory situations, demand and lead time are variables.
- Accurate analysis is difficult without simulation.
- We will look at an inventory problem with two decision variables and two probabilistic components.
- The owner of a hardware store wants to establish *order quantity* and *reorder point* decisions for a product that has probabilistic daily demand and reorder lead time.

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Simkin's Hardware Store

- The owner of a hardware store wants to find a good, low cost inventory policy for an electric drill.
- Simkin identifies two types of variables, controllable and uncontrollable inputs.
- The controllable inputs are the order quantity and reorder points.
- The uncontrollable inputs are daily demand and variable lead time.
- The demand data for the drill is shown in Table 14.6.

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Simkin's Hardware Store

Probabilities and Random Number Intervals for Daily Ace Drill Demand

(1) DEMAND FOR ACE DRILL	(2) FREQUENCY (DAYS)	(3) PROBABILITY	(4) CUMULATIVE PROBABILITY	(5) INTERVAL OF RANDOM NUMBERS
0	15	0.05	0.05	01 to 05
1	30	0.10	0.15	06 to 15
2	60	0.20	0.35	16 to 35
3	120	0.40	0.75	36 to 75
4	45	0.15	0.90	76 to 90
5	30	0.10	1.00	91 to 00
	300	1.00		

Table 14.6

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Simkin's Hardware Store

Probabilities and Random Number Intervals for Reorder Lead Time

(1) LEAD TIME (DAYS)	(2) FREQUENCY (ORDERS)	(3) PROBABILITY	(4) CUMULATIVE PROBABILITY	(5) RANDOM NUMBER INTERVAL
1	10	0.20	0.20	01 to 20
2	25	0.50	0.70	21 to 70
3	15	0.30	1.00	71 to 00
	50	1.00		

Table 14.7

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Simkin's Hardware Store

- The third step is to develop a simulation model.
- A flow diagram, or flowchart, is helpful in this process.
- The fourth step in the process is to specify the values of the variables that we wish to test.
- The first policy that Simkin wants to test is an order quantity of 10 with a reorder point of 5.
- The fifth step is to actually conduct the simulation.
- The process is simulated for a 10 day period.

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Flow Diagram for Simkin's Inventory Example

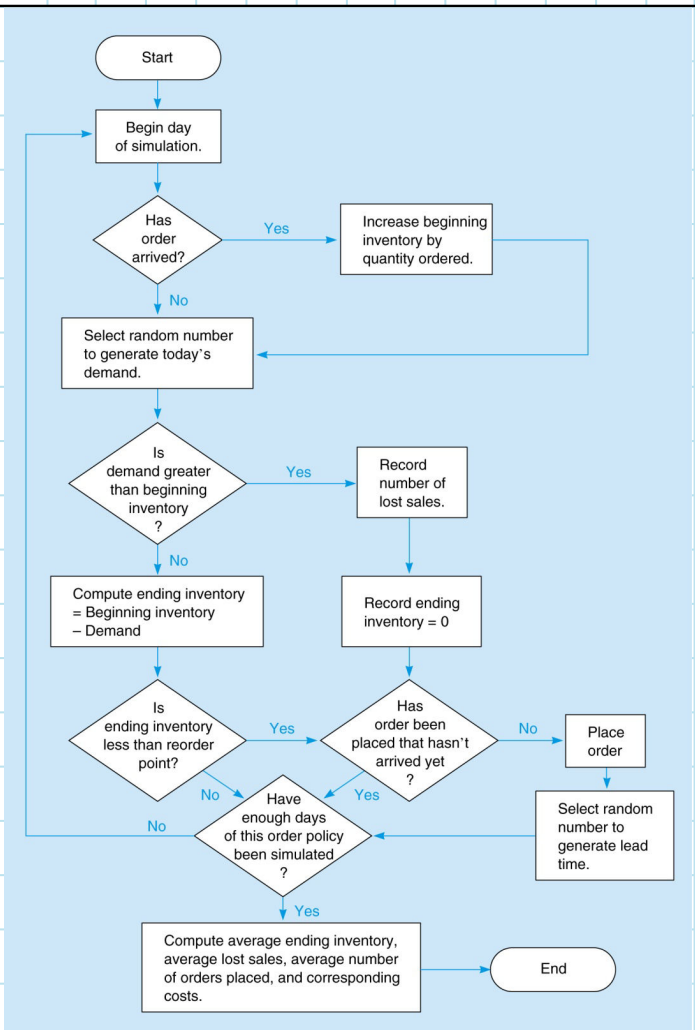

Figure 14.3

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Simkin's Hardware Store

Using the table of random numbers, the simulation is conducted using a four-step process:

1. **Begin each day by checking whether an ordered inventory has arrived. If it has, increase the current inventory by the quantity ordered.**
2. **Generate a daily demand from the demand probability by selecting a random number.**
3. **Compute the ending inventory every day. If on-hand inventory is insufficient to meet the day's demand, satisfy as much as possible and note the number of lost sales.**
4. **Determine whether the day's ending inventory has reached the reorder point. If necessary place an order.**

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Simkin's Hardware Store

Simkin Hardware's First Inventory Simulation

ORDER QUANTITY = 10 UNITS REORDER POINT = 5 UNITS

(1) DAY	(2) UNITS RECEIVED	(3) BEGINNING INVENTORY	(4) RANDOM NUMBER	(5) DEMAND	(6) ENDING INVENTORY	(7) LOST SALES	(8) ORDER	(9) RANDOM NUMBER	(10) LEAD TIME
1	…	10	06	1	9	0	No		
2	0	9	63	3	6	0	No		
3	0	6	57	3	3	0	Yes	02	1
4	0	3	94	5	0	2	No		
5	10	10	52	3	7	0	No		
6	0	7	69	3	4	0	Yes	33	2
7	0	4	32	2	2	0	No		
8	0	2	30	2	0	0	No		
9	10	10	48	3	7	0	No		
10	0	7	88	4	3	0	Yes	14	1
					Total 41	2			

Table 14.8

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Analyzing Simkin's Inventory Cost

- The objective is to find a low-cost solution so Simkin must determine the costs.
- Equations for average daily ending inventory, average lost sales, and average number of orders placed.

Average ending inventory $= \dfrac{41 \text{ total units}}{10 \text{ days}} = 4.1 \text{ units per day}$

Average lost sales $= \dfrac{2 \text{ sales lost}}{10 \text{ days}} = 0.2 \text{ unit per day}$

Average number of orders placed $= \dfrac{3 \text{ orders}}{10 \text{ days}} = 0.3 \text{ order per day}$

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Analyzing Simkin's Inventory Cost

- Simkin's store is open 200 days a year.
- Estimated ordering cost is $10 per order.
- Holding cost is $6 per drill per year.
- Lost sales cost $8.

Daily order cost = (Cost of placing one order) x (Number of orders placed per day)
= $10 per order x 0.3 order per day = $3

Daily holding cost = (Cost of holding one unit for one day) x (Average ending inventory)
= $0.03 per unit per day x 4.1 units per day
= $0.12

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Analyzing Simkin's Inventory Cost

- Simkin's store is open 200 days a year.
- Estimated ordering cost is $10 per order.
- Holding cost is $6 per drill per year.
- Lost sales cost $8.

Daily stockout cost = (Cost per lost sale) x (Average number of lost sales per day)
= $8 per lost sale x 0.2 lost sales per day
= $1.60

Total daily inventory cost = Daily order cost + Daily holding cost + Daily stockout cost
= $4.72

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Analyzing Simkin's Inventory Cost

- For the year, this policy would cost approximately $944.
- This simulation should really be extended for many more days, perhaps 100 or 1,000 days.
- Even after a larger simulation, the model must be verified and validated to make sure it truly represents the situation on which it is based.
- If we are satisfied with the model, additional simulations can be conducted using other values for the variables.
- After simulating all reasonable combinations, Simkin would select the policy that results in the lowest total cost.

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Simulation of a Queuing Problem

- Modeling waiting lines is an important application of simulation.
- The assumptions of queuing models are quite restrictive.
- Sometimes simulation is the only approach that fits.
- In this example, arrivals do not follow a Poisson distribution and unloading rates are not exponential or constant.

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Port of New Orleans

- Fully loaded barges arrive at night for unloading.
- The number of barges each night varies from 0 – 5, and the number of barges vary from day to day.
- The supervisor has information which can be used to create a probability distribution for the daily unloading rate.
- Barges are unloaded first-in, first-out.
- Barges must wait for unloading which is expensive.
- The dock superintendent wants to do a simulation study to enable him to make better staffing decisions.

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Port of New Orleans

Overnight Barge Arrival Rates and Random Number Intervals

NUMBER OF ARRIVALS	PROBABILITY	CUMULATIVE PROBABILITY	RANDOM NUMBER INTERVAL
0	0.13	0.13	01 to 13
1	0.17	0.30	14 to 30
2	0.15	0.45	31 to 45
3	0.25	0.70	46 to 70
4	0.20	0.90	71 to 90
5	0.10	1.00	91 to 00

Table 14.9

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Port of New Orleans

Unloading Rates and Random Number Intervals

DAILY UNLOADING RATE	PROBABILITY	CUMULATIVE PROBABILITY	RANDOM NUMBER INTERVAL
1	0.05	0.05	01 to 05
2	0.15	0.20	06 to 20
3	0.50	0.70	21 to 70
4	0.20	0.90	71 to 90
5	0.10	1.00	91 to 00
	1.00		

Table 14.10

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Queuing Simulation of Port of New Orleans Barge Unloadings

(1) DAY	(2) NUMBER DELAYED FROM PREVIOUS DAY	(3) RANDOM NUMBER	(4) NUMBER OF NIGHTLY ARRIVALS	(5) TOTAL TO BE UNLOADED	(6) RANDOM NUMBER	(7) NUMBER UNLOADED
1	—	52	3	3	37	3
2	0	06	0	0	63	0
3	0	50	3	3	28	3
4	0	88	4	4	02	1
5	3	53	3	6	74	4
6	2	30	1	3	35	3
7	0	10	0	0	24	0
8	0	47	3	3	03	1
9	2	99	5	7	29	3
10	4	37	2	6	60	3
11	3	66	3	6	74	4
12	2	91	5	7	85	4
13	3	35	2	5	90	4
14	1	32	2	3	73	3
15	0	00	5	5	59	3
	20		41			39
	Total delays		Total arrivals			Total unloadings

Table 14.11

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Port of New Orleans

Three important pieces of information:

Average number of barges delayed to the next day $= \dfrac{20 \text{ delays}}{15 \text{ days}}$

$= 1.33$ barges delayed per day

Average number of nightly arrivals $= \dfrac{41 \text{ arrivals}}{15 \text{ days}} = 2.73$ arrivals

Average number of barges unloaded each day $= \dfrac{39 \text{ unloadings}}{15 \text{ days}} = 2.60$ unloadings

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Excel Model for the Port of New Orleans Queuing Simulation

	A	B	C	D	E	F	G	H	I	J	K
1	Port of New Orleans Barge Unloadings										
2											
3	Day	Previously delayed	Random number	Arrivals	Total to be unoaded	Random Number	Possibly unloaded	Unloaded			
4	1	0	0.4122	2	2	0.44942	3	2			
5	2	0	0.1801	1	1	0.89542	4	1			
6	3	0	0.7886	4	4	0.83696	4	4			
7	4	0	0.6198	3	3	0.65745	3	3			
8	5	0	0.7517	4	4	0.65671	3	3			
9	6	1	0.3591	2	3	0.01266	1	1			
10	7	2	0.3651	2	4	0.60660	3	3			
11	8	1	0.2075	1	2	0.77510	4	2			
12	9	0	0.1506	1	1	0.96885	5	1			
13	10	0	0.5876	3	3	0.19753	2	2			
14											
15	Barge Arrivals						Unloading rates				
16	Demand	Probability	Lower	Cumulative	Demand		Number	Probability	Lower	Cumulative	Unloading
17	0	0.13	0.00	0.13	0		1	0.05	0.00	0.05	1
18	1	0.17	0.13	0.30	1		2	0.15	0.05	0.20	2
19	2	0.15	0.30	0.45	2		3	0.50	0.20	0.70	3
20	3	0.25	0.45	0.70	3		4	0.20	0.70	0.90	4
21	4	0.20	0.70	0.90	4		5	0.10	0.90	1.00	5
22	5	0.10	0.90	1.00	5						

Program 14.5

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Excel Model for the Port of New Orleans Queuing Simulation

	C	D
17	=0	=B17
18	=C17+B17	=D17+B18

	I	J
17	=0	=H17
18	=I17+H17	=J17+H18

	B	C	D	E	F	G	H
4	=RAND()	=VLOOKUP(B4,$C$17:$E$22,3,TRUE)	=C4	=D4	=RAND()	=VLOOKUP(F4,$I$17:$K$19,3,TRUE)	=E4+G4
5	=RAND()	=VLOOKUP(B5,$C$17:$E$22,3,TRUE)	=D4+C5	=MAX(D5,H4)	=RAND()	=VLOOKUP(F5,$I$17:$K$19,3,TRUE)	=E5+G5

Program 14.5

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Simulation Model for a Maintenance Policy

- Simulation can be used to analyze different maintenance policies before actually implementing them.
- Many options regarding staffing levels, parts replacement schedules, downtime, and labor costs can be compared.
- This can include completely shutting down factories for maintenance.

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Three Hills Power Company

- Three Hills provides power to a large city through a series of almost 200 electric generators.
- The company is concerned about generator failures because a breakdown costs about $75 per generator per hour.
- Their four repair people earn $30 per hour and work rotating 8 hour shifts.
- Management wants to evaluate the:
  1. Service maintenance cost.
  2. Simulated machine breakdown cost.
  3. Total cost.

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Three Hills Power Company

- There are two important maintenance system components:
  - Time between successive generator breakdowns which varies from 30 minutes to three hours.
  - The time it takes to repair the generators which ranges from one to three hours in one hour blocks
- A next event simulation is constructed to study this problem.

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Three Hills Flow Diagram

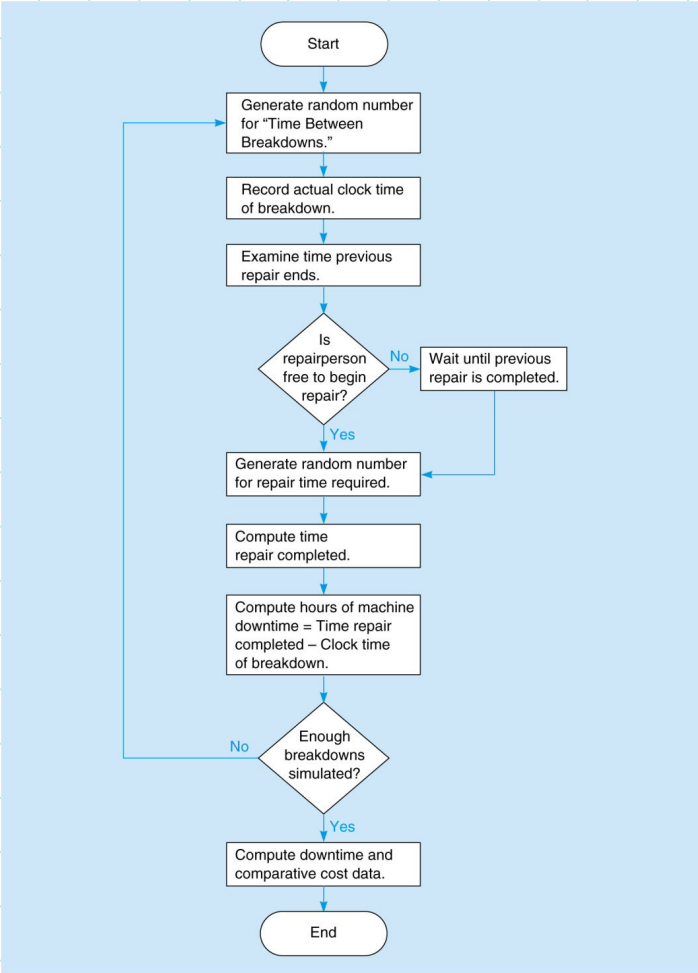

Figure 14.4

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Three Hills Power Company

Time between generator breakdowns at Three Hills Power

TIME BETWEEN RECORDED MACHINE FAILURES (HRS)	NUMBER OF TIMES OBSERVED	PROBABILITY	CUMULATIVE PROBABILITY	RANDOM NUMBER INTERVAL
0.5	5	0.05	0.05	01 to 05
1.0	6	0.06	0.11	06 to 11
1.5	16	0.16	0.27	12 to 27
2.0	33	0.33	0.60	28 to 60
2.5	21	0.21	0.81	61 to 81
3.0	19	0.19	1.00	82 to 00
Total	100	1.00		

Table 14.12

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Three Hills Power Company

Generator repair times required

REPAIR TIME REQUIRED (HRS)	NUMBER OF TIMES OBSERVED	PROBABILITY	CUMULATIVE PROBABILITY	RANDOM NUMBER INTERVAL
1	28	0.28	0.28	01 to 28
2	52	0.52	0.80	29 to 80
3	20	0.20	1.00	81 to 00
Total	100	1.00		

Table 14.13

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Three Hills Power Company

Simulation of generator breakdowns and repairs

(1) BREAKDOWN NUMBER	(2) RANDOM NUMBER FOR BREAKDOWNS	(3) TIME BETWEEN BREAKDOWNS	(4) TIME OF BREAKDOWN	(5) TIME REPAIR-PERSON IS FREE TO BEGIN THIS REPAIR	(6) RANDOM NUMBER FOR REPAIR TIME	(7) REPAIR TIME REQUIRED	(8) TIME REPAIR ENDS	(9) NUMBER OF HOURS MACHINE DOWN
1	57	2	02:00	02:00	07	1	03:00	1
2	17	1.5	03:30	03:30	60	2	05:30	2
3	36	2	05:30	05:30	77	2	07:30	2
4	72	2.5	08:00	08:00	49	2	10:00	2
5	85	3	11:00	11:00	76	2	13:00	2
6	31	2	13:00	13:00	95	3	16:00	3
7	44	2	15:00	16:00	51	2	18:00	3
8	30	2	17:00	18:00	16	1	19:00	2
9	26	1.5	18:30	19:00	14	1	20:00	1.5
10	09	1	19:30	20:00	85	3	23:00	3.5
11	49	2	21:30	23:00	59	2	01:00	3.5
12	13	1.5	23:00	01:00	85	3	04:00	5
13	33	2	01:00	04:00	40	2	06:00	5
14	89	3	04:00	06:00	42	2	08:00	4
15	13	1.5	05:30	08:00	52	2	10:00	4.5
							Total	44

Table 14.14

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Cost Analysis of the Simulation

- The simulation of 15 generator breakdowns covers 34 hours of operation.
- The analysis of this simulation is:

Service maintenance cost = 34 hours of worker service time x $30 per hour
= $1,020

Simulated machine breakdown cost = 44 total hours of breakdown x $75 lost per hour of downtime
= $3,300

Total simulated maintenance cost of the current system = Service cost + Breakdown cost
= $1,020 + $3,300
= $4,320

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Cost Analysis of the Simulation

- The cost of $4,320 should be compared with other alternative plans to see if this is a "good" value.
- The company might explore options like adding another repairperson.
- Strategies such as *preventive maintenance* might also be simulated for comparison.

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Excel Spreadsheet Model for Three Hills Power Company Maintenance Problem

Three Hills Power Company

Breakdown number	Random number	Time between breakdowns	Time of breakdowns	Time repairperson is free	Random Number	Repair time	Repair ends
1	0.4250	2	2	2	0.5263	2	4
2	0.0279	0.5	2.5	4	0.0466	1	5
3	0.9666	3	5.5	5.5	0.2342	1	6.5
4	0.9527	3	8.5	8.5	0.1482	1	9.5
5	0.2316	1.5	10	10	0.6001	2	12
6	0.3460	2	12	12	0.4528	2	14
7	0.3648	2	14	14	0.1964	1	15
8	0.8095	2.5	16.5	16.5	0.2554	1	17.5
9	0.7553	2.5	19	19	0.6737	2	21
10	0.4003	2	21	21	0.4339	2	23

Demand Table

Time between breakdowns	Probability	Lower	Cumulative	Demand
0.5	0.05	0	0.05	0.5
1.0	0.06	0.05	0.11	1
1.5	0.16	0.11	0.27	1.5
2.0	0.33	0.27	0.6	2
2.5	0.21	0.6	0.81	2.5
3.0	0.19	0.81	1	3

Repair times

Time	Probability	Lower	Cumulative	Lead time
1	0.28	0.00	0.28	1
2	0.52	0.28	0.80	2
3	0.20	0.80	1.00	3

Program 14.6

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Excel Spreadsheet Model for Three Hills Power Company Maintenance Problem

	C	D	E
17	=0	=B17	=A17
18	=C17+B17	=D17+B18	=A18

	I	J	K
17	=0	=H17	=G17
18	=I17+H17	=J17+H18	=G18
19	=I18+H18	=J18+H19	=G19

	B	C	D	E	F	G	H
4	=RAND()	=VLOOKUP(B4,$C$17:$E$22,3,TRUE)	=C4	=D4	=RAND()	=VLOOKUP(F4,$I$17:$K$19,3,TRUE)	=E4+G4
5	=RAND()	=VLOOKUP(B5,$C$17:$E$22,3,TRUE)	=D4+C5	=MAX(D5,H4)	=RAND()	=VLOOKUP(F5,$I$17:$K$19,3,TRUE)	=E5+G5

Program 14.6

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Other Simulation Models

- **Simulation models are often broken into three categories:**
  - **The Monte Carlo method.**
  - **Operational gaming.**
  - **Systems simulation.**
- **Though theoretically different, computerized simulation has tended to blur the differences.**

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Operational Gaming

- *Operational gaming* refers to simulation involving two or more competing players.
- The best examples of this are military games and business games.
- These types of simulation allow the testing of skills and decision-making in a competitive environment.

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Systems Simulation

- *Systems simulation* is similar in that allows users to test various managerial policies and decisions to evaluate their effect on the operating environment.
- This models the dynamics of *large* systems.
- A *corporate operating system* might model sales, production levels, marketing policies, investments, union contracts, utility rates, financing, and other factors.
- *Economic simulations*, often called econometric models, are used by governments, bankers, and large organizations to predict inflation rates, domestic and foreign money supplies, and unemployment levels.

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Systems Simulation

Inputs and Outputs of a Typical Economic System Simulation

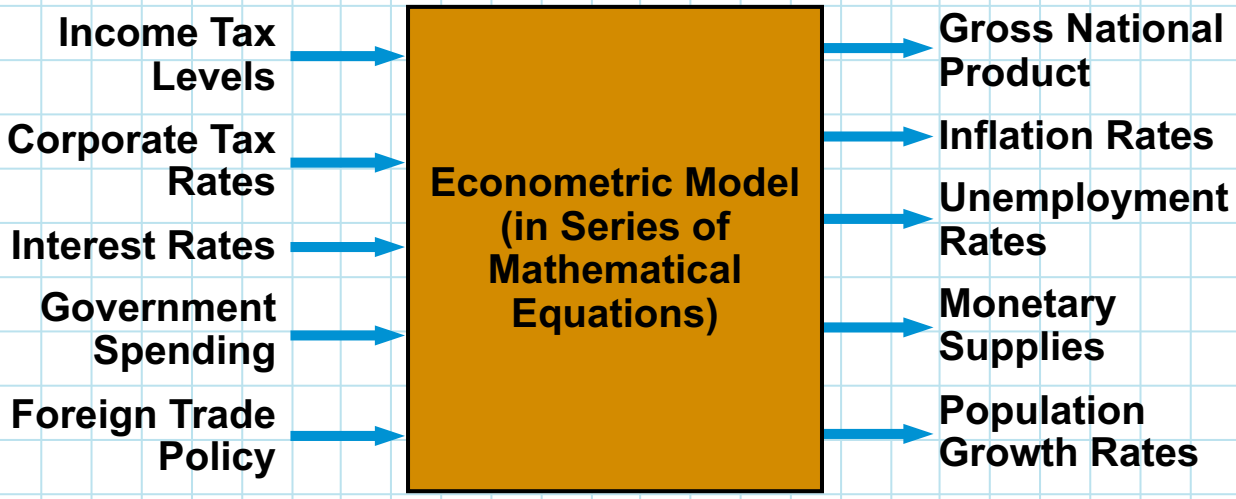

Figure 14.5

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Verification and Validation

- It is important that a simulation model be checked to see that it is working properly and providing good representation of the real world situation.
- The *verification* process involves determining that the computer model is internally consistent and following the logic of the conceptual model.
- Verification answers the question "Did we build the model right?"
- *Validation* is the process of comparing a simulation model to the real system it represents to make sure it is accurate.
- Validation answers the question "Did we build the right model?"

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Role of Computers in Simulation

- Computers are critical in simulating complex tasks.
- General-purpose programming languages can be used for simulation, but a variety of simulation software tools have been developed to make the process easier:
  - Arena
  - ProModel
  - SIMUL8
  - ExtendSim
  - Proof 5
- Excel and add-ins can also be used for simulation problems

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