# Chapter 1

# Introduction to Quantitative Analysis

To accompany
*Quantitative Analysis for Management, Eleventh Edition*,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

## Learning Objectives

**After completing this chapter, students will be able to:**

1. Describe the quantitative analysis approach
2. Understand the application of quantitative analysis in a real situation
3. Describe the use of modeling in quantitative analysis
4. Use computers and spreadsheet models to perform quantitative analysis
5. Discuss possible problems in using quantitative analysis
6. Perform a break-even analysis

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# Chapter Outline


1.1 Introduction
1.2 What Is Quantitative Analysis?
1.3 The Quantitative Analysis Approach
1.4 How to Develop a Quantitative Analysis Model
1.5 The Role of Computers and Spreadsheet Models in the Quantitative Analysis Approach
1.6 Possible Problems in the Quantitative Analysis Approach
1.7 Implementation — Not Just the Final Step


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# Introduction

- Mathematical tools have been used for thousands of years.
- Quantitative analysis can be applied to a wide variety of problems.
  - It's not enough to just know the mathematics of a technique.
  - One must understand the specific applicability of the technique, its limitations, and its assumptions.

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# Examples of Quantitative Analyses

- In the mid 1990s, Taco Bell saved over $150 million using forecasting and scheduling quantitative analysis models.
- NBC television increased revenues by over $200 million between 1996 and 2000 by using quantitative analysis to develop better sales plans.
- Continental Airlines saved over $40 million in 2001 using quantitative analysis models to quickly recover from weather delays and other disruptions.

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# What is Quantitative Analysis?

***Quantitative analysis*** **is a scientific approach to managerial decision making in which raw data are processed and manipulated to produce meaningful information.**

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# What is Quantitative Analysis?

- ***Quantitative factors*** **are data that can be accurately calculated. Examples include:**
  - **Different investment alternatives**
  - **Interest rates**
  - **Inventory levels**
  - **Demand**
  - **Labor cost**
- ***Qualitative factors*** **are more difficult to quantify but affect the decision process. Examples include:**
  - **The weather**
  - **State and federal legislation**
  - **Technological breakthroughs.**

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# The Quantitative Analysis Approach

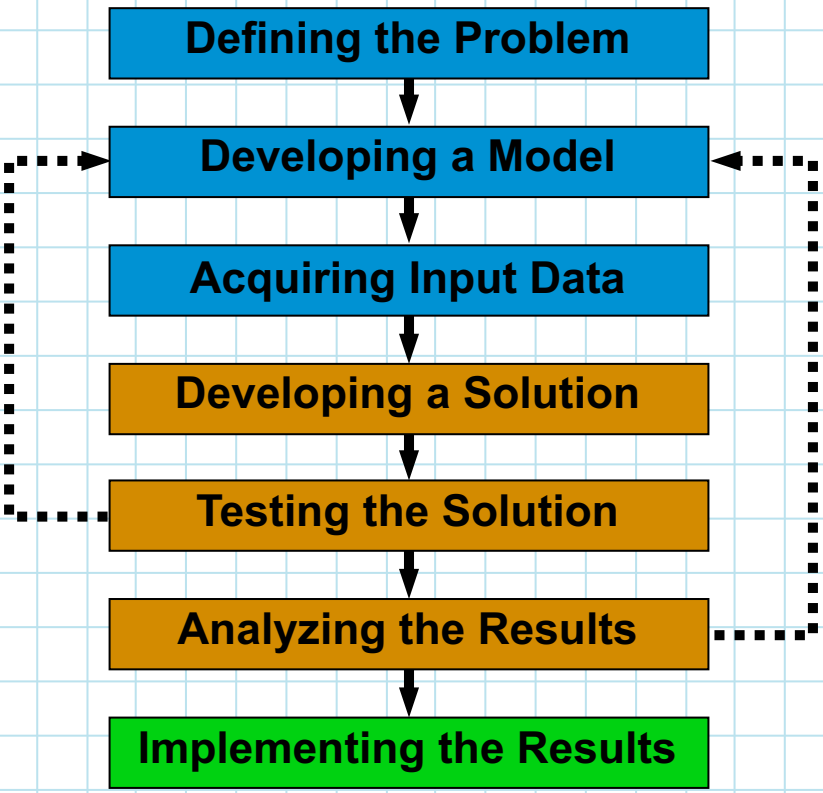


**Figure 1.1**

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# Defining the Problem

**Develop a clear and concise statement that gives direction and meaning to subsequent steps.**

- **This may be the most important and difficult step.**
- **It is essential to go beyond symptoms and identify true causes.**
- **It may be necessary to concentrate on only a few of the problems – selecting the right problems is very important**
- **Specific and measurable objectives may have to be developed.**

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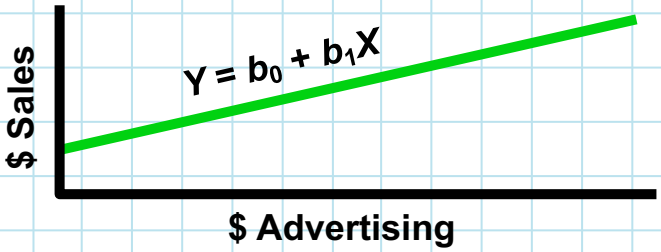
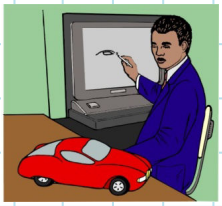
# Developing a Model

**Quantitative analysis models are realistic, solvable, and understandable mathematical representations of a situation.**

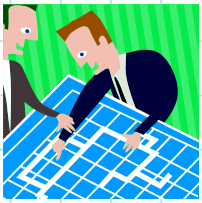



**There are different types of models:**

**Scale models**



**Schematic models**



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# Developing a Model

Models generally contain variables (controllable and uncontrollable) and parameters.

- Controllable variables are the decision variables and are generally unknown.
  - How many items should be ordered for inventory?
- Parameters are known quantities that are a part of the model.
  - What is the holding cost of the inventory?

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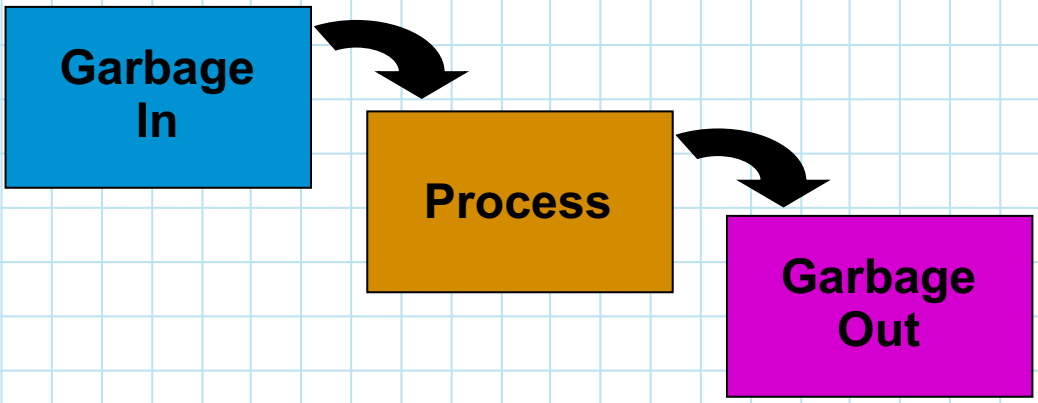
# Acquiring Input Data

Input data must be accurate – GIGO rule:




Data may come from a variety of sources such as company reports, company documents, interviews, on-site direct measurement, or statistical sampling.

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# Developing a Solution

The best (optimal) solution to a problem is found by manipulating the model variables until a solution is found that is practical and can be implemented.

Common techniques are

- *Solving* equations.
- *Trial and error* – trying various approaches and picking the best result.
- *Complete enumeration* – trying all possible values.
- Using an *algorithm* – a series of repeating steps to reach a solution.

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# Testing the Solution

Both input data and the model should be tested for accuracy before analysis and implementation.

- New data can be collected to test the model.
- Results should be logical, consistent, and represent the real situation.

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# Analyzing the Results

**Determine the implications of the solution:**

- **Implementing results often requires change in an organization.**
- **The impact of actions or changes needs to be studied and understood before implementation.**

***Sensitivity analysis*** **determines how much the results will change if the model or input data changes.**

- **Sensitive models should be very thoroughly tested.**

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# Implementing the Results

**Implementation incorporates the solution into the company.**

- **Implementation can be very difficult.**
- **People may be resistant to changes.**
- **Many quantitative analysis efforts have failed because a good, workable solution was not properly implemented.**

**Changes occur over time, so even successful implementations must be monitored to determine if modifications are necessary.**

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# Modeling in the Real World

Quantitative analysis models are used extensively by real organizations to solve real problems.

- In the real world, quantitative analysis models can be complex, expensive, and difficult to sell.
- Following the steps in the process is an important component of success.

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# How To Develop a Quantitative Analysis Model

A mathematical model of profit:

Profit = Revenue – Expenses

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# How To Develop a Quantitative Analysis Model

Expenses can be represented as the sum of fixed and variable costs. Variable costs are the product of unit costs times the number of units.

Profit = Revenue – (Fixed cost + Variable cost)

Profit = (Selling price per unit)(number of units sold) – [Fixed cost + (Variable costs per unit)(Number of units sold)]

Profit = $sX - [f + vX]$

Profit = $sX - f - vX$

where

$s$ = selling price per unit

$f$ = fixed cost

$v$ = variable cost per unit

$X$ = number of units sold

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# How To Develop a Quantitative Analysis Model

Expenses can be r
variable costs and
unit costs times th

Profit = Revenue

Profit = (Selling
sold) – [
unit)(Nu

Profit = $sX - [f + vX]$

Profit = $sX - f - vX$

where

$s$ = selling price per unit

$f$ = fixed cost

$v$ = variable cost per unit

$X$ = number of units sold

The *parameters* of this model are $f$, $v$, and $s$ as these are the inputs inherent in the model

The *decision variable* of interest is $X$

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# Pritchett's Precious Time Pieces

The company buys, sells, and repairs old clocks. Rebuilt springs sell for $10 per unit. Fixed cost of equipment to build springs is $1,000. Variable cost for spring material is $5 per unit.

$s = 10 \qquad f = 1{,}000 \qquad v = 5$

Number of spring sets sold = $X$

Profits = $sX - f - vX$

If sales = 0, profits = $-f$ = –$1,000.

If sales = 1,000, profits = [(10)(1,000) – 1,000 – (5)(1,000)]

= $4,000

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# Pritchett's Precious Time Pieces

Companies are often interested in the *break-even point* (BEP). The BEP is the number of units sold that will result in $0 profit.

$0 = sX - f - vX, \quad \text{or} \quad 0 = (s - v)X - f$

Solving for $X$, we have

$$f = (s - v)X$$

$$X = \frac{f}{s - v}$$

$$\text{BEP} = \frac{\textit{Fixed cost}}{\textit{(Selling price per unit)} - \textit{(Variable cost per unit)}}$$

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# Pritchett's Precious Time Pieces

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p
th

**BEP for Pritchett's Precious Time Pieces**

**BEP = $1,000/($10 – $5) = 200 units**

**Sales of less than 200 units of rebuilt springs will result in a loss.**

**Sales of over 200 units of rebuilt springs will result in a profit.**

$$\text{BEP} = \frac{\textit{Fixed cost}}{\textit{(Selling price per unit)} - \textit{(Variable cost per unit)}}$$

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# Advantages of Mathematical Modeling

1. Models can accurately represent reality.
2. Models can help a decision maker formulate problems.
3. Models can give us insight and information.
4. Models can save time and money in decision making and problem solving.
5. A model may be the only way to solve large or complex problems in a timely fashion.
6. A model can be used to communicate problems and solutions to others.

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# Models Categorized by Risk

- Mathematical models that do not involve risk are called *deterministic* models.
  - All of the values used in the model are known with complete certainty.
- Mathematical models that involve risk, chance, or uncertainty are called *probabilistic* models.
  - Values used in the model are estimates based on probabilities.

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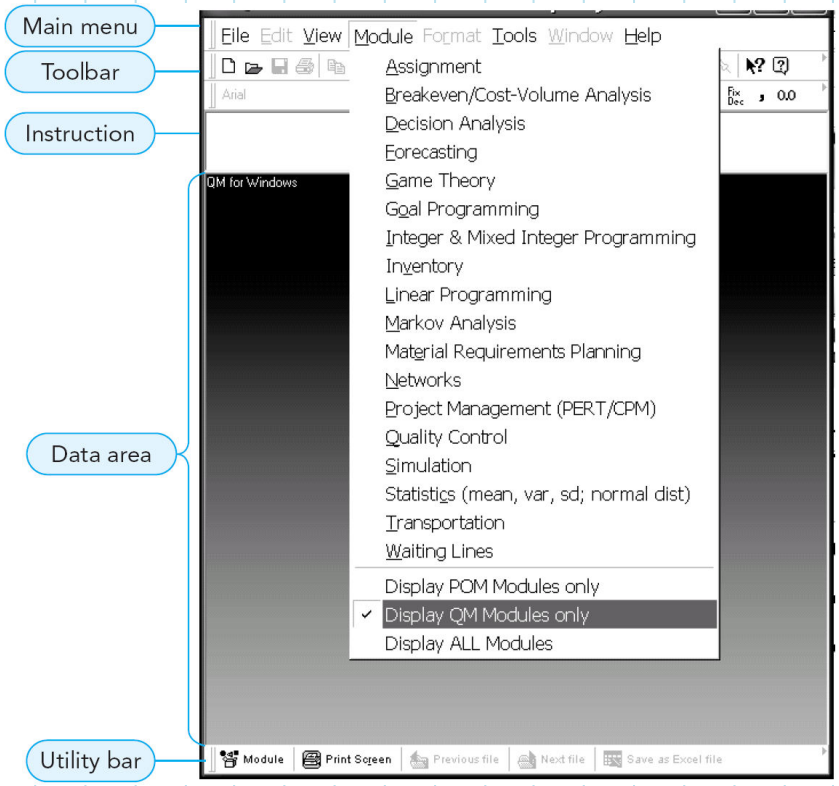
# Computers and Spreadsheet Models

## QM for Windows

- An easy to use decision support system for use in POM and QM courses
- This is the main menu of quantitative models




Program 1.1

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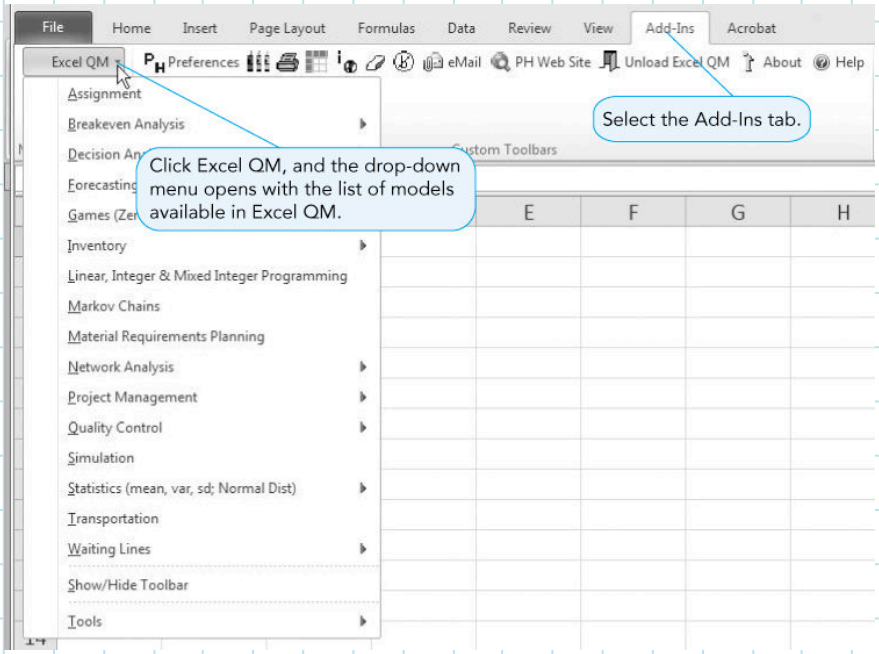
# Computers and Spreadsheet Models

## Excel QM's Main Menu (2010)

- **Works automatically within Excel spreadsheets**

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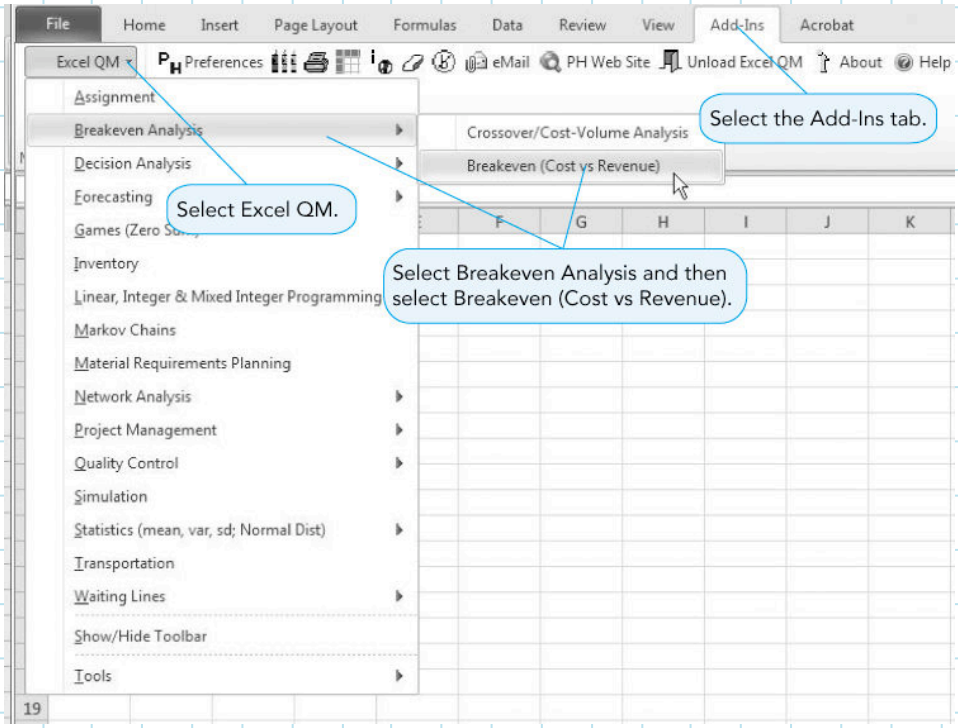
# Computers and Spreadsheet Models

## Selecting Break-Even Analysis in Excel QM

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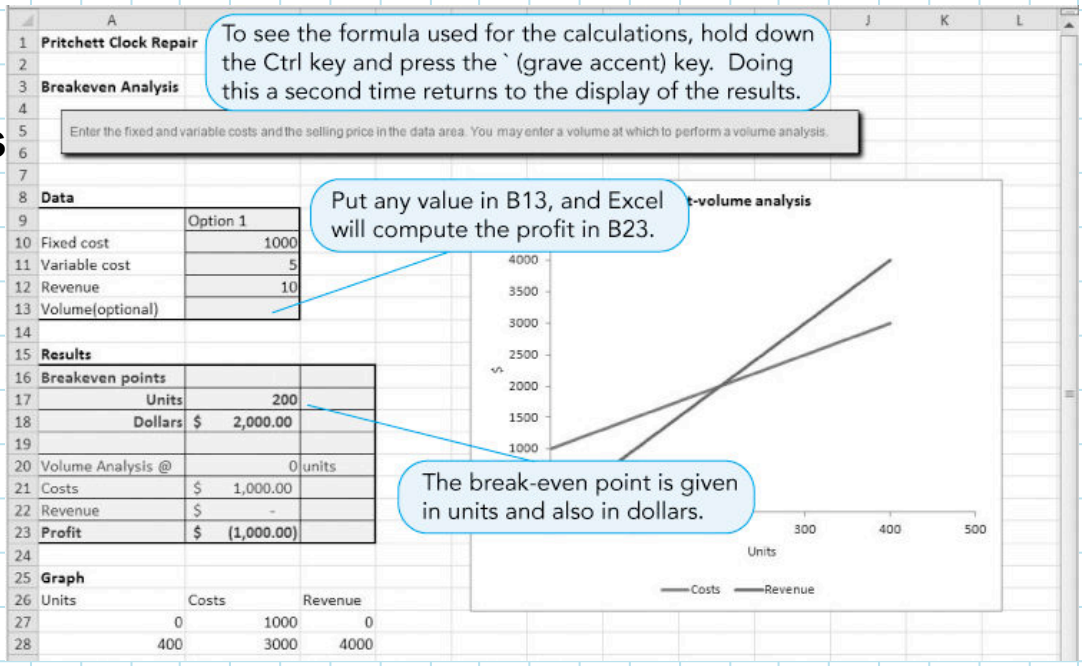
# Computers and Spreadsheet Models

## Break-Even Analysis in Excel QM

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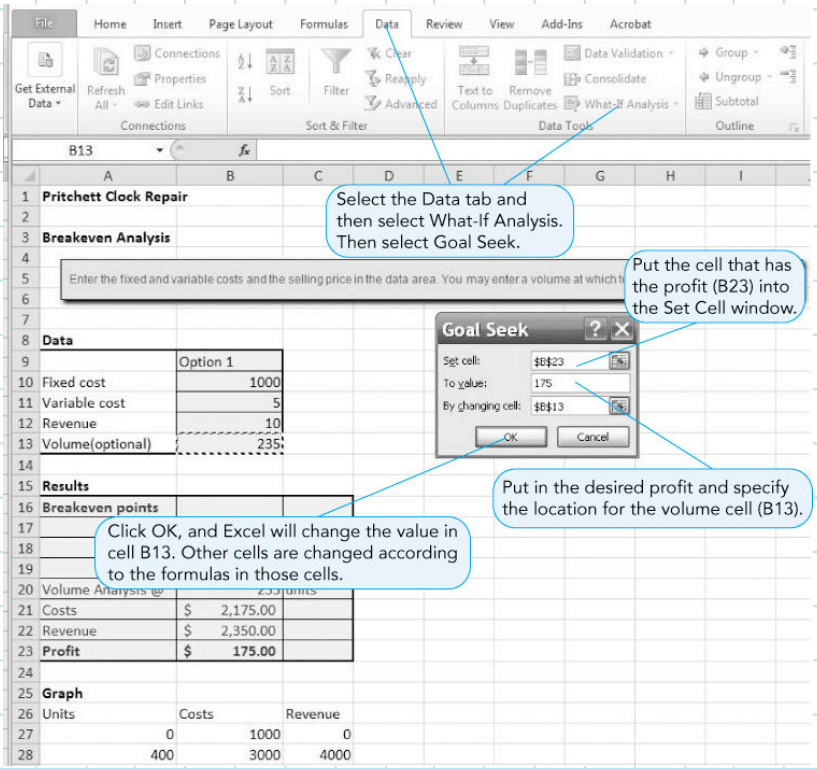
# Computers and Spreadsheet Models

## Using Goal Seek in the Break-Even Problem




**Program 1.4**

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# Possible Problems in the Quantitative Analysis Approach

## Defining the problem

- Problems may not be easily identified.
- There may be conflicting viewpoints
- There may be an impact on other departments.
- Beginning assumptions may lead to a particular conclusion.
- The solution may be outdated.

## Developing a model

- Manager's perception may not fit a textbook model.
- There is a trade-off between complexity and ease of understanding.

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# Possible Problems in the Quantitative Analysis Approach

## Acquiring accurate input data

- Accounting data may not be collected for quantitative problems.
- The validity of the data may be suspect.

## Developing an appropriate solution

- The mathematics may be hard to understand.
- Having only one answer may be limiting.

## Testing the solution for validity

## Analyzing the results in terms of the whole organization

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## *Implementation – Not Just the Final Step*

**There may be an institutional lack of commitment and resistance to change.**

- **Management may fear the use of formal analysis processes will reduce their decision-making power.**
- **Action-oriented managers may want "quick and dirty" techniques.**
- **Management support and user involvement are important.**

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## *Implementation – Not Just the Final Step*

**There may be a lack of commitment by quantitative analysts.**

- **Analysts should be involved with the problem and care about the solution.**
- **Analysts should work with users and take their feelings into account.**

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# Chapter 3

# Decision Analysis

To accompany
*Quantitative Analysis for Management, Eleventh Edition,*
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

# Learning Objectives

**After completing this chapter, students will be able to:**

1. List the steps of the decision-making process.
2. Describe the types of decision-making environments.
3. Make decisions under uncertainty.
4. Use probability values to make decisions under risk.

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# Learning Objectives

**After completing this chapter, students will be able to:**

5. Develop accurate and useful decision trees.
6. Revise probabilities using Bayesian analysis.
7. Use computers to solve basic decision-making problems.
8. Understand the importance and use of utility theory in decision making.

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# Chapter Outline


3.1 Introduction
3.2 The Six Steps in Decision Making
3.3 Types of Decision-Making Environments
3.4 Decision Making under Uncertainty
3.5 Decision Making under Risk
3.6 Decision Trees
3.7 How Probability Values Are Estimated by Bayesian Analysis
3.8 Utility Theory


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# Introduction

- What is involved in making a good decision?
- Decision theory is an analytic and systematic approach to the study of decision making.
- A good decision is one that is based on logic, considers all available data and possible alternatives, and the quantitative approach described here.

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# The Six Steps in Decision Making

1. Clearly define the problem at hand.
2. List the possible alternatives.
3. Identify the possible outcomes or states of nature.
4. List the payoff (typically profit) of each combination of alternatives and outcomes.
5. Select one of the mathematical decision theory models.
6. Apply the model and make your decision.

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# Thompson Lumber Company

**Step 1 –** Define the problem.

- The company is considering expanding by manufacturing and marketing a new product – backyard storage sheds.

**Step 2 –** List alternatives.

- Construct a large new plant.
- Construct a small new plant.
- Do not develop the new product line at all.

**Step 3 –** Identify possible outcomes.

- The market could be favorable or unfavorable.

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# Thompson Lumber Company

**Step 4 –** List the payoffs.

- Identify *conditional values* for the profits for large plant, small plant, and no development for the two possible market conditions.

**Step 5 –** Select the decision model.

- This depends on the environment and amount of risk and uncertainty.

**Step 6 –** Apply the model to the data.

- Solution and analysis are then used to aid in decision-making.

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# Thompson Lumber Company

Decision Table with Conditional Values for Thompson Lumber

| | STATE OF NATURE | |
|---|---|---|
| ALTERNATIVE | FAVORABLE MARKET ($) | UNFAVORABLE MARKET ($) |
| Construct a large plant | 200,000 | –180,000 |
| Construct a small plant | 100,000 | –20,000 |
| Do nothing | 0 | 0 |

Table 3.1

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# *Types of Decision-Making Environments*

**Type 1:** Decision making under certainty

- The decision maker ***knows with certainty*** the consequences of every alternative or decision choice.

**Type 2:** Decision making under uncertainty

- The decision maker ***does not know*** the probabilities of the various outcomes.

**Type 3:** Decision making under risk

- The decision maker ***knows the probabilities*** of the various outcomes.

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# *Decision Making Under Uncertainty*

There are several criteria for making decisions under uncertainty:

1. Maximax (optimistic)
2. Maximin (pessimistic)
3. Criterion of realism (Hurwicz)
4. Equally likely (Laplace)
5. Minimax regret

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# Maximax

**Used to find the alternative that maximizes the maximum payoff.**

- **Locate the maximum payoff for each alternative.**
- **Select the alternative with the maximum number.**

| ALTERNATIVE | STATE OF NATURE: FAVORABLE MARKET ($) | STATE OF NATURE: UNFAVORABLE MARKET ($) | MAXIMUM IN A ROW ($) |
|---|---|---|---|
| Construct a large plant | 200,000 | –180,000 | 200,000 Maximax |
| Construct a small plant | 100,000 | –20,000 | 100,000 |
| Do nothing | 0 | 0 | 0 |

Table 3.2

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# Maximin

**Used to find the alternative that maximizes the minimum payoff.**

- **Locate the minimum payoff for each alternative.**
- **Select the alternative with the maximum number.**

| ALTERNATIVE | STATE OF NATURE: FAVORABLE MARKET ($) | STATE OF NATURE: UNFAVORABLE MARKET ($) | MINIMUM IN A ROW ($) |
|---|---|---|---|
| Construct a large plant | 200,000 | –180,000 | –180,000 |
| Construct a small plant | 100,000 | –20,000 | –20,000 |
| Do nothing | 0 | 0 | 0 Maximin |

Table 3.3

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# Criterion of Realism (Hurwicz)

This is a *weighted average* compromise between optimism and pessimism.

- Select a coefficient of realism $\alpha$, with $0 \le \alpha \le 1$.
- A value of 1 is perfectly optimistic, while a value of 0 is perfectly pessimistic.
- Compute the weighted averages for each alternative.
- Select the alternative with the highest value.

Weighted average = $\alpha$(maximum in row)
+ (1 – $\alpha$)(minimum in row)

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# Criterion of Realism (Hurwicz)

- For the large plant alternative using $\alpha$ = 0.8:
  (0.8)(200,000) + (1 – 0.8)(–180,000) = 124,000
- For the small plant alternative using $\alpha$ = 0.8:
  (0.8)(100,000) + (1 – 0.8)(–20,000) = 76,000

| ALTERNATIVE | STATE OF NATURE: FAVORABLE MARKET ($) | STATE OF NATURE: UNFAVORABLE MARKET ($) | CRITERION OF REALISM ($\alpha$ = 0.8) $ |
|---|---|---|---|
| Construct a large plant | 200,000 | –180,000 | 124,000 Realism |
| Construct a small plant | 100,000 | –20,000 | 76,000 |
| Do nothing | 0 | 0 | 0 |

Table 3.4

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# Equally Likely (Laplace)

Considers all the payoffs for each alternative

- Find the average payoff for each alternative.
- Select the alternative with the highest average.

| ALTERNATIVE | STATE OF NATURE: FAVORABLE MARKET ($) | STATE OF NATURE: UNFAVORABLE MARKET ($) | ROW AVERAGE ($) |
|---|---|---|---|
| Construct a large plant | 200,000 | –180,000 | 10,000 |
| Construct a small plant | 100,000 | –20,000 | 40,000 |
| Do nothing | 0 | 0 | 0 |

Equally likely

Table 3.5

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# Minimax Regret

Based on *opportunity loss* or *regret*, this is the difference between the optimal profit and actual payoff for a decision.

- Create an opportunity loss table by determining the opportunity loss from not choosing the best alternative.
- Opportunity loss is calculated by subtracting each payoff in the column from the best payoff in the column.
- Find the maximum opportunity loss for each alternative and pick the alternative with the minimum number.

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# Minimax Regret

## Determining Opportunity Losses for Thompson Lumber

| STATE OF NATURE | |
|---|---|
| FAVORABLE MARKET ($) | UNFAVORABLE MARKET ($) |
| 200,000 – 200,000 | 0 – (–180,000) |
| 200,000 – 100,000 | 0 – (–20,000) |
| 200,000 – 0 | 0 – 0 |

Table 3.6

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# Minimax Regret

## Opportunity Loss Table for Thompson Lumber

| | STATE OF NATURE | |
|---|---|---|
| ALTERNATIVE | FAVORABLE MARKET ($) | UNFAVORABLE MARKET ($) |
| Construct a large plant | 0 | 180,000 |
| Construct a small plant | 100,000 | 20,000 |
| Do nothing | 200,000 | 0 |

Table 3.7

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# Minimax Regret

## Thompson's Minimax Decision Using Opportunity Loss

| ALTERNATIVE | STATE OF NATURE | | |
|---|---|---|---|
| | FAVORABLE MARKET ($) | UNFAVORABLE MARKET ($) | MAXIMUM IN A ROW ($) |
| Construct a large plant | 0 | 180,000 | 180,000 |
| Construct a small plant | 100,000 | 20,000 | 100,000 |
| Do nothing | 200,000 | 0 | 200,000 |

Minimax

Table 3.8

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# Decision Making Under Risk

- This is decision making when there are several possible states of nature, and the probabilities associated with each possible state are known.
- The most popular method is to choose the alternative with the highest *expected monetary value (EMV).*
  - This is very similar to the *expected value* calculated in the last chapter.

EMV (alternative $i$) = (payoff of first state of nature)
x (probability of first state of nature)
+ (payoff of second state of nature)
x (probability of second state of nature)
+ … + (payoff of last state of nature)
x (probability of last state of nature)

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# *EMV* for Thompson Lumber

- Suppose each market outcome has a probability of occurrence of 0.50.
- Which alternative would give the highest EMV?
- The calculations are:

EMV (large plant) = ($200,000)(0.5) + (–$180,000)(0.5)
= $10,000

EMV (small plant) = ($100,000)(0.5) + (–$20,000)(0.5)
= $40,000

EMV (do nothing) = ($0)(0.5) + ($0)(0.5)
= $0

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# *EMV* for Thompson Lumber

| ALTERNATIVE | STATE OF NATURE: FAVORABLE MARKET ($) | STATE OF NATURE: UNFAVORABLE MARKET ($) | EMV ($) |
|---|---|---|---|
| Construct a large plant | 200,000 | –180,000 | 10,000 |
| Construct a small plant | 100,000 | –20,000 | 40,000 |
| Do nothing | 0 | 0 | 0 |
| Probabilities | 0.50 | 0.50 | |

Table 3.9

Largest EMV

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# Expected Value of Perfect Information (EVPI)

- EVPI places an upper bound on what you should pay for additional information.

$$EVPI = EVwPI - \text{Maximum } EMV$$

- EVwPI is the long run average return if we have perfect information before a decision is made.

EVwPI = (best payoff for first state of nature)
x (probability of first state of nature)
+ (best payoff for second state of nature)
x (probability of second state of nature)
+ … + (best payoff for last state of nature)
x (probability of last state of nature)

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# Expected Value of Perfect Information (EVPI)

- Suppose Scientific Marketing, Inc. offers analysis that will provide certainty about market conditions (favorable).
- Additional information will cost $65,000.
- Should Thompson Lumber purchase the information?

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# Expected Value of Perfect Information (EVPI)

## Decision Table with Perfect Information

| ALTERNATIVE | STATE OF NATURE: FAVORABLE MARKET ($) | STATE OF NATURE: UNFAVORABLE MARKET ($) | EMV ($) |
|---|---|---|---|
| Construct a large plant | 200,000 | -180,000 | 10,000 |
| Construct a small plant | 100,000 | -20,000 | 40,000 |
| Do nothing | 0 | 0 | 0 |
| With perfect information | 200,000 | 0 | 100,000 EVwPI |
| Probabilities | 0.5 | 0.5 | |

Table 3.10

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# Expected Value of Perfect Information (EVPI)

The maximum EMV without additional information is $40,000.

EVPI = EVwPI – Maximum EMV

= $100,000 - $40,000

= $60,000

So the maximum Thompson should pay for the additional information is $60,000.

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# Expected Value of Perfect Information (EVPI)

The maximum EMV without additional information is $40,000.

EVPI = EVwPI – Maximum EMV
= $100,000 - $40,000
= $60,000

So the maximum Thompson should pay for the additional information is $60,000.

Therefore, Thompson should not pay $65,000 for this information.

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# Expected Opportunity Loss

- *Expected opportunity loss* (EOL) is the cost of not picking the best solution.
- First construct an opportunity loss table.
- For each alternative, multiply the opportunity loss by the probability of that loss for each possible outcome and add these together.
- Minimum EOL will always result in the same decision as maximum EMV.
- Minimum EOL will always equal EVPI.

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# Expected Opportunity Loss

| ALTERNATIVE | STATE OF NATURE: FAVORABLE MARKET ($) | STATE OF NATURE: UNFAVORABLE MARKET ($) | EOL |
|---|---|---|---|
| Construct a large plant | 0 | 180,000 | 90,000 |
| Construct a small plant | 100,000 | 20,000 | 60,000 |
| Do nothing | 200,000 | 0 | 100,000 |
| Probabilities | 0.50 | 0.50 | |

Table 3.11

Minimum EOL

EOL (large plant) = (0.50)($0) + (0.50)($180,000)
= $90,000

EOL (small plant) = (0.50)($100,000) + (0.50)($20,000)
= $60,000

EOL (do nothing) = (0.50)($200,000) + (0.50)($0)
= $100,000

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# Sensitivity Analysis

- Sensitivity analysis examines how the decision might change with different input data.
- For the Thompson Lumber example:

$P$ = probability of a favorable market

$(1 - P)$ = probability of an unfavorable market

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# Sensitivity Analysis

$$\text{EMV(Large Plant)} = \$200{,}000P - \$180{,}000)(1 - P)$$
$$= \$200{,}000P - \$180{,}000 + \$180{,}000P$$
$$= \$380{,}000P - \$180{,}000$$

$$\text{EMV(Small Plant)} = \$100{,}000P - \$20{,}000)(1 - P)$$
$$= \$100{,}000P - \$20{,}000 + \$20{,}000P$$
$$= \$120{,}000P - \$20{,}000$$

$$\text{EMV(Do Nothing)} = \$0P + 0(1 - P)$$
$$= \$0$$

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# Sensitivity Analysis

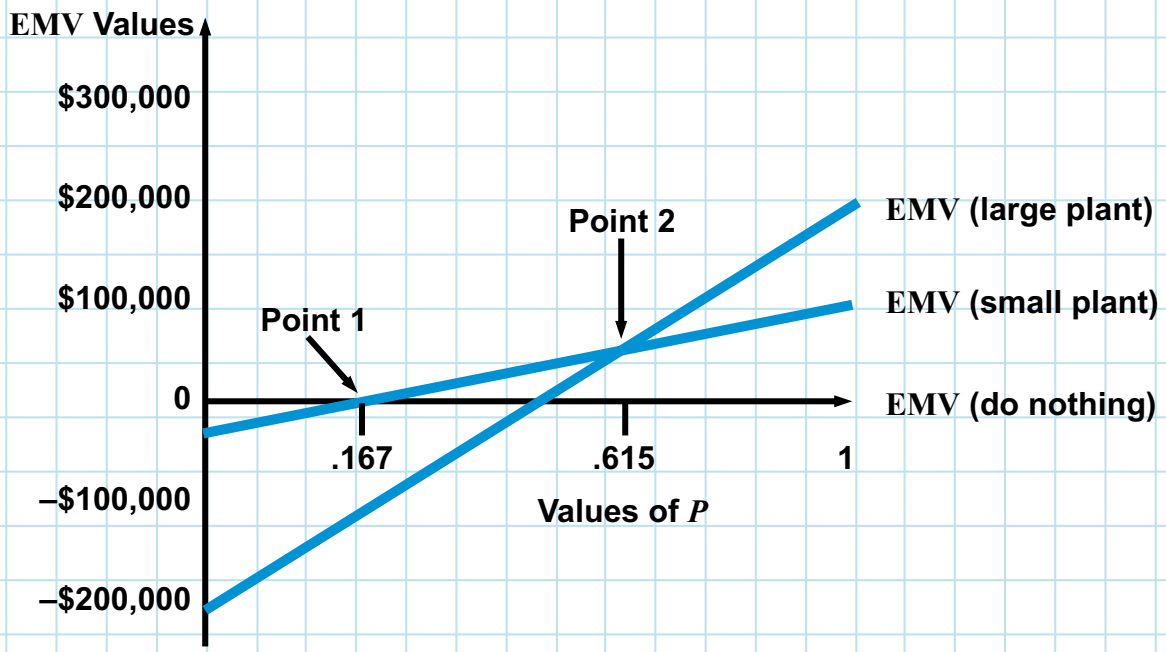


Figure 3.1

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# Sensitivity Analysis

**Point 1:**

EMV(do nothing) = EMV(small plant)

$$0 = \$120{,}000P - \$20{,}000 \qquad P = \frac{20{,}000}{120{,}000} = 0.167$$

**Point 2:**

EMV(small plant) = EMV(large plant)

$$\$120{,}000P - \$20{,}000 = \$380{,}000P - \$180{,}000$$

$$P = \frac{160{,}000}{260{,}000} = 0.615$$

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# Sensitivity Analysis

| BEST ALTERNATIVE | RANGE OF *P* VALUES |
|---|---|
| Do nothing | Less than 0.167 |
| Construct a small plant | 0.167 – 0.615 |
| Construct a large plant | Greater than 0.615 |

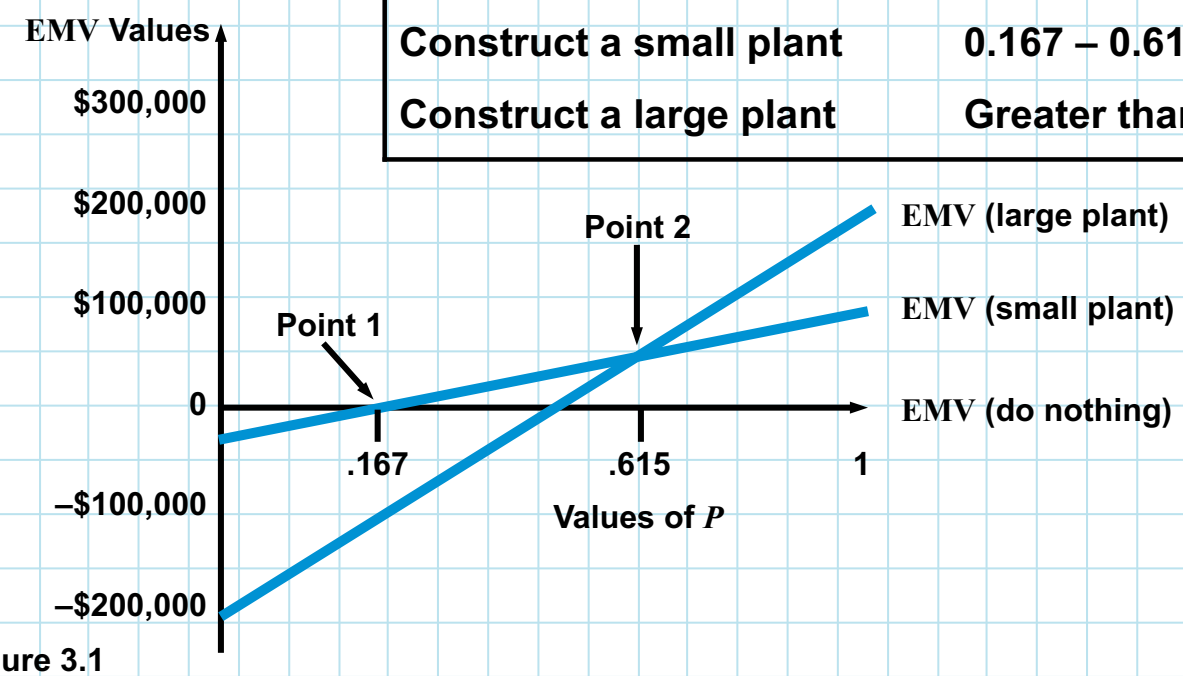


Figure 3.1

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# Using Excel

## Input Data for the Thompson Lumber Problem Using Excel QM

Compute the EMV for each alternative using the SUMPRODUCT function, the worst case using the MIN function, and the best case using the MAX function.

| | A | B | C | D | E | F | G | I |
|---|---|---|---|---|---|---|---|---|
| 1 | **Thompson Lumber** | | | | | | | |
| 2 | | | | | | | | |
| 3 | **Decision Tables** | | | | | | | |
| 4 | Enter the profits or costs in the main body of the data table. Enter probabilities in the first row if you want | | | | | | | |
| 5 | to compute the expected value. | | | | | | | |
| 6 | Data | | | | Results | | | |
| 7 | Profit | Favorable Market | Unfavorable Market | | EMV | Minimum | Maximum | Hurwicz |
| 8 | Probability | 0.5 | 0.5 | | | | | 0.8 |
| 9 | Large Plant | 200000 | -180000 | | =SUMPRODUCT(B$8:C$8,B9:C9) | =MIN(B9:C9) | =MAX(B9:C9) | =$I$8*G9+(1-$I$8)*F9 |
| 10 | Small plant | 100000 | -20000 | | =SUMPRODUCT(B$8:C$8,B10:C10) | =MIN(B10:C10) | =MAX(B10:C10) | =$I$8*G10+(1-$I$8)*F10 |
| 11 | Do nothing | 0 | 0 | | =SUMPRODUCT(B$8:C$8,B11:C11) | =MIN(B11:C11) | =MAX(B11:C11) | |
| 12 | | | | | =MAX(E9:E11) | =MAX(F9:F11) | =MAX(G9:G11) | =MAX(I9:I11) |
| 13 | | | | | | | | |
| 14 | **Expected Value of Perfect Information** | | | | | | | |
| 15 | Column best | =MAX(B9:B11) | =MAX(C9:C11 | | =SUMPRODUCT(B$8:C$8,B15:C15) | <-Expected value under certainty | | |
| 16 | | | | | =E12 | <-Best expected value | | |
| 17 | | | | | =E15-E12 | <-Expected value of perfect information | | |
| 18 | | | | | | | | |
| 19 | **Regret** | | | | | | | |
| 20 | | =B7 | =C7 | | Expected | Maximum | | |
| 21 | =A8 | =B8 | =C8 | | | | | |
| 22 | =A9 | =B15 - B9 | =C15 - C9 | | =SUMPRODUCT(B$8:C$8,B22:C22) | =MAX(B22:C22) | | |
| 23 | =A10 | =B15 - B10 | =C15 - C10 | | =SUMPRODUCT(B$8:C$8,B23:C23) | =MAX(B23:C23) | | |
| 24 | =A11 | =B15 - B11 | =C15 - C11 | | =SUMPRODUCT(B$8:C$8,B24:C24) | =MAX(B24:C24) | | |
| 25 | | | | | =MIN(E22:E24) | =MIN(F22:F24) | | |
| 26 | | | | | | | | |
| 27 | | | | | | | | |
| 28 | | | | | | | | |
| 29 | | | | | | | | |

To calculate the EVPI, find the best outcome for each scenario.

Find the best outcome for each measure using the MAX function.

Use SUMPRODUCT to compute the product of the best outcomes by the probabilities and find the difference between this and the best expected value yielding the EVPI.

**Program 3.1A**

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# Using Excel

## Output Results for the Thompson Lumber Problem Using Excel QM

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **Thompson Lumber** | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | **Decision Tables** | | | | | | | | |
| 4 | Enter the profits or costs in the main body of the data table. Enter probabilities in the | | | | | | | | |
| 5 | first row if you want to compute the expected value. | | | | | | | | |
| 6 | Data | | | | Results | | | | |
| 7 | Profit | Favorable Market | Unfavorable Market | | EMV | Minimum | Maximum | | Hurwicz |
| 8 | Probability | 0.5 | 0.5 | | | | | coefficient | 0.8 |
| 9 | Large Plant | 200000 | -180000 | | 10000 | -180000 | 200000 | | 124000 |
| 10 | Small plant | 100000 | -20000 | | 40000 | -20000 | 100000 | | 76000 |
| 11 | Do nothing | 0 | 0 | | 0 | 0 | 0 | | |
| 12 | | | | **Maximum** | **40000** | **0** | **200000** | | **124000** |
| 13 | | | | | | | | | |
| 14 | **Expected Value of Perfect Information** | | | | | | | | |
| 15 | Column best | 200000 | 0 | | 100000 | <-Expected value under certainty | | | |
| 16 | | | | | 40000 | <-Best expected value | | | |
| 17 | | | | | 60000 | <-Expected value of perfect information | | | |
| 18 | | | | | | | | | |
| 19 | **Regret** | | | | | | | | |
| 20 | | Favorable Market | Unfavorable Market | | Expected | Maximum | | | |
| 21 | Probability | 0.5 | 0.5 | | | | | | |
| 22 | Large Plant | 0 | 180000 | | 90000 | 180000 | | | |
| 23 | Small plant | 100000 | 20000 | | 60000 | 100000 | | | |
| 24 | Do nothing | 200000 | 0 | | 100000 | 200000 | | | |
| 25 | | | | **Minimum** | **60000** | **100000** | | | |

**Program 3.1B**

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# Decision Trees

- Any problem that can be presented in a decision table can also be graphically represented in a *decision tree.*
- Decision trees are most beneficial when a sequence of decisions must be made.
- All decision trees contain *decision points* or *nodes,* from which one of several alternatives may be chosen.
- All decision trees contain *state-of-nature points* or *nodes,* out of which one state of nature will occur.

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# Five Steps of Decision Tree Analysis

1. Define the problem.
2. Structure or draw the decision tree.
3. Assign probabilities to the states of nature.
4. Estimate payoffs for each possible combination of alternatives and states of nature.
5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node.

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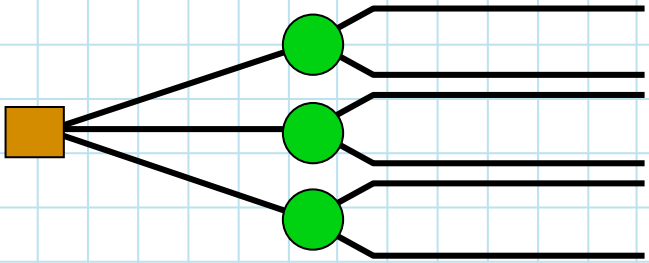
# Structure of Decision Trees

- Trees start from left to right.
- Trees represent decisions and outcomes in sequential order.
  - Squares represent decision nodes.
  - Circles represent states of nature nodes.
  - Lines or branches connect the decisions nodes and the states of nature.



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# Thompson's Decision Tree

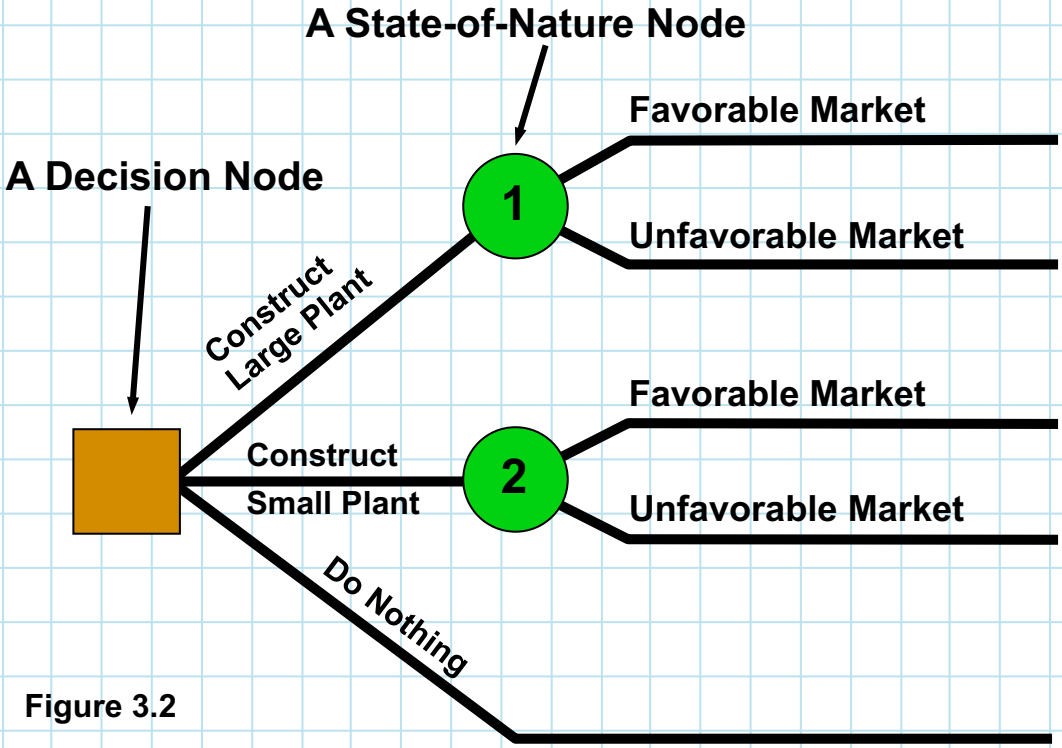


Figure 3.2

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# Thompson's Decision Tree

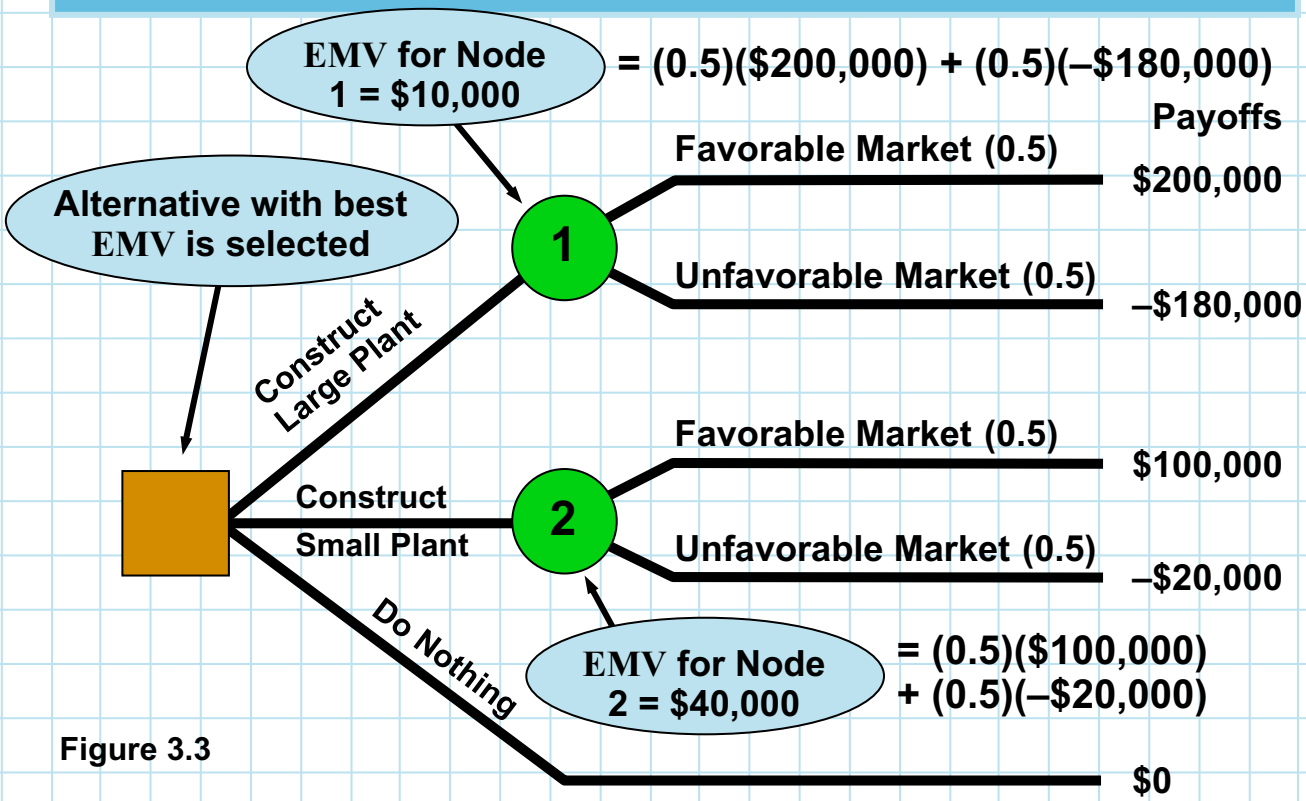


Figure 3.3

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# Thompson's Complex Decision Tree

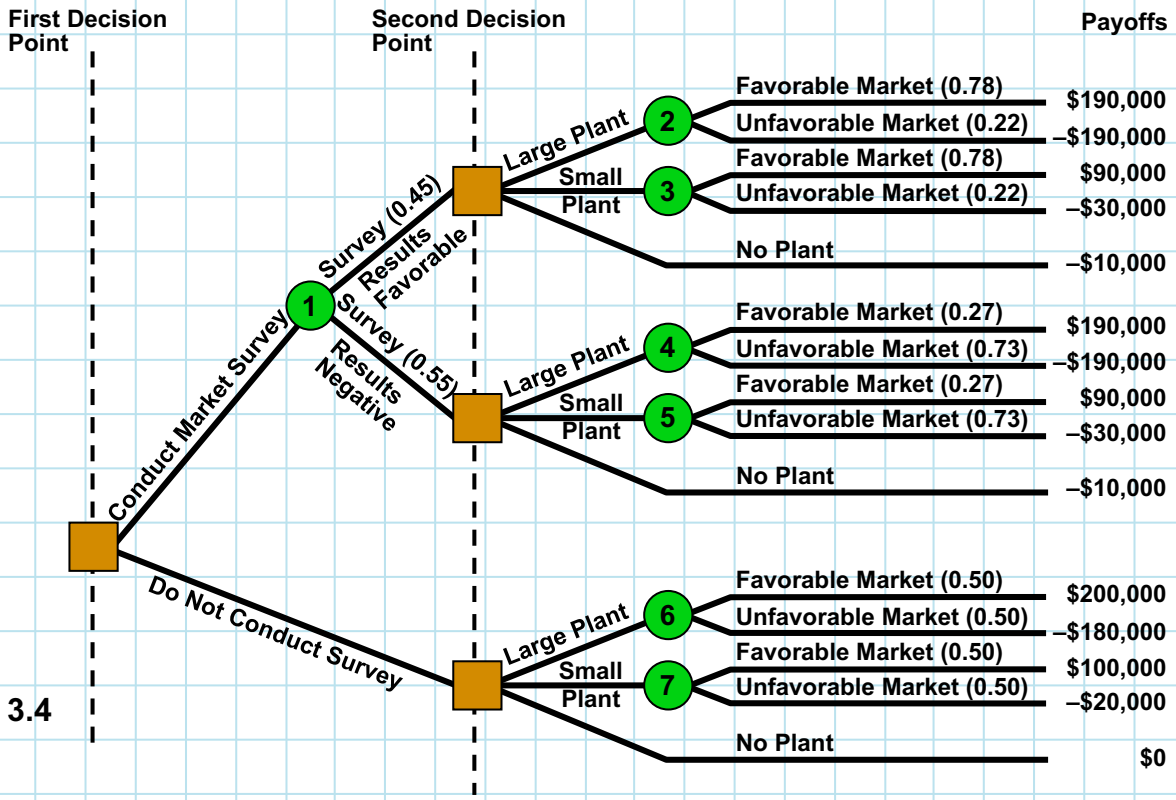


Figure 3.4

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# Thompson's Complex Decision Tree

1. Given favorable survey results,

EMV(node 2) = EMV(large plant | positive survey)
= (0.78)($190,000) + (0.22)(–$190,000) = $106,400

EMV(node 3) = EMV(small plant | positive survey)
= (0.78)($90,000) + (0.22)(–$30,000) = $63,600

EMV for no plant = –$10,000

2. Given negative survey results,

EMV(node 4) = EMV(large plant | negative survey)
= (0.27)($190,000) + (0.73)(–$190,000) = –$87,400

EMV(node 5) = EMV(small plant | negative survey)
= (0.27)($90,000) + (0.73)(–$30,000) = $2,400

EMV for no plant = –$10,000

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# Thompson's Complex Decision Tree

3. Compute the expected value of the market survey,

EMV(node 1) = EMV(conduct survey)
= (0.45)($106,400) + (0.55)($2,400)
= $47,880 + $1,320 = $49,200

4. If the market survey is not conducted,

EMV(node 6) = EMV(large plant)
= (0.50)($200,000) + (0.50)(–$180,000) = $10,000

EMV(node 7) = EMV(small plant)
= (0.50)($100,000) + (0.50)(–$20,000) = $40,000

EMV for no plant = $0

5. The best choice is to seek marketing information.

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# Thompson's Complex Decision Tree

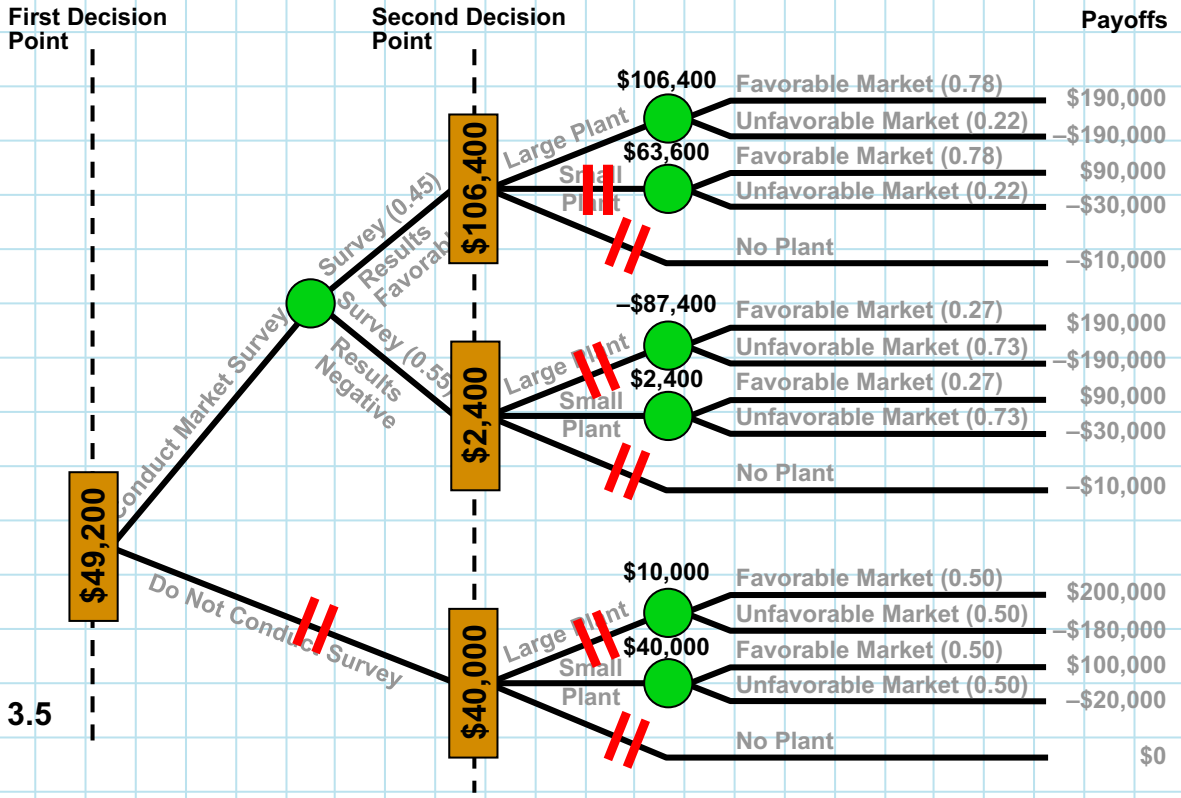


Figure 3.5

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# Expected Value of Sample Information

- **Suppose Thompson wants to know the actual value of doing the survey.**

$$\text{EVSI} = \left(\begin{array}{c}\text{Expected value}\\ \textit{with}\text{ sample}\\ \text{information, assuming}\\ \text{no cost to gather it}\end{array}\right) - \left(\begin{array}{c}\text{Expected value}\\ \text{of best decision}\\ \textit{without}\text{ sample}\\ \text{information}\end{array}\right)$$

$$= (\text{EV with sample information + cost}) - (\text{EV without sample information})$$

$$\text{EVSI} = (\$49{,}200 + \$10{,}000) - \$40{,}000 = \$19{,}200$$

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# Sensitivity Analysis

- How sensitive are the decisions to changes in the probabilities?
  - How sensitive is our decision to the probability of a favorable survey result?
  - That is, if the probability of a favorable result ($p$ = .45) where to change, would we make the same decision?
  - How much could it change before we would make a different decision?

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# Sensitivity Analysis

$p$ = probability of a favorable survey result

$(1 - p)$ = probability of a negative survey result

$$\text{EMV(node 1)} = (\$106{,}400)p + (\$2{,}400)(1 - p)$$
$$= \$104{,}000p + \$2{,}400$$

We are indifferent when the EMV of node 1 is the same as the EMV of not conducting the survey, \$40,000

$$\$104{,}000p + \$2{,}400 = \$40{,}000$$
$$\$104{,}000p = \$37{,}600$$
$$p = \$37{,}600/\$104{,}000 = 0.36$$

If $p<0.36$, do not conduct the survey. If $p>0.36$, conduct the survey.

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# Bayesian Analysis

- There are many ways of getting probability data. It can be based on:
  - Management's experience and intuition.
  - Historical data.
  - Computed from other data using Bayes' theorem.
- Bayes' theorem incorporates initial estimates and information about the accuracy of the sources.
- It also allows the revision of initial estimates based on new information.

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# Calculating Revised Probabilities

- In the Thompson Lumber case we used these four conditional probabilities:

$P$ (favorable market(FM) | survey results positive) = 0.78
$P$ (unfavorable market(UM) | survey results positive) = 0.22
$P$ (favorable market(FM) | survey results negative) = 0.27
$P$ (unfavorable market(UM) | survey results negative) = 0.73

- But how were these calculated?
- The prior probabilities of these markets are:

$P$ (FM) = 0.50
$P$ (UM) = 0.50

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# Calculating Revised Probabilities

- Through discussions with experts Thompson has learned the information in the table below.
- He can use this information and Bayes' theorem to calculate posterior probabilities.

| RESULT OF SURVEY | STATE OF NATURE | |
|---|---|---|
| | FAVORABLE MARKET (FM) | UNFAVORABLE MARKET (UM) |
| Positive (predicts favorable market for product) | $P$ (survey positive \| FM) = 0.70 | $P$ (survey positive \| UM) = 0.20 |
| Negative (predicts unfavorable market for product) | $P$ (survey negative \| FM) = 0.30 | $P$ (survey negative \| UM) = 0.80 |

Table 3.12

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# Calculating Revised Probabilities

- Recall Bayes' theorem:

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B \mid A) \times P(A) + P(B \mid A') \times P(A')}$$

where

$$A, B = \text{any two events}$$
$$A' = \text{complement of } A$$

For this example, $A$ will represent a favorable market and $B$ will represent a positive survey.

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# Calculating Revised Probabilities

- $P(FM \mid \text{survey positive})$

$$= \frac{P(\text{survey positive} \mid FM) \times P(FM)}{P(\text{survey positive} \mid FM) \times P(FM) + P(\text{survey positive} \mid UM) \times P(UM)}$$

$$= \frac{(0.70)(0.50)}{(0.70)(0.50) + (0.20)(0.50)} = \frac{0.35}{0.45} = 0.78$$

- $P(UM \mid \text{survey positive})$

$$= \frac{P(\text{survey positive} \mid UM) \times P(UM)}{P(\text{survey positive} \mid UM) \times P(UM) + P(\text{survey positive} \mid FM) \times P(FM)}$$

$$= \frac{(0.20)(0.50)}{(0.20)(0.50) + (0.70)(0.50)} = \frac{0.10}{0.45} = 0.22$$

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# Calculating Revised Probabilities

## Probability Revisions Given a Positive Survey

| STATE OF NATURE | CONDITIONAL PROBABILITY $P$(SURVEY POSITIVE \| STATE OF NATURE) | PRIOR PROBABILITY | | JOINT PROBABILITY | POSTERIOR PROBABILITY: $P$(STATE OF NATURE \| SURVEY POSITIVE) | |
|---|---|---|---|---|---|---|
| FM | 0.70 | X 0.50 | = | 0.35 | 0.35/0.45 = | 0.78 |
| UM | 0.20 | X 0.50 | = | 0.10 | 0.10/0.45 = | 0.22 |
| | | $P$(survey results positive) = | | 0.45 | | 1.00 |

Table 3.13

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# Calculating Revised Probabilities

- $P(FM \mid \text{survey negative})$

$$= \frac{P(\text{survey negative} \mid FM) \times P(FM)}{P(\text{survey negative} \mid FM) \times P(FM) + P(\text{survey negative} \mid UM) \times P(UM)}$$

$$= \frac{(0.30)(0.50)}{(0.30)(0.50) + (0.80)(0.50)} = \frac{0.15}{0.55} = 0.27$$

- $P(UM \mid \text{survey negative})$

$$= \frac{P(\text{survey negative} \mid UM) \times P(UM)}{P(\text{survey negative} \mid UM) \times P(UM) + P(\text{survey negative} \mid FM) \times P(FM)}$$

$$= \frac{(0.80)(0.50)}{(0.80)(0.50) + (0.30)(0.50)} = \frac{0.40}{0.55} = 0.73$$

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# Calculating Revised Probabilities

## Probability Revisions Given a Negative Survey

| STATE OF NATURE | CONDITIONAL PROBABILITY $P$(SURVEY NEGATIVE \| STATE OF NATURE) | PRIOR PROBABILITY | | POSTERIOR PROBABILITY: JOINT PROBABILITY | POSTERIOR PROBABILITY: $P$(STATE OF NATURE \| SURVEY NEGATIVE) |
|---|---|---|---|---|---|
| FM | 0.30 | X 0.50 | = | 0.15 | 0.15/0.55 = 0.27 |
| UM | 0.80 | X 0.50 | = | 0.40 | 0.40/0.55 = 0.73 |
| | | $P$(survey results positive) | = | 0.55 | 1.00 |

Table 3.14

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# Using Excel

## Formulas Used for Bayes' Calculations in Excel

A1 = Bayes Theorem for Thompson Lumber Example

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **Bayes Theorem for Thompson Lumber Example** | | | | |
| 2 | | | | | |
| 3 | **Fill in cells B7, B8, and C7.** | | | | |
| 4 | | | | | |
| 5 | **Probability Revisions Given a Positive Survey** | | | | |
| 6 | State of Nature | P(Sur.Pos.\|state of nature) | Prior Prob. | Joint Prob. | Posterior Probability |
| 7 | FM | 0.7 | 0.5 | =B7*C7 | =D7/$D$9 |
| 8 | UM | 0.2 | =1-C7 | =B8*C8 | =D8/$D$9 |
| 9 | | | P(Sur.pos.)= | =SUM(D7:D8) | |
| 10 | | | | | |
| 11 | **Probability Revisions Given a Negative Survey** | | | | |
| 12 | State of Nature | P(Sur.Pos.\|state of nature) | Prior Prob. | Joint Prob. | Posterior Probability |
| 13 | FM | =1-B7 | =C7 | =B13*C13 | =D13/$D$15 |
| 14 | UM | =1-B8 | =C8 | =B14*C14 | =D14/$D$15 |
| 15 | | | P(Sur.neg.)= | =SUM(D13:D14) | |
| 16 | | | | | |

Enter *P*(Favorable Market) in cell C7.

Enter *P*(Survey positive | Favorable Market) in cell B7.

Enter *P*(Survey positive | Unfavorable Market) in cell B8.

**Program 3.2A**

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# Using Excel

## Results of Bayes' Calculations in Excel

A1 = Bayes Theorem for Thompson Lumber Example

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **Bayes Theorem for Thompson Lumber Example** | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | **Fill in cells B7, B8, and C7.** | | | | | | | | |
| 4 | | | | | | | | | |
| 5 | **Probability Revisions Given a Positive Survey** | | | | | | | | |
| 6 | State of Nature | P(Sur.Pos.\|state of nature) | Prior Prob. | Joint Prob. | Posterior Probability | | | | |
| 7 | FM | 0.70 | 0.50 | 0.35 | 0.78 | | | | |
| 8 | UM | 0.20 | 0.50 | 0.10 | 0.22 | | | | |
| 9 | | | P(Sur.pos.)= | 0.45 | | | | | |
| 10 | | | | | | | | | |
| 11 | **Probability Revisions Given a Negative Survey** | | | | | | | | |
| 12 | State of Nature | P(Sur.Pos.\|state of nature) | Prior Prob. | Joint Prob. | Posterior Probability | | | | |
| 13 | FM | 0.30 | 0.50 | 0.15 | 0.27 | | | | |
| 14 | UM | 0.80 | 0.50 | 0.40 | 0.73 | | | | |
| 15 | | | P(Sur.neg.)= | 0.55 | | | | | |
| 16 | | | | | | | | | |

**Program 3.2B**

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## *Potential Problems Using Survey Results*

- We can not always get the necessary data for analysis.
- Survey results may be based on cases where an action was taken.
- Conditional probability information may not be as accurate as we would like.

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## *Utility Theory*

- Monetary value is not always a true indicator of the overall value of the result of a decision.
- The overall value of a decision is called *utility.*
- Economists assume that rational people make decisions to maximize their utility.

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# Utility Theory

## Your Decision Tree for the Lottery Ticket

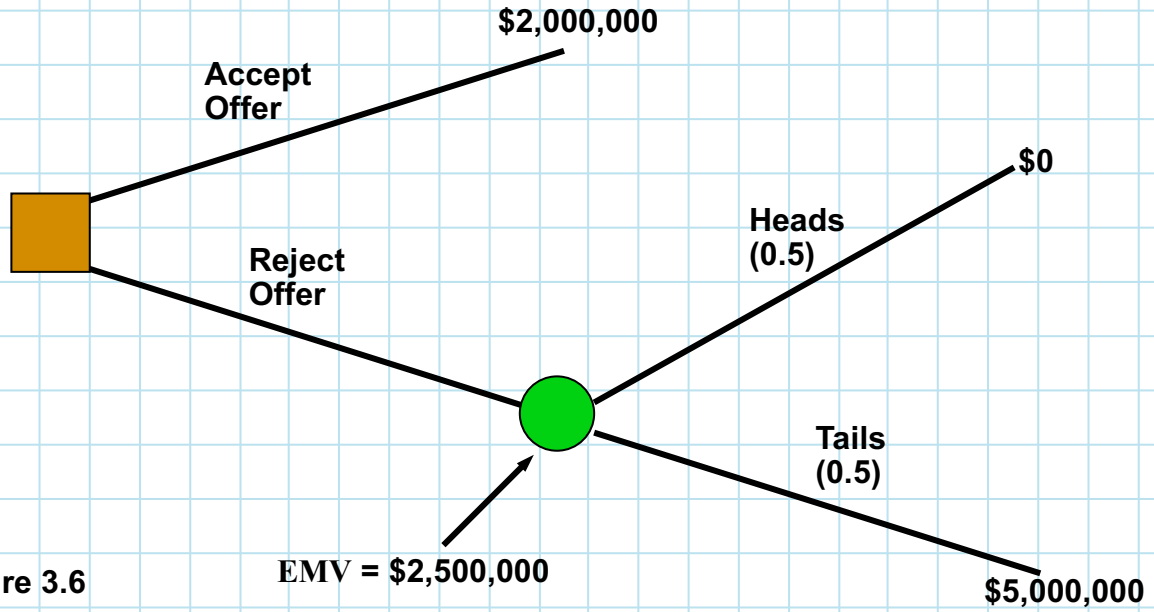


Figure 3.6

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# Utility Theory

- ***Utility assessment*** **assigns the worst outcome a utility of 0, and the best outcome, a utility of 1.**
- **A *standard gamble* is used to determine utility values.**
- **When you are indifferent, your utility values are equal.**

**Expected utility of alternative 2 = Expected utility of alternative 1**

**Utility of other outcome = ($p$)(utility of best outcome, which is 1) + (1 – $p$)(utility of the worst outcome, which is 0)**

**Utility of other outcome = ($p$)(1) + (1 – $p$)(0) = $p$**

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# Standard Gamble for Utility Assessment

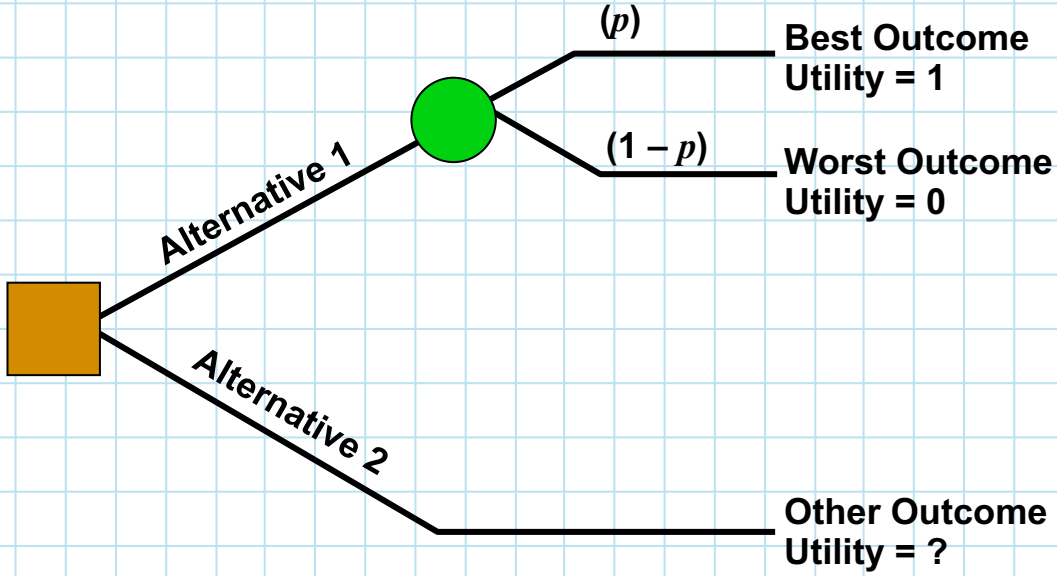


Figure 3.7

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# Investment Example

- Jane Dickson wants to construct a utility curve revealing her preference for money between $0 and $10,000.
- A utility curve plots the utility value versus the monetary value.
- An investment in a bank will result in $5,000.
- An investment in real estate will result in $0 or $10,000.
- Unless there is an 80% chance of getting $10,000 from the real estate deal, Jane would prefer to have her money in the bank.
- So if $p$ = 0.80, Jane is indifferent between the bank or the real estate investment.

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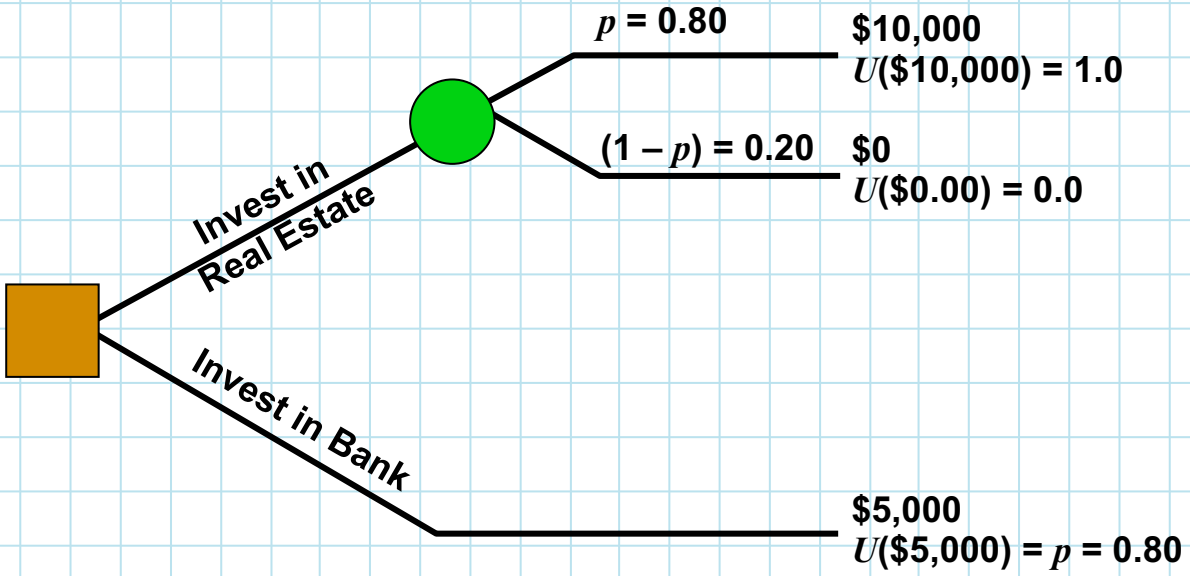
# Investment Example




Utility for \$5,000 = $U$(\$5,000) = $pU$(\$10,000) + (1 – $p$)$U$(\$0)
= (0.8)(1) + (0.2)(0) = 0.8

Figure 3.8

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# Investment Example

- We can assess other utility values in the same way.
- For Jane these are:

Utility for \$7,000 = 0.90
Utility for \$3,000 = 0.50

- Using the three utilities for different dollar amounts, she can construct a utility curve.

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# Utility Curve

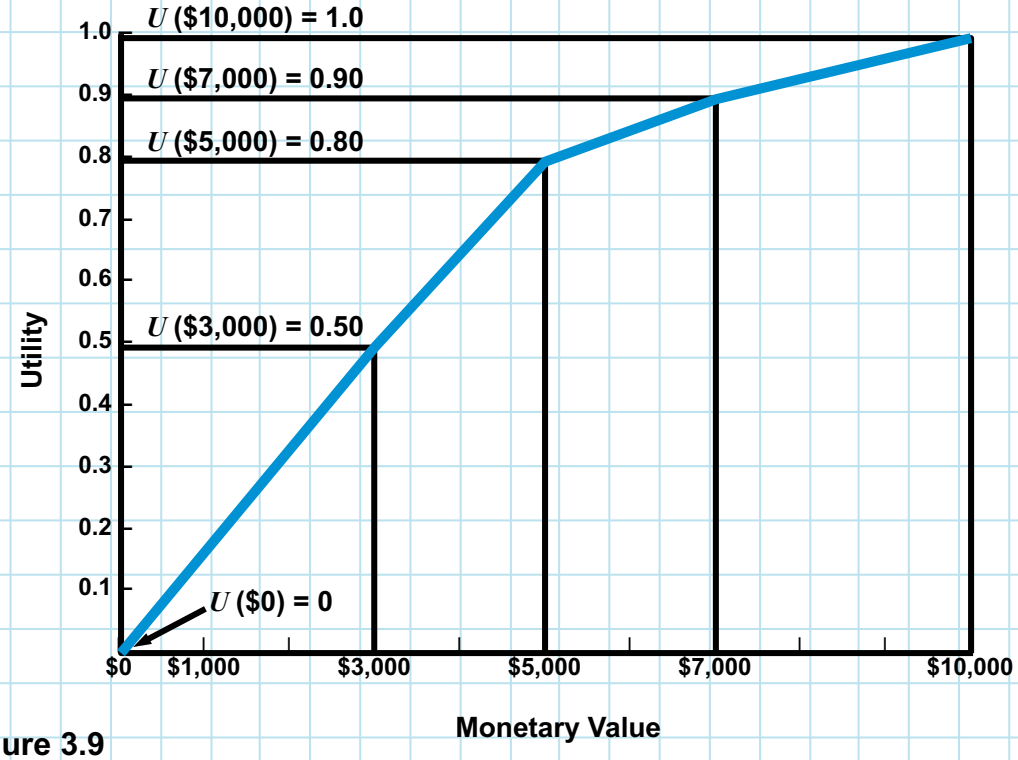


Figure 3.9

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# Utility Curve

- Jane's utility curve is typical of a risk avoider.
  - She gets less utility from greater risk.
  - She avoids situations where high losses might occur.
  - As monetary value increases, her utility curve increases at a slower rate.
- A risk seeker gets more utility from greater risk
  - As monetary value increases, the utility curve increases at a faster rate.
- Someone with risk indifference will have a linear utility curve.

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# Preferences for Risk

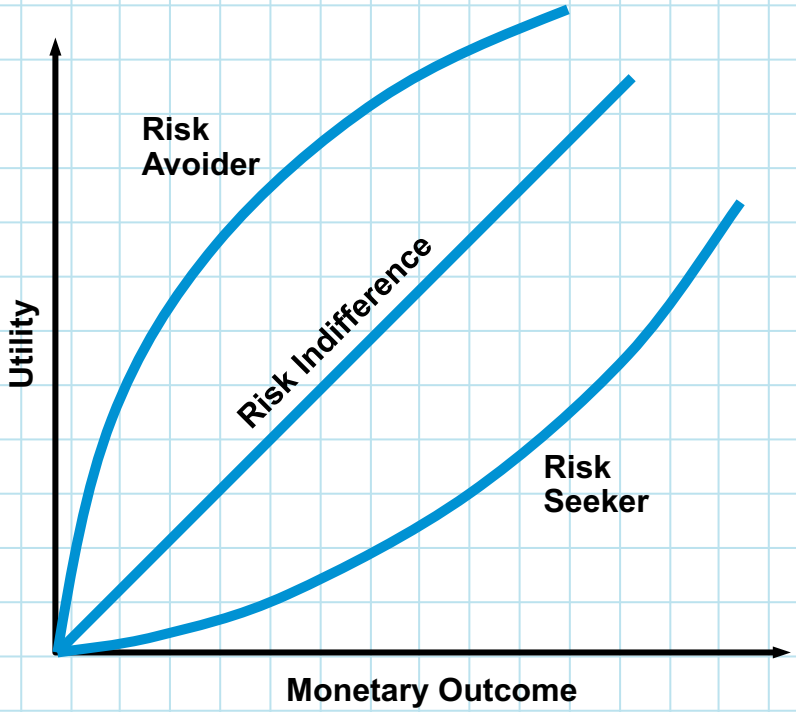


Figure 3.10

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# Utility as a Decision-Making Criteria

- Once a utility curve has been developed it can be used in making decisions.
- This replaces monetary outcomes with utility values.
- The expected utility is computed instead of the EMV.

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# Utility as a Decision-Making Criteria

- Mark Simkin loves to gamble.
- He plays a game tossing thumbtacks in the air.
- If the thumbtack lands point up, Mark wins $10,000.
- If the thumbtack lands point down, Mark loses $10,000.
- Mark believes that there is a 45% chance the thumbtack will land point up.
- Should Mark play the game (alternative 1)?

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# Utility as a Decision-Making Criteria

## Decision Facing Mark Simkin

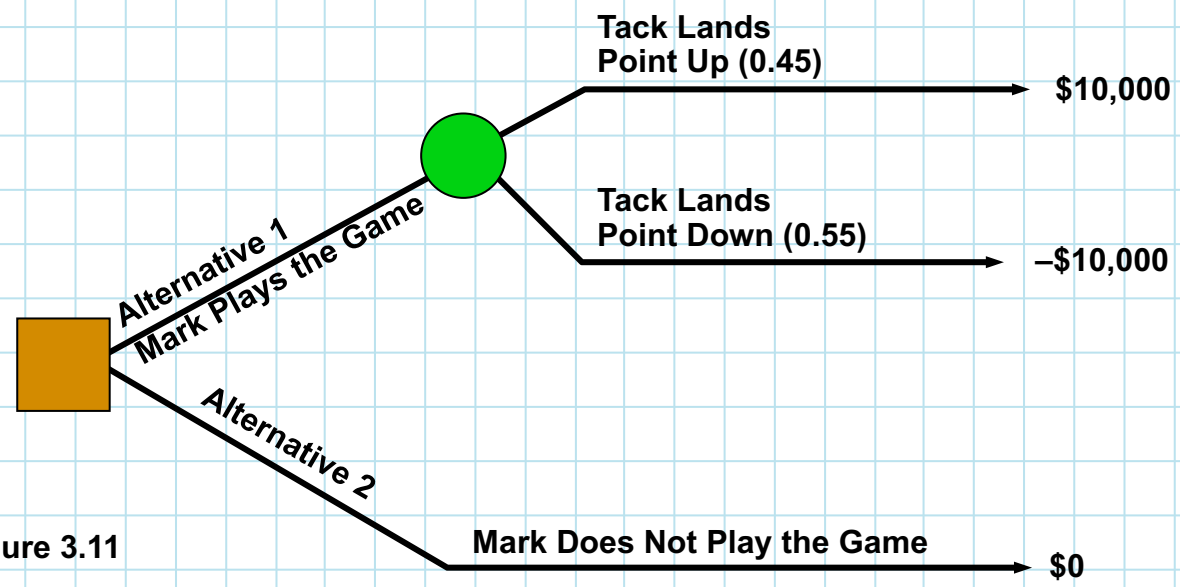


Figure 3.11

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# Utility as a Decision-Making Criteria

- **Step 1– Define Mark's utilities.**

$$U(-\$10{,}000) = 0.05$$
$$U(\$0) = 0.15$$
$$U(\$10{,}000) = 0.30$$

- **Step 2 – Replace monetary values with utility values.**

$$E(\text{alternative 1: play the game}) = (0.45)(0.30) + (0.55)(0.05)$$
$$= 0.135 + 0.027 = 0.162$$
$$E(\text{alternative 2: don't play the game}) = 0.15$$

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# Utility Curve for Mark Simkin

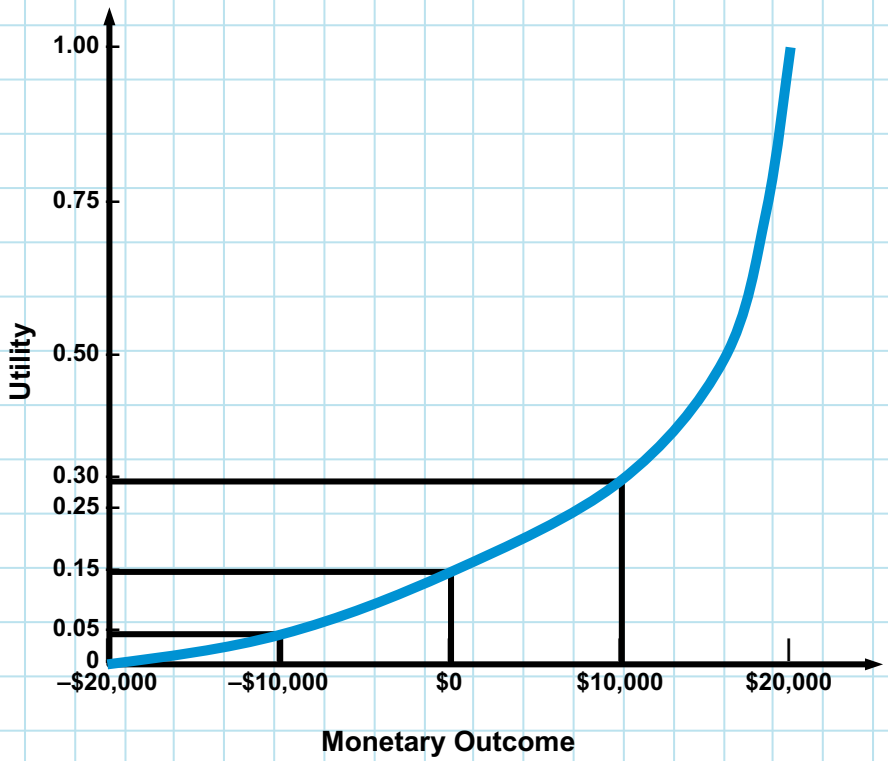


Figure 3.12

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# Utility as a Decision-Making Criteria

## Using Expected Utilities in Decision Making

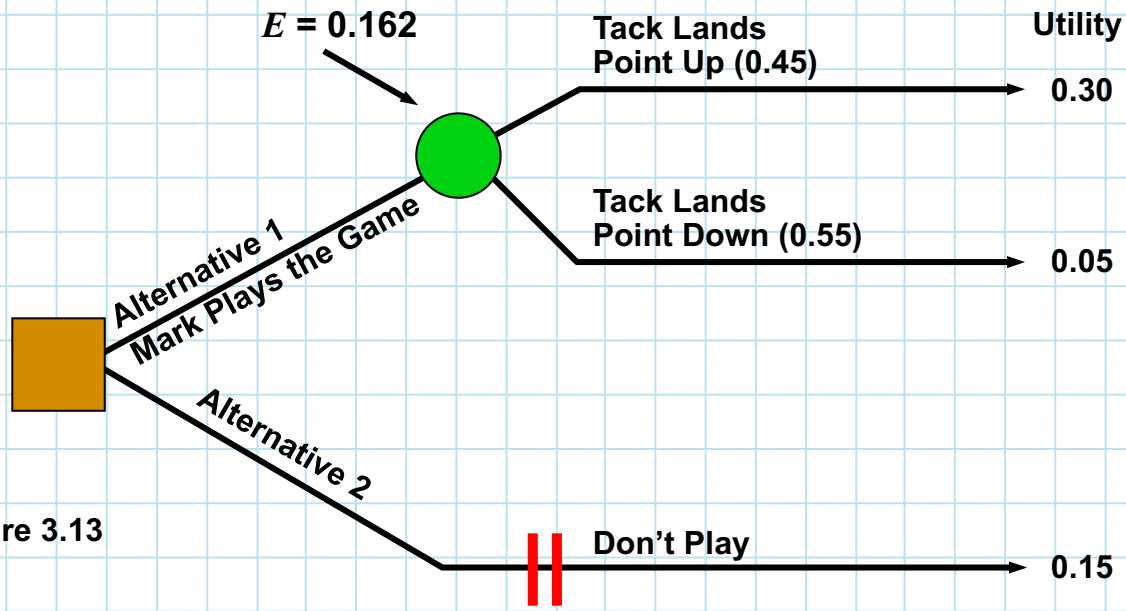


Figure 3.13



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# Chapter 5

# Forecasting

To accompany
*Quantitative Analysis for Management, Eleventh Edition*,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

## Learning Objectives

**After completing this chapter, students will be able to:**

1. Understand and know when to use various families of forecasting models.
2. Compare moving averages, exponential smoothing, and other time-series models.
3. Seasonally adjust data.
4. Understand Delphi and other qualitative decision-making approaches.
5. Compute a variety of error measures.

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# Chapter Outline

5.1 Introduction
5.2 Types of Forecasts
5.3 Scatter Diagrams and Time Series
5.4 Measures of Forecast Accuracy
5.5 Time-Series Forecasting Models
5.6 Monitoring and Controlling Forecasts

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# Introduction

- Managers are always trying to reduce uncertainty and make better estimates of what will happen in the future.
  - This is the main purpose of forecasting.
  - Some firms use subjective methods: seat-of-the pants methods, intuition, experience.
  - There are also several quantitative techniques, including:
    - Moving averages
    - Exponential smoothing
    - Trend projections
    - Least squares regression analysis

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# Introduction

- Eight steps to forecasting:
  1. Determine the use of the forecast—what objective are we trying to obtain?
  2. Select the items or quantities that are to be forecasted.
  3. Determine the time horizon of the forecast.
  4. Select the forecasting model or models.
  5. Gather the data needed to make the forecast.
  6. Validate the forecasting model.
  7. Make the forecast.
  8. Implement the results.

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# Introduction

- These steps are a systematic way of initiating, designing, and implementing a forecasting system.
- When used regularly over time, data is collected routinely and calculations performed automatically.
- There is seldom one superior forecasting system.
  - Different organizations may use different techniques.
  - Whatever tool works best for a firm is the one that should be used.

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# Forecasting Models

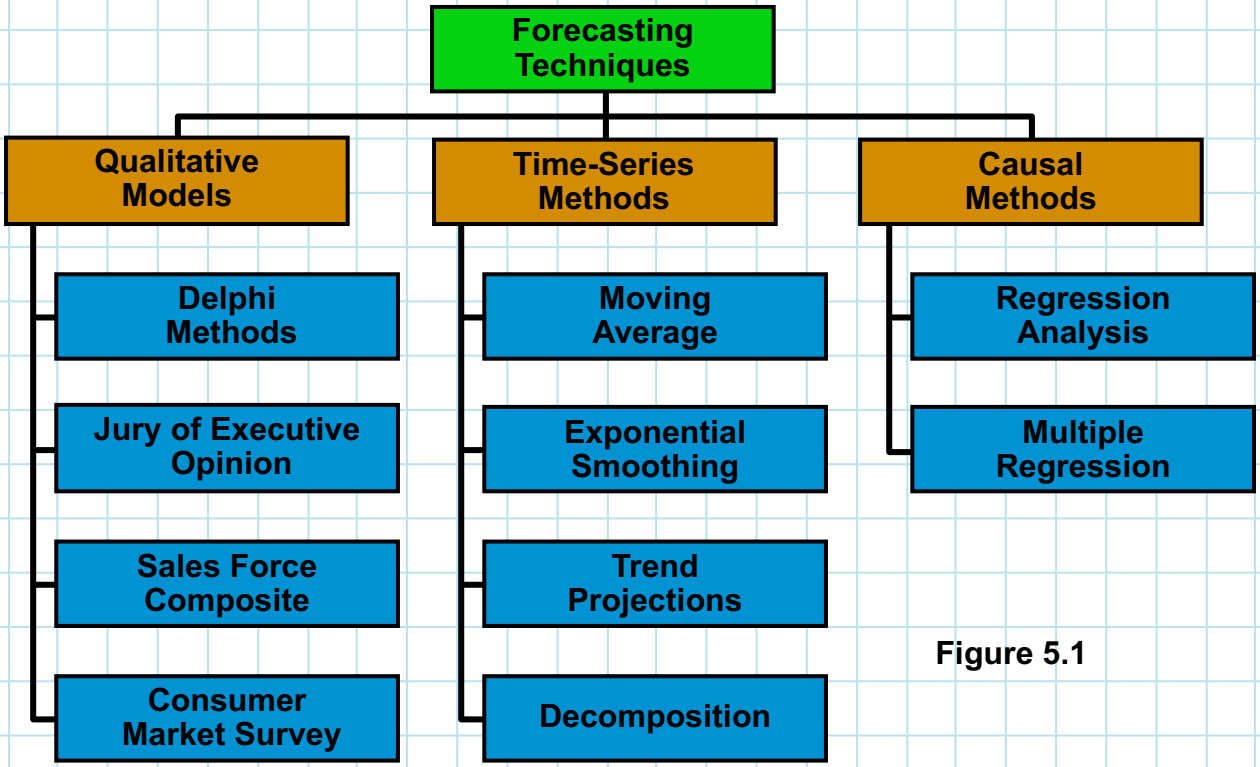


Figure 5.1

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# Qualitative Models

- *Qualitative models* incorporate judgmental or subjective factors.
- These are useful when subjective factors are thought to be important or when accurate quantitative data is difficult to obtain.
- Common qualitative techniques are:
  - Delphi method.
  - Jury of executive opinion.
  - Sales force composite.
  - Consumer market surveys.

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# Qualitative Models

- *Delphi Method* – This is an iterative group process where (possibly geographically dispersed) *respondents* provide input to *decision makers.*
- *Jury of Executive Opinion* – This method collects opinions of a small group of high-level managers, possibly using statistical models for analysis.
- *Sales Force Composite* – This allows individual salespersons estimate the sales in their region and the data is compiled at a district or national level.
- *Consumer Market Survey* – Input is solicited from customers or potential customers regarding their purchasing plans.

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# Time-Series Models

- *Time-series models* attempt to predict the future based on the past.
- Common time-series models are:
  - Moving average.
  - Exponential smoothing.
  - Trend projections.
  - Decomposition.
- Regression analysis is used in trend projections and one type of decomposition model.

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# Causal Models

- *Causal models* use variables or factors that might influence the quantity being forecasted.
- The objective is to build a model with the best statistical relationship between the variable being forecast and the independent variables.
- Regression analysis is the most common technique used in causal modeling.

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# Scatter Diagrams

Wacker Distributors wants to forecast sales for three different products (annual sales in the table, in units):

| YEAR | TELEVISION SETS | RADIOS | COMPACT DISC PLAYERS |
|---|---|---|---|
| 1 | 250 | 300 | 110 |
| 2 | 250 | 310 | 100 |
| 3 | 250 | 320 | 120 |
| 4 | 250 | 330 | 140 |
| 5 | 250 | 340 | 170 |
| 6 | 250 | 350 | 150 |
| 7 | 250 | 360 | 160 |
| 8 | 250 | 370 | 190 |
| 9 | 250 | 380 | 200 |
| 10 | 250 | 390 | 190 |

Table 5.1

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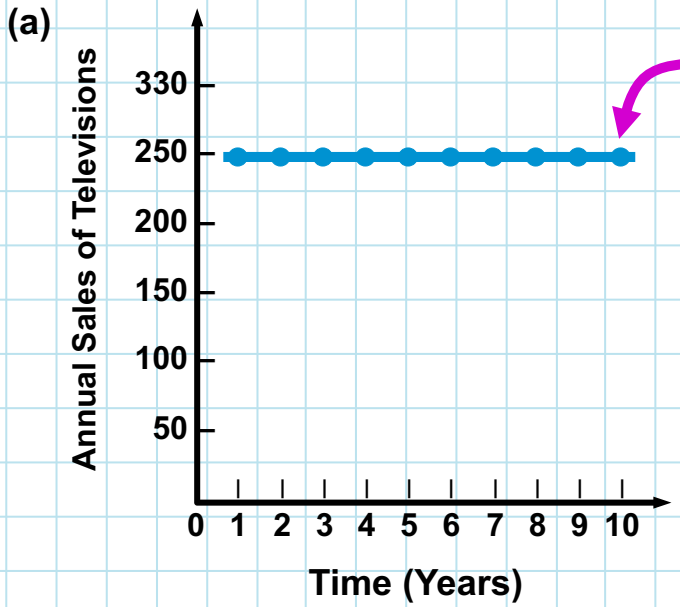
# Scatter Diagram for TVs




- Sales appear to be constant over time

  Sales = 250
- A good estimate of sales in year 11 is 250 televisions

Figure 5.2a

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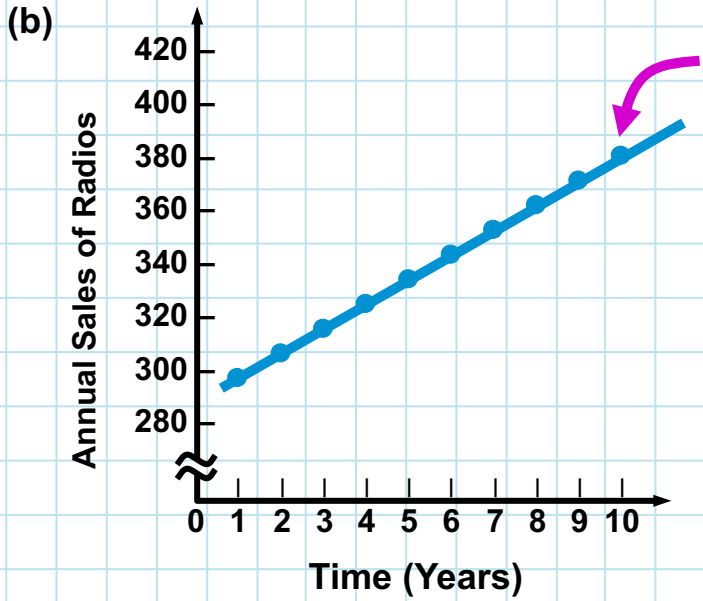
# Scatter Diagram for Radios




- Sales appear to be increasing at a constant rate of 10 radios per year

Sales = 290 + 10(Year)

- A reasonable estimate of sales in year 11 is 400 radios.

Figure 5.2b

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# Scatter Diagram for CD Players

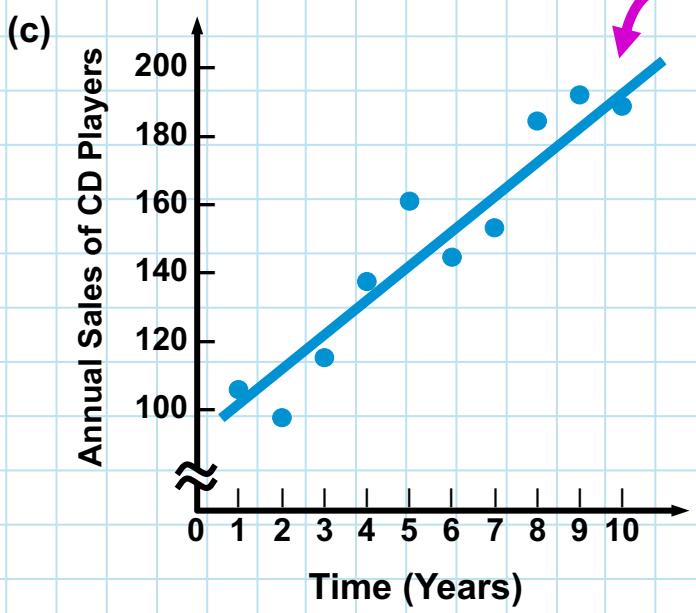


Figure 5.2c

- This trend line may not be perfectly accurate because of variation from year to year
- Sales appear to be increasing
- A forecast would probably be a larger figure each year

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# Measures of Forecast Accuracy

- We compare forecasted values with actual values to see how well one model works or to compare models.

Forecast error = Actual value – Forecast value

- One measure of accuracy is the *mean absolute deviation* (*MAD*):

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n}$$

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# Measures of Forecast Accuracy

## Using a *naïve* forecasting model we can compute the MAD:

| YEAR | ACTUAL SALES OF CD PLAYERS | FORECAST SALES | ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST) |
|---|---|---|---|
| 1 | 110 | — | — |
| 2 | 100 | 110 | \|100 – 110\| = 10 |
| 3 | 120 | 100 | \|120 – 110\| = 20 |
| 4 | 140 | 120 | \|140 – 120\| = 20 |
| 5 | 170 | 140 | \|170 – 140\| = 30 |
| 6 | 150 | 170 | \|150 – 170\| = 20 |
| 7 | 160 | 150 | \|160 – 150\| = 10 |
| 8 | 190 | 160 | \|190 – 160\| = 30 |
| 9 | 200 | 190 | \|200 – 190\| = 10 |
| 10 | 190 | 200 | \|190 – 200\| = 10 |
| 11 | — | 190 | — |
| | | | Sum of \|errors\| = 160 |
| | | | MAD = 160/9 = 17.8 |

Table 5.2

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# Measures of Forecast Accuracy

## Using a *naïve* forecasting model we can compute the MAD:

| YEAR | ACTUAL SALES OF CD PLAYERS | FORECAST SALES | ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST) |
|---|---|---|---|
| 1 | 110 | — | — |
| | | | – 110\| = 10 |
| | | | – 110\| = 20 |
| | | | – 120\| = 20 |
| | | | – 140\| = 30 |
| | | | – 170\| = 20 |
| | | | – 150\| = 10 |
| | | | – 160\| = 30 |
| 9 | 200 | 190 | \|200 – 190\| = 10 |
| 10 | 190 | 200 | \|190 – 200\| = 10 |
| 11 | — | 190 | — |
| | | | Sum of \|errors\| = 160 |
| | | | MAD = 160/9 = 17.8 |

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n} = \frac{160}{9} = 17.8$$

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Table 5.2

# Measures of Forecast Accuracy

- There are other popular measures of forecast accuracy.
- The *mean squared error:*

$$\text{MSE} = \frac{\sum(\text{error})^2}{n}$$

- The *mean absolute percent error:*

$$\text{MAPE} = \frac{\sum\left|\frac{\text{error}}{\text{actual}}\right|}{n}100\%$$

- And *bias* is the average error.

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# Time-Series Forecasting Models

- A time series is a sequence of evenly spaced events.
- Time-series forecasts predict the future based solely on the past values of the variable, and other variables are ignored.

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# Components of a Time-Series

A time series typically has four components:

1. *Trend* (*T*) is the gradual upward or downward movement of the data over time.
2. *Seasonality* (*S*) is a pattern of demand fluctuations above or below the trend line that repeats at regular intervals.
3. *Cycles* (*C*) are patterns in annual data that occur every several years.
4. *Random variations* (*R*) are "blips" in the data caused by chance or unusual situations, and follow no discernible pattern.

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# Decomposition of a Time-Series

Product Demand Charted over 4 Years, with Trend and Seasonality Indicated

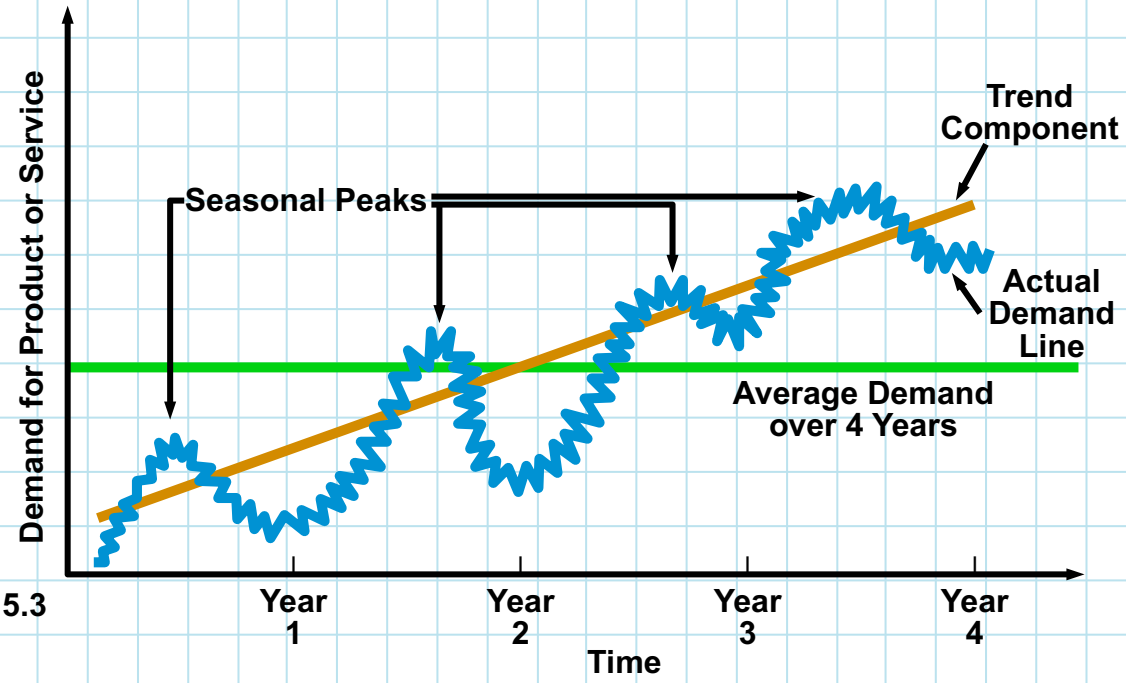


Figure 5.3

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# Decomposition of a Time-Series

- There are two general forms of time-series models:
  - The multiplicative model:

$$\text{Demand} = T \times S \times C \times R$$

  - The additive model:

$$\text{Demand} = T + S + C + R$$

- Models may be combinations of these two forms.
- Forecasters often assume errors are normally distributed with a mean of zero.

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# Moving Averages

- *Moving averages* can be used when demand is relatively steady over time.
- The next forecast is the average of the most recent $n$ data values from the time series.
- This methods tends to smooth out short-term irregularities in the data series.

$$\text{Moving average forecast} = \frac{\text{Sum of demands in previous } n \text{ periods}}{n}$$

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# Moving Averages

- Mathematically:

$$F_{t+1} = \frac{Y_t + Y_{t-1} + \ldots + Y_{t-n+1}}{n}$$

Where:

$F_{t+1}$ = forecast for time period $t$ + 1

$Y_t$ = actual value in time period $t$

$n$ = number of periods to average

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# Wallace Garden Supply

- Wallace Garden Supply wants to forecast demand for its Storage Shed.
- They have collected data for the past year.
- They are using a three-month moving average to forecast demand ($n$ = 3).

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# Wallace Garden Supply

| MONTH | ACTUAL SHED SALES | THREE-MONTH MOVING AVERAGE |
|---|---|---|
| January | 10 | |
| February | 12 | |
| March | 13 | |
| April | 16 | (10 + 12 + 13)/3 = 11.67 |
| May | 19 | (12 + 13 + 16)/3 = 13.67 |
| June | 23 | (13 + 16 + 19)/3 = 16.00 |
| July | 26 | (16 + 19 + 23)/3 = 19.33 |
| August | 30 | (19 + 23 + 26)/3 = 22.67 |
| September | 28 | (23 + 26 + 30)/3 = 26.33 |
| October | 18 | (26 + 30 + 28)/3 = 28.00 |
| November | 16 | (30 + 28 + 18)/3 = 25.33 |
| December | 14 | (28 + 18 + 16)/3 = 20.67 |
| January | — | (18 + 16 + 14)/3 = 16.00 |

Table 5.3

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# Weighted Moving Averages

- *Weighted moving averages* use weights to put more emphasis on previous periods.
- This is often used when a trend or other pattern is emerging.

$$F_{t+1} = \frac{\sum(\text{Weight in period } i)(\text{Actual value in period})}{\sum(\text{Weights})}$$

- Mathematically:

$$F_{t+1} = \frac{w_1 Y_t + w_2 Y_{t-1} + \ldots + w_n Y_{t-n+1}}{w_1 + w_2 + \ldots + w_n}$$

where

$w_i$ = weight for the $i^{th}$ observation

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# Wallace Garden Supply

- Wallace Garden Supply decides to try a weighted moving average model to forecast demand for its Storage Shed.
- They decide on the following weighting scheme:

| WEIGHTS APPLIED | PERIOD |
|---|---|
| 3 | Last month |
| 2 | Two months ago |
| 1 | Three months ago |

3 x Sales last month + 2 x Sales two months ago + 1 X Sales three months ago

6 — Sum of the weights

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# Wallace Garden Supply

| MONTH | ACTUAL SHED SALES | THREE-MONTH WEIGHTED MOVING AVERAGE |
|---|---|---|
| January | 10 | |
| February | 12 | |
| March | 13 | |
| April | 16 | [(3 X 13) + (2 X 12) + (10)]/6 = 12.17 |
| May | 19 | [(3 X 16) + (2 X 13) + (12)]/6 = 14.33 |
| June | 23 | [(3 X 19) + (2 X 16) + (13)]/6 = 17.00 |
| July | 26 | [(3 X 23) + (2 X 19) + (16)]/6 = 20.50 |
| August | 30 | [(3 X 26) + (2 X 23) + (19)]/6 = 23.83 |
| September | 28 | [(3 X 30) + (2 X 26) + (23)]/6 = 27.50 |
| October | 18 | [(3 X 28) + (2 X 30) + (26)]/6 = 28.33 |
| November | 16 | [(3 X 18) + (2 X 28) + (30)]/6 = 23.33 |
| December | 14 | [(3 X 16) + (2 X 18) + (28)]/6 = 18.67 |
| January | — | [(3 X 14) + (2 X 16) + (18)]/6 = 15.33 |

Table 5.4

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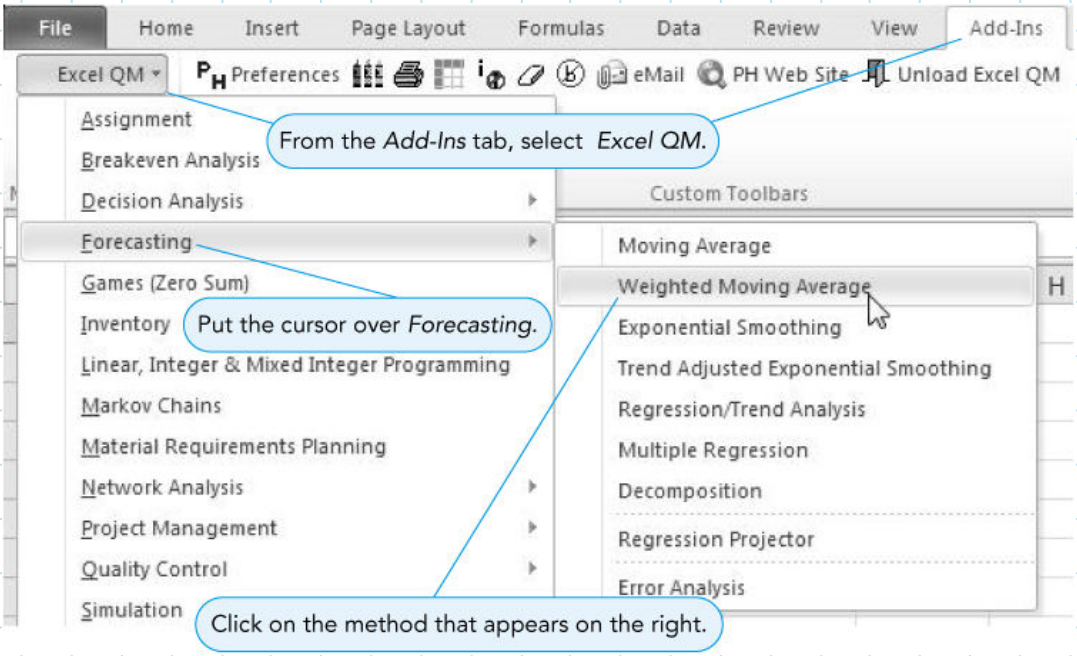
# Wallace Garden Supply

## Selecting the Forecasting Module in Excel QM

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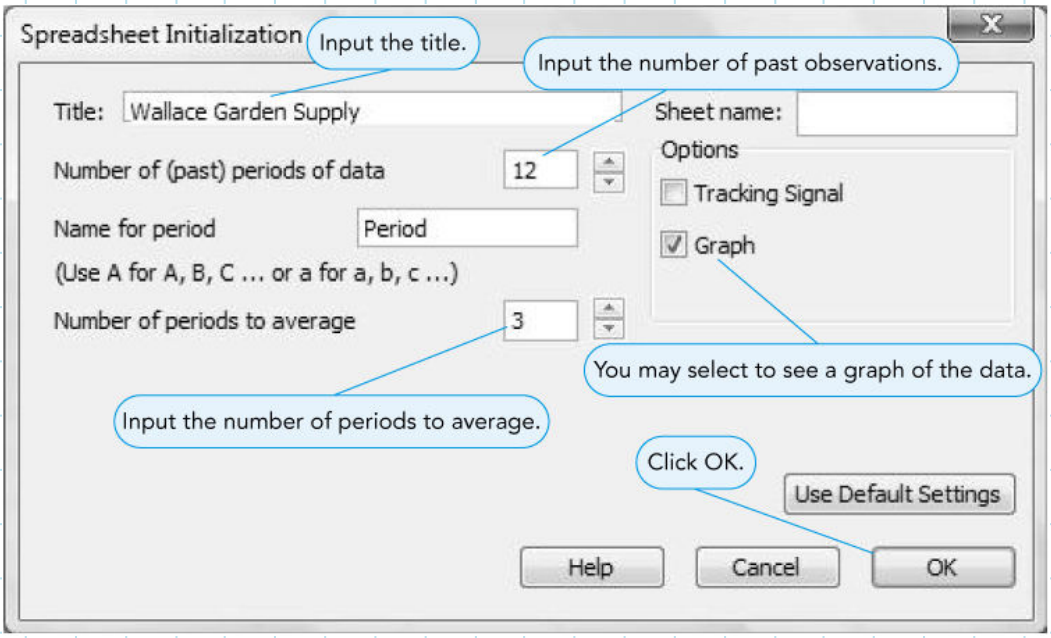
**Program 5.1A**



# Wallace Garden Supply

## Initialization Screen for Weighted Moving Average




**Program 5.1B**

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# Wallace Garden Supply

## Weighted Moving Average in Excel QM for Wallace Garden Supply

Input the past observations.

The names of the periods can be changed.

Past forecasts, errors, and measures of accuracy are shown.

Input the weight. Note that the highest weight is for the most recent observation.

The forecast for the next period is here.

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Wallace Garden | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | Forecasting | | Weighted moving averages - 3 period moving average | | | | | | |
| 6 | | | | | | | | | |
| 7 | Data | | | | Forecasts and Error Analysis | | | | |
| 8 | Period | Demand | Weights | | Forecast | Error | Absolute | Squared | Abs Pct Err |
| 9 | January | 10 | 1 | | | | | | |
| 10 | February | 12 | 2 | | | | | | |
| 11 | March | 13 | 3 | | | | | | |
| 12 | April | 16 | | | 12.1667 | 3.8333 | 3.8333 | 14.6944 | 23.96% |
| 13 | May | [illegible] | | | [illegible] | 4.6667 | 4.6667 | 21.7778 | 24.56% |
| 14 | June | [illegible] | | | [illegible] | 6 | 6 | 36 | 26.09% |
| 15 | July | [illegible] | | | [illegible] | 5.5 | 5.5 | 30.25 | 21.15% |
| 16 | August | [illegible] | | | [illegible] | 6.1667 | 6.1667 | 38.0278 | 20.56% |
| 17 | September | [illegible] | | | 27.5 | 0.5 | 0.5 | 0.25 | 01.79% |
| 18 | October | 18 | | | 28.3333 | -10.3333 | 10.3333 | 106.7778 | 57.41% |
| 19 | November | 16 | | | 23.3333 | -7.3333 | 7.3333 | 53.7778 | 45.83% |
| 20 | December | 14 | | | 18.6667 | -4.6667 | 4.6667 | 21.7778 | 33.33% |
| 21 | | | | | Total | 4.3333 | 49.0000 | 323.3333 | 254.68% |
| 22 | | | | | Average | 0.4815 | 5.4444 | 35.9259 | 28.30% |
| 23 | | | | | | Bias | MAD | MSE | MAPE |
| 24 | | | | | | | SE | 6.79636 | |
| 25 | Next period | 15.3333333 | | | | | | | |

Program 5.1C

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# Exponential Smoothing

- ***Exponential smoothing*** **is a type of moving average that is easy to use and requires little record keeping of data.**

**New forecast = Last period's forecast**
**+ $\alpha$(Last period's actual demand**
**– Last period's forecast)**

**Here $\alpha$ is a weight (or *smoothing constant*) in which $0 \le \alpha \le 1$.**

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# Exponential Smoothing

**Mathematically:**

$$F_{t+1} = F_t + \alpha(Y_t - F_t)$$

**Where:**

$F_{t+1}$ = new forecast (for time period $t$ + 1)
$F_t$ = pervious forecast (for time period t)
$\alpha$ = smoothing constant ($0 \leq \alpha \leq 1$)
$Y_t$ = pervious period's actual demand

**The idea is simple – the new estimate is the old estimate plus some fraction of the error in the last period.**

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# Exponential Smoothing Example

- In January, February's demand for a certain car model was predicted to be 142.
- Actual February demand was 153 autos
- Using a smoothing constant of $\alpha$ = 0.20, what is the forecast for March?

New forecast (for March demand) = 142 + 0.2(153 – 142)
= 144.2 or 144 autos

- If actual demand in March was 136 autos, the April forecast would be:

New forecast (for April demand) = 144.2 + 0.2(136 – 144.2)
= 142.6 or 143 autos

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# Selecting the Smoothing Constant

- Selecting the appropriate value for $\alpha$ is key to obtaining a good forecast.
- The objective is always to generate an accurate forecast.
- The general approach is to develop trial forecasts with different values of $\alpha$ and select the $\alpha$ that results in the lowest *MAD*.

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# Exponential Smoothing

**Port of Baltimore Exponential Smoothing Forecast for $\alpha$=0.1 and $\alpha$=0.5.**

| QUARTER | ACTUAL TONNAGE UNLOADED | FORECAST USING $\alpha$ =0.10 | FORECAST USING $\alpha$ =0.50 |
|---|---|---|---|
| 1 | 180 | 175 | 175 |
| 2 | 168 | 175.5 = 175.00 + 0.10(180 – 175) | 177.5 |
| 3 | 159 | 174.75 = 175.50 + 0.10(168 – 175.50) | 172.75 |
| 4 | 175 | 173.18 = 174.75 + 0.10(159 – 174.75) | 165.88 |
| 5 | 190 | 173.36 = 173.18 + 0.10(175 – 173.18) | 170.44 |
| 6 | 205 | 175.02 = 173.36 + 0.10(190 – 173.36) | 180.22 |
| 7 | 180 | 178.02 = 175.02 + 0.10(205 – 175.02) | 192.61 |
| 8 | 182 | 178.22 = 178.02 + 0.10(180 – 178.02) | 186.30 |
| 9 | ? | 178.60 = 178.22 + 0.10(182 – 178.22) | 184.15 |

Table 5.5

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# Exponential Smoothing

## Absolute Deviations and MADs for the Port of Baltimore Example

| QUARTER | ACTUAL TONNAGE UNLOADED | FORECAST WITH $\alpha = 0.10$ | ABSOLUTE DEVIATIONS FOR $\alpha = 0.10$ | FORECAST WITH $\alpha = 0.50$ | ABSOLUTE DEVIATIONS FOR $\alpha = 0.50$ |
|---|---|---|---|---|---|
| 1 | 180 | 175 | 5 | 175 | 5 |
| 2 | 168 | 175.5 | 7.5 | 177.5 | 9.5 |
| 3 | 159 | 174.75 | 15.75 | 172.75 | 13.75 |
| 4 | 175 | 173.18 | 1.82 | 165.88 | 9.12 |
| 5 | 190 | 173.36 | 16.64 | 170.44 | 19.56 |
| 6 | 205 | 175.02 | 29.98 | 180.22 | 24.78 |
| 7 | 180 | 178.02 | 1.98 | 192.61 | 12.61 |
| 8 | 182 | 178.22 | 3.78 | 186.30 | 4.3 |
| Sum of absolute deviations | | | 82.45 | | 98.63 |
| | | $MAD = \frac{\Sigma\lvert\text{deviations}\rvert}{n} =$ | 10.31 | $MAD =$ | 12.33 |

Table 5.6

Best choice

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# Port of Baltimore Exponential Smoothing Example in Excel QM

If initial forecast is given, enter it here. If you do not want to include the error for this initial forecast, cells E10:H10.

Port of Baltimore

Forecasting — Exponential smoothing

Enter alpha (between 0 and 1), enter the past demands in the shaded column then enter a starting forecast. If the starting forecast is not in the first period then delete the error analysis for all rows above the starting forecast.

Alpha

Enter the data and alpha.

| Data | | Forecasts and Error Analysis | | | | |
|---|---|---|---|---|---|---|
| Period | Demand | Forecast | Error | Absolut | Squared | Abs Pct E |
| Quarter 1 | 180 | 175 | 5 | 5 | 25 | 02.78% |
| Quarter 2 | 168 | 175.5 | -7.5 | 7.5 | 56.25 | 04.46% |
| Quarter 3 | 159 | 174.75 | -15.75 | 15.75 | 248.06 | 09.91% |
| Quarter 4 | 175 | 173.175 | 1.825 | 1.825 | 3.3306 | 01.04% |
| Quarter 5 | 190 | 173.358 | 16.643 | 16.643 | 276.97 | 08.76% |
| Quarter 6 | 205 | 175.022 | 29.978 | 29.978 | 898.7 | 14.62% |
| Quarter 7 | 180 | 178.02 | 1.9804 | 1.9804 | 3.9221 | 01.10% |
| Quarter 8 | 182 | 178.218 | 3.7824 | 3.7824 | 14.306 | 0.02078 |
| | | Total | 35.959 | 82.459 | 1526.5 | 44.75% |
| | | Average | 4.4948 | 10.307 | 190.82 | 05.59% |
| | | | Bias | MAD | MSE | MAPE |
| | | | | SE | 15.951 | |
| Next peri | 178.596 | | | | | |

The forecast for quarter 9 is here.

Program 5.2

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# Exponential Smoothing with Trend Adjustment

- Like all averaging techniques, exponential smoothing does not respond to trends.
- A more complex model can be used that adjusts for trends.
- The basic approach is to develop an exponential smoothing forecast, and then adjust it for the trend.

Forecast including trend ($FIT_{t+1}$) = Smoothed forecast ($F_{t+1}$) + Smoothed Trend ($T_{t+1}$)

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# Exponential Smoothing with Trend Adjustment

- The equation for the trend correction uses a new smoothing constant $\beta$.
- $T_t$ must be given or estimated. $T_{t+1}$ is computed by:

$$T_{t+1} = (1-\beta)T_t + \beta(F_{t+1} - FIT_t)$$

where

$T_t$ = smoothed trend for time period $t$

$F_t$ = smoothed forecast for time period $t$

$FIT_t$ = forecast including trend for time period $t$

$\alpha$ =smoothing constant for forecasts

$\beta$ = smoothing constant for trend

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# Selecting a Smoothing Constant

- As with exponential smoothing, a high value of $\beta$ makes the forecast more responsive to changes in trend.
- A low value of $\beta$ gives less weight to the recent trend and tends to smooth out the trend.
- Values are generally selected using a trial-and-error approach based on the value of the $MAD$ for different values of $\beta$.

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# Midwestern Manufacturing

- Midwest Manufacturing has a demand for electrical generators from 2004 – 2010 as given in the table below.
- To forecast demand, Midwest assumes:
  - $F_1$ is perfect.
  - $T_1 = 0$.
  - $\alpha = 0.3$
  - $\beta = 0.4$.

| YEAR | ELECTRICAL GENERATORS SOLD |
|---|---|
| 2004 | 74 |
| 2005 | 79 |
| 2006 | 80 |
| 2007 | 90 |
| 2008 | 105 |
| 2009 | 142 |
| 2010 | 122 |

Table 5.7

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# Midwestern Manufacturing

- According to the assumptions,

  $FIT_1 = F_1 + T_1 = 74 + 0 = 74.$

- Step 1: Compute $F_{t+1}$ by:

  $FIT_{t+1} = F_t + \alpha(Y_t - FIT_t)$

  $= 74 + 0.3(74-74) = 74$

- Step 2: Update the trend using:

  $T_{t+1} = T_t + \beta(F_{t+1} - FIT_t)$

  $T_2 = T_1 + .4(F_2 - FIT_1)$

  $= 0 + .4(74 - 74) = 0$

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# Midwestern Manufacturing

- Step 3: Calculate the trend-adjusted exponential smoothing forecast ($F_{t+1}$) using the following:

  $FIT_2 = F_2 + T_2$

  $= 74 + 0 = 74$

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# Midwestern Manufacturing

- For 2006 (period 3) we have:
- Step 1: $F_3 = FIT_2 + 0.3(Y_2 - FIT_2)$
  $= 74 + .3(79 - 74)$
  $= 75.5$
- Step 2: $T_3 = T_2 + 0.4(F_3 - FIT_2)$
  $= 0 + 0.4(75.5 - 74)$
  $= 0.6$
- Step 3: $FIT_3 = F_3 + T_3$
  $= 75.5 + 0.6$
  $= 76.1$

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# Midwestern Manufacturing Exponential Smoothing with Trend Forecasts

| Time (t) | Demand ($Y_t$) | $FIT_{t+1} = F_t + 0.3(Y_t - FIT_t)$ | $T_{t+1} = T_t + 0.4(F_{t+1} - FIT_t)$ | $FIT_{t+1} = F_{t+1} + T_{t+1}$ |
|---|---|---|---|---|
| 1 | 74 | 74 | 0 | 74 |
| 2 | 79 | 74=74+0.3(74-74) | 0 = 0+0.4(74-74) | 74 = 74+0 |
| 3 | 80 | 75.5=74+0.3(79-74) | 0.6 = 0+0.4(75.5-74) | 76.1 = 75.5+0.6 |
| 4 | 90 | 77.270=76.1+0.3(80-76.1) | 1.068 = 0.6+0.4(77.27-76.1) | 78.338 = 77.270+1.068 |
| 5 | 105 | 81.837=78.338+0.3(90-78.338) | 2.468 = 1.068+0.4(81.837-78.338) | 84.305 = 81.837+2.468 |
| 6 | 142 | 90.514=84.305+0.3(105-84.305) | 4.952 = 2.468+0.4(90.514-84.305) | 95.466 = 90.514+4.952 |
| 7 | 122 | 109.426=95.466+0.3(142-95.466) | 10.536 = 4.952+0.4(109.426-95.466) | 119.962 = 109.426+10.536 |
| 8 | | 120.573=119.962+0.3(122-119.962) | 10.780 = 10.536+0.4(120.573-119.962) | 131.353 = 120.573+10.780 |

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Table 5.8

# Midwestern Manufacturing

## Midwestern Manufacturing Trend-Adjusted Exponential Smoothing in Excel QM

Input values for the smoothing constants.

You may enter initial values for $F_1$ and $T_1$.

Enter the past observations.

The forecast for the next year is given here.

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **Midwestern Manufacturing** | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | **Forecasting** | | **Trend adjusted exponential smoothing** | | | | | | | |
| 4–6 | Enter alpha and beta (between 0 and 1), enter the past demands in the shaded column then enter a starting forecast. If the starting forecast is not in the first period then delete the error analysis for all rows above the starting forecast. | | | | | | | | | |
| 7 | Alpha | 0.3 | | | | | | | | |
| 8 | Beta | 0.4 | | | | | | | | |
| 9 | **Data** | | | **Forecasts and Error Analysis** | | | | | | |
| 10 | Period | Demand | | Smoothed Forecast, $F_t$ | Smoothed Trend, $T_t$ | Forecast Including Trend, $FIT_t$ | Error | Absolute | Squared | Abs Pct Err |
| 11 | Period 1 | 74 | | 74 | | 74 | 0 | 0 | 0 | 00.00% |
| 12 | Period 2 | 79 | | 74 | 0 | 74 | 5 | 5 | 25 | 06.33% |
| 13 | Period 3 | 80 | | 75.5 | 0.6 | 76.1 | 4.5 | 4.5 | 20.25 | 05.63% |
| 14 | Period 4 | 90 | | 77.27 | 1.068 | 78.338 | 12.73 | 12.73 | 162.053 | 14.14% |
| 15 | Period 5 | 105 | | 81.8366 | 2.46744 | 84.30404 | 23.1634 | 23.1634 | 536.543 | 22.06% |
| 16 | Period 6 | 142 | | 90.512828 | 4.9509552 | 95.4637832 | 51.4872 | 51.4872 | 2650.93 | 36.26% |
| 17 | Period 7 | 122 | | 109.424648 | 10.5353012 | 119.9599495 | 12.5754 | 12.5754 | 158.139 | 0.1030767 |
| 18 | | **Next period** | | **120.571965** | **10.7801073** | **131.3520719** | | | | |
| 19 | | | | Total | | | 109.456 | 109.456 | 3552.91 | 94.73% |
| 20 | | | | **Average** | | | **15.6366** | **15.6366** | **507.559** | **13.53%** |
| 21 | | | | | | | **Bias** | **MAD** | **MSE** | **MAPE** |
| 22 | | | | | | | | **SE** | **26.6568** | |

**Program 5.3**

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# Trend Projections

- Trend projection fits a trend line to a series of historical data points.
- The line is projected into the future for medium- to long-range forecasts.
- Several trend equations can be developed based on exponential or quadratic models.
- The simplest is a linear model developed using regression analysis.

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# Trend Projection

The mathematical form is

$$\hat{Y} = b_0 + b_1 X$$

Where

$\hat{Y}$ = predicted value
$b_0$ = intercept
$b_1$ = slope of the line
$X$ = time period (i.e., $X$ = 1, 2, 3, …, $n$)

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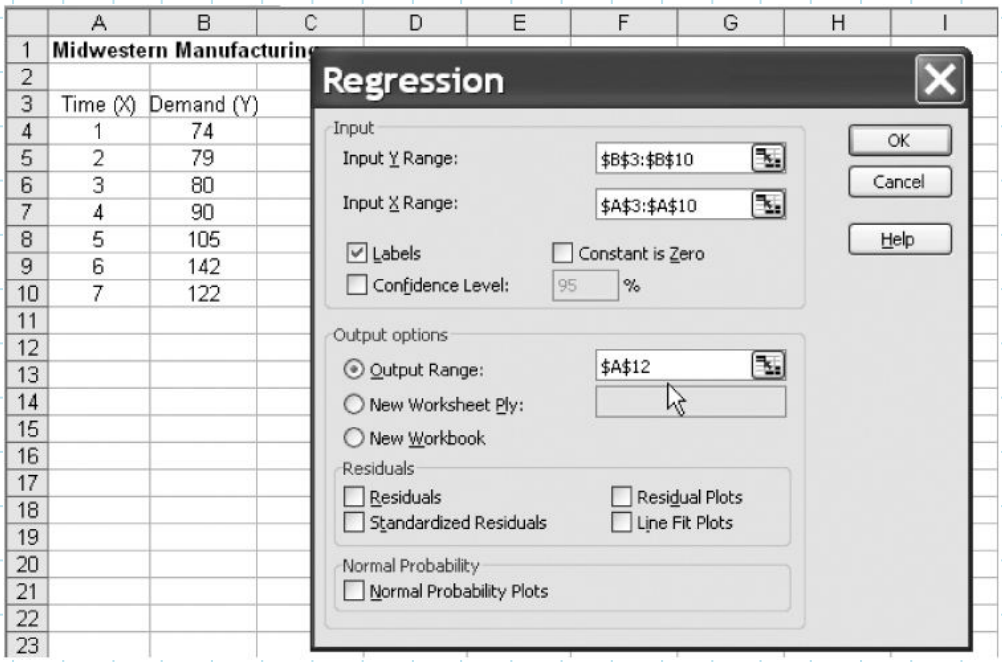
# Midwestern Manufacturing

Excel Input Screen for Midwestern Manufacturing Trend Line




Program 5.4A

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# Midwestern Manufacturing

## Excel Output for Midwestern Manufacturing Trend Line

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Midwestern Manufacturing | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | Time (X) | Demand (Y) | | | | | | | |
| 4 | 1 | 74 | | | | | | | |
| 5 | 2 | 79 | | | | | | | |
| 6 | 3 | 80 | | | | | | | |
| 7 | 4 | 90 | | | | | | | |
| 8 | 5 | 105 | | | | | | | |
| 9 | 6 | 142 | | | | | | | |
| 10 | 7 | 122 | | | | | | | |
| 11 | | | | | | | | | |
| 12 | SUMMARY OUTPUT | | | | | | | | |
| 13 | | | | | | | | | |
| 14 | *Regression Statistics* | | | | | | | | |
| 15 | Multiple R | 0.89491 | | | | | | | |
| 16 | R Square | 0.80086 | | | | | | | |
| 17 | Adjusted R | 0.76104 | | | | | | | |
| 18 | Standard E | 12.43239 | | | | | | | |
| 19 | Observatio | 7 | | | | | | | |
| 20 | | | | | | | | | |
| 21 | ANOVA | | | | | | | | |
| 22 | | *df* | *SS* | *MS* | *F* | *Significance F* | | | |
| 23 | Regressior | 1 | 3108.0357 | 3108.0357 | 20.1084 | 0.0065 | | | |
| 24 | Residual | 5 | 772.8214 | 154.5643 | | | | | |
| 25 | Total | 6 | 3880.8571 | | | | | | |
| 26 | | | | | | | | | |
| 27 | | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *ower 95.0%* | *Upper 95.0%* |
| 28 | Intercept | 56.71429 | 10.50729 | 5.39762 | 0.00295 | 29.70449 | 83.72408 | 29.70449 | 83.72408 |
| 29 | Time (X) | 10.53571 | 2.34950 | 4.48424 | 0.00649 | 4.49614 | 16.57529 | 4.49614 | 16.57529 |

The next year will be time period 8.

The slope of the trend line is 10.54.

**Program 5.4B**

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# Midwestern Manufacturing Company Example

- **The forecast equation is**

$$\hat{Y} = 56.71 + 10.54X$$

- **To project demand for 2011, we use the coding system to define $X$ = 8**

**(sales in 2011) = 56.71 + 10.54(8)**
**= 141.03, or 141 generators**

- **Likewise for $X$ = 9**

**(sales in 2012) = 56.71 + 10.54(9)**
**= 151.57, or 152 generators**

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# Midwestern Manufacturing

## Electrical Generators and the Computed Trend Line

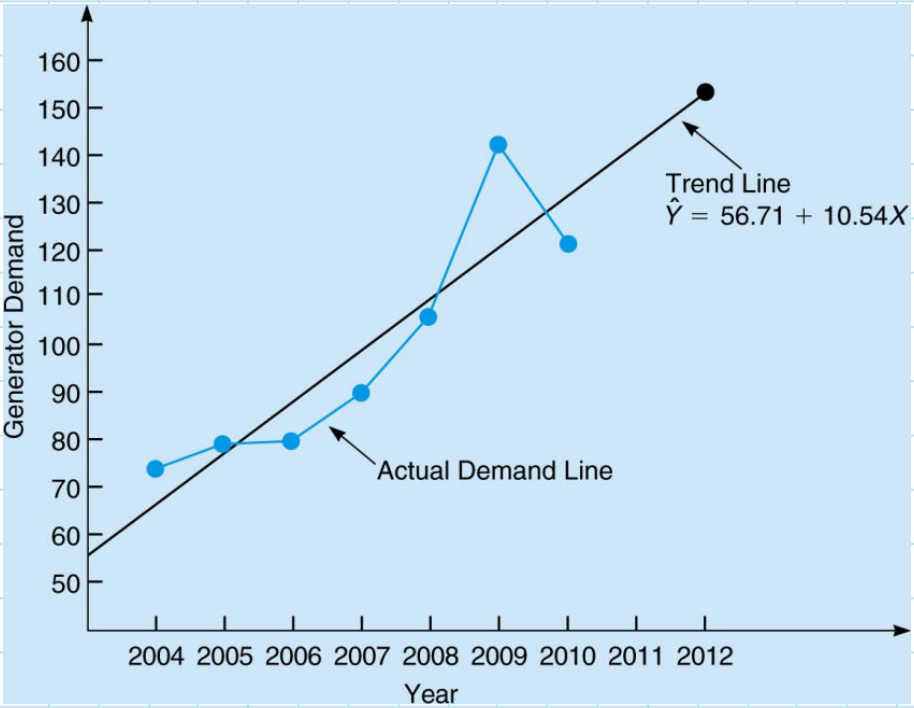


**Figure 5.4**

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# Midwestern Manufacturing

## Excel QM Trend Projection Model

**Midwestern Manufacturing**

Past forecasts and errors are shown here.

**Forecasting** **Regression/Trend analysis**

Input the past data and the time periods.

**Data**

| Period | Demand (y) | Period(x) |
|---|---|---|
| Year 1 | 74 | 1 |
| Year 2 | 79 | 2 |
| Year 3 | 80 | 3 |
| Year 4 | 90 | 4 |
| Year 5 | 105 | 5 |
| Year 6 | 142 | 6 |
| Year 7 | | |
| Intercept | 56.71429 | |
| Slope | 10.53571 | |
| Next perioc | 141 | 8 |

**Forecasts and Error Analysis**

| Forecast | Error | Absolute | Squared | Abs Pct Err |
|---|---|---|---|---|
| 67.25 | 6.75 | 6.75 | 45.5625 | 09.12% |
| 77.7857 | 1.2143 | 1.2143 | 1.4745 | 01.54% |
| 88.3214 | -8.3214 | 8.3214 | 69.2462 | 10.40% |
| 98.8571 | -8.8571 | 8.8571 | 78.4490 | 09.84% |
| 109.3929 | -4.3929 | 4.3929 | 19.2972 | 04.18% |
| 119.9286 | 22.0714 | 22.0714 | 487.1480 | 15.54% |
| | -8.4643 | 8.4643 | 71.6441 | 06.94% |
| | 326E-14 | 60.0714 | 772.8214 | 57.57% |
| Average | -6.09037E-15 | 8.5816 | 110.4031 | 08.22% |
| | | | | |
| | | Correlatic | 0.89491 | |

The intercept ($b_0$) and slope ($b_1$) are found here.

To obtain a forecast for a future period, enter the time period here.

**Program 5.5**

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# Seasonal Variations

- Recurring variations over time may indicate the need for seasonal adjustments in the trend line.
- A seasonal index indicates how a particular season compares with an average season.
- When no trend is present, the seasonal index can be found by dividing the average value for a particular season by the average of all the data.

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# Eichler Supplies

- Eichler Supplies sells telephone answering machines.
- Sales data for the past two years has been collected for one particular model.
- The firm wants to create a forecast that includes seasonality.

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# Eichler Supplies Answering Machine Sales and Seasonal Indices

| MONTH | SALES DEMAND YEAR 1 | SALES DEMAND YEAR 2 | AVERAGE TWO-YEAR DEMAND | MONTHLY DEMAND | AVERAGE SEASONAL INDEX |
|---|---|---|---|---|---|
| January | 80 | 100 | 90 | 94 | 0.957 |
| February | 85 | 75 | 80 | 94 | 0.851 |
| March | 80 | 90 | 85 | 94 | 0.904 |
| April | 110 | 90 | 100 | 94 | 1.064 |
| May | 115 | 131 | 123 | 94 | 1.309 |
| June | 120 | 110 | 115 | 94 | 1.223 |
| July | 100 | 110 | 105 | 94 | 1.117 |
| August | 110 | 90 | 100 | 94 | 1.064 |
| September | 85 | 95 | 90 | 94 | 0.957 |
| October | 75 | 85 | 80 | 94 | 0.851 |
| November | 85 | 75 | 80 | 94 | 0.851 |
| December | 80 | 80 | 80 | 94 | 0.851 |
| | | | Total average demand = 1,128 | | |

$$\text{Average monthly demand} = \frac{1{,}128}{12 \text{ months}} = 94$$

$$\text{Seasonal index} = \frac{\text{Average two-year demand}}{\text{Average monthly demand}}$$

Table 5.9

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# Seasonal Variations

- The calculations for the seasonal indices are

Jan. $\frac{1{,}200}{12} \times 0.957 = 96$

Feb. $\frac{1{,}200}{12} \times 0.851 = 85$

Mar. $\frac{1{,}200}{12} \times 0.904 = 90$

Apr. $\frac{1{,}200}{12} \times 1.064 = 106$

May $\frac{1{,}200}{12} \times 1.309 = 131$

June $\frac{1{,}200}{12} \times 1.223 = 122$

July $\frac{1{,}200}{12} \times 1.117 = 112$

Aug. $\frac{1{,}200}{12} \times 1.064 = 106$

Sept. $\frac{1{,}200}{12} \times 0.957 = 96$

Oct. $\frac{1{,}200}{12} \times 0.851 = 85$

Nov. $\frac{1{,}200}{12} \times 0.851 = 85$

Dec. $\frac{1{,}200}{12} \times 0.851 = 85$

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# Seasonal Variations with Trend

- When both trend and seasonal components are present, the forecasting task is more complex.
- Seasonal indices should be computed using a *centered moving average* (*CMA*) approach.
- There are four steps in computing CMAs:
  1. Compute the CMA for each observation (where possible).
  2. Compute the seasonal ratio = Observation/CMA for that observation.
  3. Average seasonal ratios to get seasonal indices.
  4. If seasonal indices do not add to the number of seasons, multiply each index by (Number of seasons)/(Sum of indices).

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# Turner Industries

- The following table shows Turner Industries' quarterly sales figures for the past three years, in millions of dollars:

| QUARTER | YEAR 1 | YEAR 2 | YEAR 3 | AVERAGE |
|---|---|---|---|---|
| 1 | 108 | 116 | 123 | 115.67 |
| 2 | 125 | 134 | 142 | 133.67 |
| 3 | 150 | 159 | 168 | 159.00 |
| 4 | 141 | 152 | 165 | 152.67 |
| Average | 131.00 | 140.25 | 149.50 | 140.25 |

Table 5.10

Definite trend

Seasonal pattern

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# Turner Industries

- To calculate the CMA for quarter 3 of year 1 we compare the actual sales with an average quarter centered on that time period.
- We will use 1.5 quarters before quarter 3 and 1.5 quarters after quarter 3 – that is we take quarters 2, 3, and 4 and one half of quarters 1, year 1 and quarter 1, year 2.

$$\text{CMA}(q3, y1) = \frac{0.5(108) + 125 + 150 + 141 + 0.5(116)}{4} = 132.00$$

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# Turner Industries

Compare the actual sales in quarter 3 to the CMA to find the seasonal ratio:

$$\text{Seasonal ratio} = \frac{\text{Sales in quarter 3}}{\text{CMA}} = \frac{150}{132} = 1.136$$

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# Turner Industries

| YEAR | QUARTER | SALES | *CMA* | SEASONAL RATIO |
|---|---|---|---|---|
| 1 | 1 | 108 | | |
| | 2 | 125 | | |
| | 3 | 150 | 132.000 | 1.136 |
| | 4 | 141 | 134.125 | 1.051 |
| 2 | 1 | 116 | 136.375 | 0.851 |
| | 2 | 134 | 138.875 | 0.965 |
| | 3 | 159 | 141.125 | 1.127 |
| | 4 | 152 | 143.000 | 1.063 |
| 3 | 1 | 123 | 145.125 | 0.848 |
| | 2 | 142 | 147.875 | 0.960 |
| | 3 | 168 | | |
| | 4 | 165 | | |

Table 5.11

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# Turner Industries

**There are two seasonal ratios for each quarter so these are averaged to get the seasonal index:**

**Index for quarter 1 = $I_1$ = (0.851 + 0.848)/2 = 0.85**

**Index for quarter 2 = $I_2$ = (0.965 + 0.960)/2 = 0.96**

**Index for quarter 3 = $I_3$ = (1.136 + 1.127)/2 = 1.13**

**Index for quarter 4 = $I_4$ = (1.051 + 1.063)/2 = 1.06**

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# Turner Industries

**Scatterplot of Turner Industries Sales Data and Centered Moving Average**

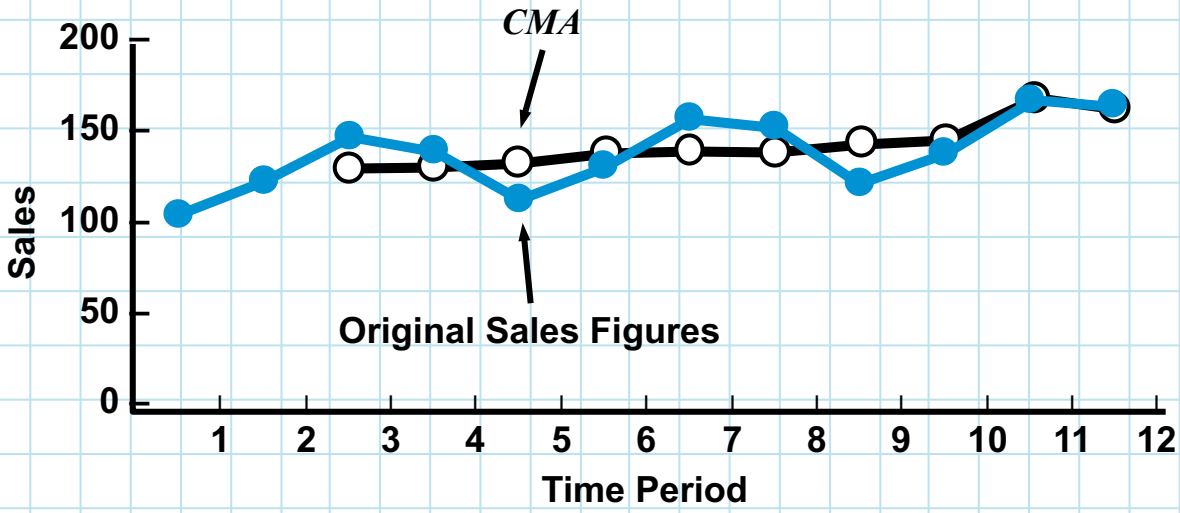


Figure 5.5

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# The Decomposition Method of Forecasting with Trend and Seasonal Components

- *Decomposition* is the process of isolating linear trend and seasonal factors to develop more accurate forecasts.
- There are five steps to decomposition:
  1. Compute seasonal indices using CMAs.
  2. Deseasonalize the data by dividing each number by its seasonal index.
  3. Find the equation of a trend line using the deseasonalized data.
  4. Forecast for future periods using the trend line.
  5. Multiply the trend line forecast by the appropriate seasonal index.

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# Deseasonalized Data for Turner Industries

- Find a trend line using the deseasonalized data:

$$b_1 = 2.34 \qquad b_0 = 124.78$$

- Develop a forecast using this trend and multiply the forecast by the appropriate seasonal index.

$$\hat{Y} = 124.78 + 2.34X$$
$$= 124.78 + 2.34(13)$$
$$= 155.2$$ (forecast before adjustment for seasonality)

$$\hat{Y} \times I_1 = 155.2 \times 0.85 = 131.92$$

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# Deseasonalized Data for Turner Industries

| SALES ($1,000,000s) | SEASONAL INDEX | DESEASONALIZED SALES ($1,000,000s) |
|---|---|---|
| 108 | 0.85 | 127.059 |
| 125 | 0.96 | 130.208 |
| 150 | 1.13 | 132.743 |
| 141 | 1.06 | 133.019 |
| 116 | 0.85 | 136.471 |
| 134 | 0.96 | 139.583 |
| 159 | 1.13 | 140.708 |
| 152 | 1.06 | 143.396 |
| 123 | 0.85 | 144.706 |
| 142 | 0.96 | 147.917 |
| 168 | 1.13 | 148.673 |
| 165 | 1.06 | 155.660 |

Table 5.12

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# San Diego Hospital

A San Diego hospital used 66 months of adult inpatient days to develop the following seasonal indices.

| MONTH | SEASONALITY INDEX | MONTH | SEASONALITY INDEX |
|---|---|---|---|
| January | 1.0436 | July | 1.0302 |
| February | 0.9669 | August | 1.0405 |
| March | 1.0203 | September | 0.9653 |
| April | 1.0087 | October | 1.0048 |
| May | 0.9935 | November | 0.9598 |
| June | 0.9906 | December | 0.9805 |

Table 5.13

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# San Diego Hospital

Using this data they developed the following equation:

$$\hat{Y} = 8{,}091 + 21.5X$$

where

$\hat{Y}$ = forecast patient days

$X$ = time in months

Based on this model, the forecast for patient days for the next period (67) is:

Patient days = 8,091 + (21.5)(67) = 9,532 (trend only)

Patient days = (9,532)(1.0436)

= 9,948 (trend and seasonal)

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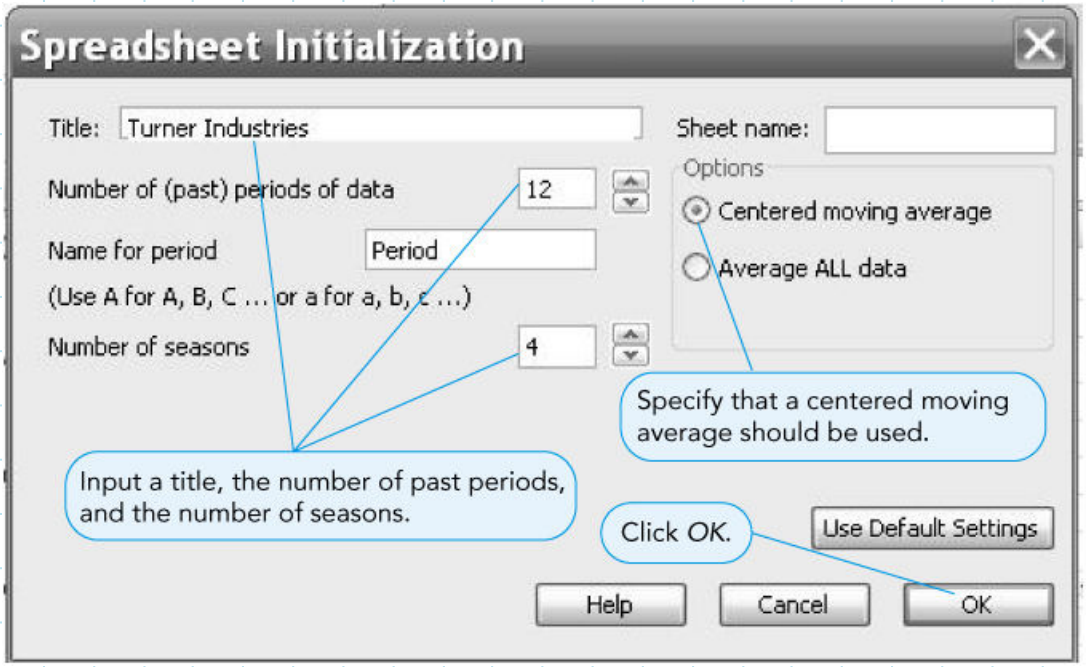
# San Diego Hospital

## Initialization Screen for the Decomposition method in Excel QM




**Program 5.6A**

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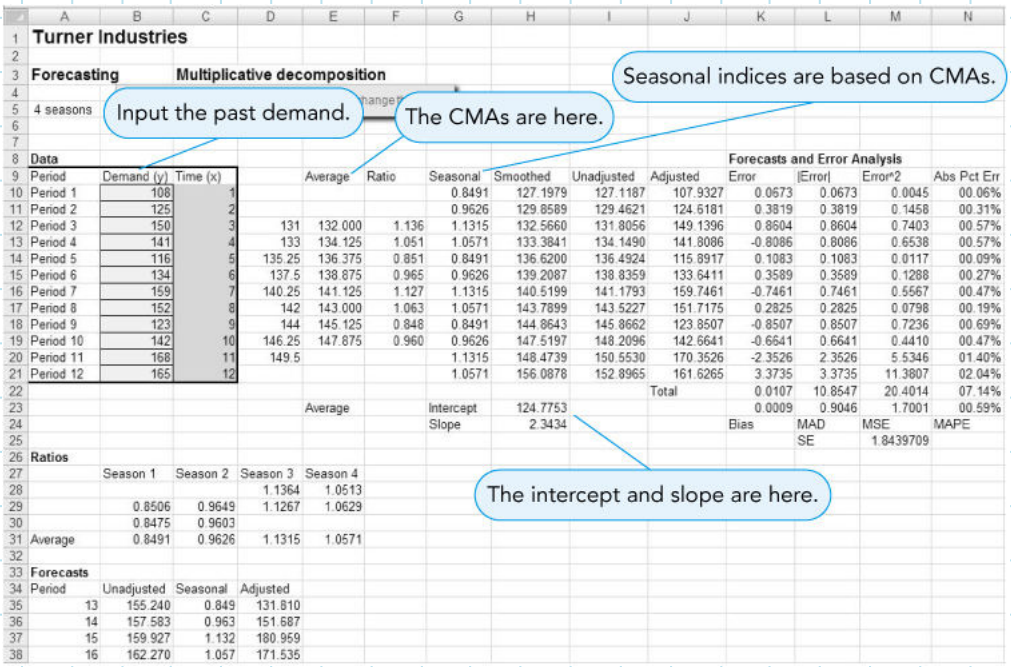
# San Diego Hospital

## Turner Industries Forecast Using the Decomposition Method in Excel QM



Turner Industries

Forecasting — Multiplicative decomposition

4 seasons

Data

| Period | Demand (y) | Time (x) | | Average | Ratio | Seasonal | Smoothed | Unadjusted | Adjusted | Forecasts and Error Analysis: Error | \|Error\| | Error^2 | Abs Pct Err |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Period 1 | 108 | 1 | | | | 0.8491 | 127.1979 | 127.1187 | 107.9327 | 0.0673 | 0.0673 | 0.0045 | 00.06% |
| Period 2 | 125 | 2 | | | | 0.9626 | 129.8589 | 129.4621 | 124.6181 | 0.3819 | 0.3819 | 0.1458 | 00.31% |
| Period 3 | 150 | 3 | 131 | 132.000 | 1.136 | 1.1315 | 132.5660 | 131.8056 | 149.1396 | 0.8604 | 0.8604 | 0.7403 | 00.57% |
| Period 4 | 141 | 4 | 133 | 134.125 | 1.051 | 1.0571 | 133.3841 | 134.1490 | 141.8086 | -0.8086 | 0.8086 | 0.6538 | 00.57% |
| Period 5 | 116 | 5 | 135.25 | 136.375 | 0.851 | 0.8491 | 136.6200 | 136.4924 | 115.8917 | 0.1083 | 0.1083 | 0.0117 | 00.09% |
| Period 6 | 134 | 6 | 137.5 | 138.875 | 0.965 | 0.9626 | 139.2087 | 138.8359 | 133.6411 | 0.3589 | 0.3589 | 0.1288 | 00.27% |
| Period 7 | 159 | 7 | 140.25 | 141.125 | 1.127 | 1.1315 | 140.5199 | 141.1793 | 159.7461 | -0.7461 | 0.7461 | 0.5567 | 00.47% |
| Period 8 | 152 | 8 | 142 | 143.000 | 1.063 | 1.0571 | 143.7899 | 143.5227 | 151.7175 | 0.2825 | 0.2825 | 0.0798 | 00.19% |
| Period 9 | 123 | 9 | 144 | 145.125 | 0.848 | 0.8491 | 144.8643 | 145.8662 | 123.8507 | -0.8507 | 0.8507 | 0.7236 | 00.69% |
| Period 10 | 142 | 10 | 146.25 | 147.875 | 0.960 | 0.9626 | 147.5197 | 148.2096 | 142.6641 | -0.6641 | 0.6641 | 0.4410 | 00.47% |
| Period 11 | 168 | 11 | 149.5 | | | 1.1315 | 148.4739 | 150.5530 | 170.3526 | -2.3526 | 2.3526 | 5.5346 | 01.40% |
| Period 12 | 165 | 12 | | | | 1.0571 | 156.0878 | 152.8965 | 161.6265 | 3.3735 | 3.3735 | 11.3807 | 02.04% |
| | | | | | | | | | Total | 0.0107 | 10.8547 | 20.4014 | 07.14% |
| | | | | Average | | Intercept | 124.7753 | | | 0.0009 | 0.9046 | 1.7001 | 00.59% |
| | | | | | | Slope | 2.3434 | | | Bias | MAD | MSE | MAPE |
| | | | | | | | | | | | SE | 1.8439709 | |

Ratios

| | Season 1 | Season 2 | Season 3 | Season 4 |
|---|---|---|---|---|
| | | | 1.1364 | 1.0513 |
| | 0.8506 | 0.9649 | 1.1267 | 1.0629 |
| | 0.8475 | 0.9603 | | |
| Average | 0.8491 | 0.9626 | 1.1315 | 1.0571 |

Forecasts

| Period | Unadjusted | Seasonal | Adjusted |
|---|---|---|---|
| 13 | 155.240 | 0.849 | 131.810 |
| 14 | 157.583 | 0.963 | 151.687 |
| 15 | 159.927 | 1.132 | 180.959 |
| 16 | 162.270 | 1.057 | 171.535 |



**Program 5.6B**

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# Using Regression with Trend and Seasonal Components

- ***Multiple regression*** **can be used to forecast both trend and seasonal components in a time series.**
  - **One independent variable is time.**
  - **Dummy independent variables are used to represent the seasons.**
- **The model is an additive decomposition model:**

$$\hat{Y} = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4$$

**where**

$X_1$ = time period
$X_2$ = 1 if quarter 2, 0 otherwise
$X_3$ = 1 if quarter 3, 0 otherwise
$X_4$ = 1 if quarter 4, 0 otherwise

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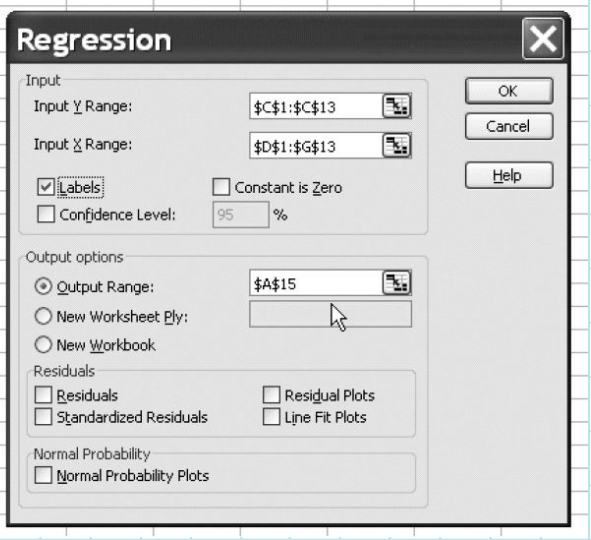
# Regression with Trend and Seasonal Components

## Excel Input for the Turner Industries Example Using Multiple Regression

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | Year | Quarter | Sales | Time Peri | X2 Qtr 2 | X3 Qtr 3 | X4 Qtr 4 |
| 2 | 1 | 1 | 108 | 1 | 0 | 0 | 0 |
| 3 | | 2 | 125 | 2 | 1 | 0 | 0 |
| 4 | | 3 | 150 | 3 | 0 | 1 | 0 |
| 5 | | 4 | 141 | 4 | 0 | 0 | 1 |
| 6 | 2 | 1 | 116 | 5 | 0 | 0 | 0 |
| 7 | | 2 | 134 | 6 | 1 | 0 | 0 |
| 8 | | 3 | 159 | 7 | 0 | 1 | 0 |
| 9 | | 4 | 152 | 8 | 0 | 0 | 1 |
| 10 | 3 | 1 | 123 | 9 | 0 | 0 | 0 |
| 11 | | 2 | 142 | 10 | 1 | 0 | 0 |
| 12 | | 3 | 168 | 11 | 0 | 1 | 0 |
| 13 | | 4 | 165 | 12 | 0 | 0 | 1 |




**Program 5.7A**

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# Using Regression with Trend and Seasonal Components

**Excel Output for the Turner Industries Example Using Multiple Regression**

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Year | Quarter | Sales | X1 Time Period | X2 Qtr 2 | X3 Qtr 3 | X4 Qtr4 | | |
| 2 | 1 | 1 | 108 | 1 | 0 | 0 | 0 | | |
| 3 | | 2 | 125 | 2 | 1 | 0 | 0 | | |
| 4 | | 3 | 150 | 3 | 0 | 1 | 0 | | |
| 5 | | 4 | 141 | 4 | 0 | 0 | 1 | | |
| 6 | 2 | 1 | 116 | 5 | 0 | 0 | 0 | | |
| 7 | | 2 | 134 | 6 | 1 | 0 | 0 | | |
| 8 | | 3 | 159 | 7 | 0 | 1 | 0 | | |
| 9 | | 4 | 152 | 8 | 0 | 0 | 1 | | |
| 10 | 3 | 1 | 123 | 9 | 0 | 0 | 0 | | |
| 11 | | 2 | 142 | 10 | 1 | 0 | 0 | | |
| 12 | | 3 | 168 | 11 | 0 | 1 | 0 | | |
| 13 | | 4 | 165 | 12 | 0 | 0 | 1 | | |
| 14 | | | | | | | | | |
| 15 | SUMMARY OUTPUT | | | | | | | | |
| 16 | | | | | | | | | |
| 17 | *Regression Statistics* | | | | | | | | |
| 18 | Multiple R | 0.99718 | | | | | | | |
| 19 | R Square | 0.99436 | | | | | | | |
| 20 | Adjusted R | 0.99114 | | | | | | | |
| 21 | Standard E | 1.83225 | | | | | | | |
| 22 | Observatio | 12 | | | | | | | |
| 23 | | | | | | | | | |
| 24 | ANOVA | | | | | | | | |
| 25 | | *df* | *SS* | *MS* | *F* | *Significance F* | | | |
| 26 | Regressior | 4 | 4144.75 | 1.0362E+03 | 3.0865E+02 | 6.0284E-08 | | | |
| 27 | Residual | 7 | 23.5 | 3.3571E+00 | | | | | |
| 28 | Total | 11 | 4168.25 | | | | | | |
| 29 | | | | | | | | | |
| 30 | | *Coefficient* | *standard Erro* | *t Stat* | *p-value* | *Lower 95%* | *Upper 95%* | *ower 95.0%* | *Upper 95.0%* |
| 31 | Intercept | 104.1042 | 1.3322 | 78.1449 | 0.0000 | 100.9540 | 107.2543 | 100.9540 | 107.2543 |
| 32 | X1 Time Pe | 2.3125 | 0.1619 | 14.2791 | 0.0000 | 1.9296 | 2.6954 | 1.9296 | 2.6954 |
| 33 | X2 Qtr 2 | 15.6875 | 1.5048 | 10.4252 | 0.0000 | 12.1293 | 19.2457 | 12.1293 | 19.2457 |
| 34 | X3 Qtr 3 | 38.7083 | 1.5307 | 25.2882 | 0.0000 | 35.0888 | 42.3278 | 35.0888 | 42.3278 |
| 35 | X4 Qtr4 | 30.0625 | 1.5729 | 19.1123 | 0.0000 | 26.3431 | 33.7819 | 26.3431 | 33.7819 |

Quarter 1 is indicated by letting $X_2 = X_3 = X_4 = 0$.

**Program 5.7B**

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# Using Regression with Trend and Seasonal Components

- **The resulting regression equation is:**

$$\hat{Y} = 104.1 + 2.3X_1 + 15.7X_2 + 38.7X_3 + 30.1X_4$$

- **Using the model to forecast sales for the first two quarters of next year:**

$$\hat{Y} = 104.1 + 2.3(13) + 15.7(0) + 38.7(0) + 30.1(0) = 134$$

$$\hat{Y} = 104.1 + 2.3(14) + 15.7(1) + 38.7(0) + 30.1(0) = 152$$

- **These are different from the results obtained using the multiplicative decomposition method.**
- **Use MAD or MSE to determine the best model.**

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# Monitoring and Controlling Forecasts

- *Tracking signals* can be used to monitor the performance of a forecast.
- A tracking signal is computed as the running sum of the forecast errors (RSFE), and is computed using the following equation:

$$\text{Tracking signal} = \frac{\text{RSFE}}{\text{MAD}}$$

where

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n}$$

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# Monitoring and Controlling Forecasts

## Plot of Tracking Signals

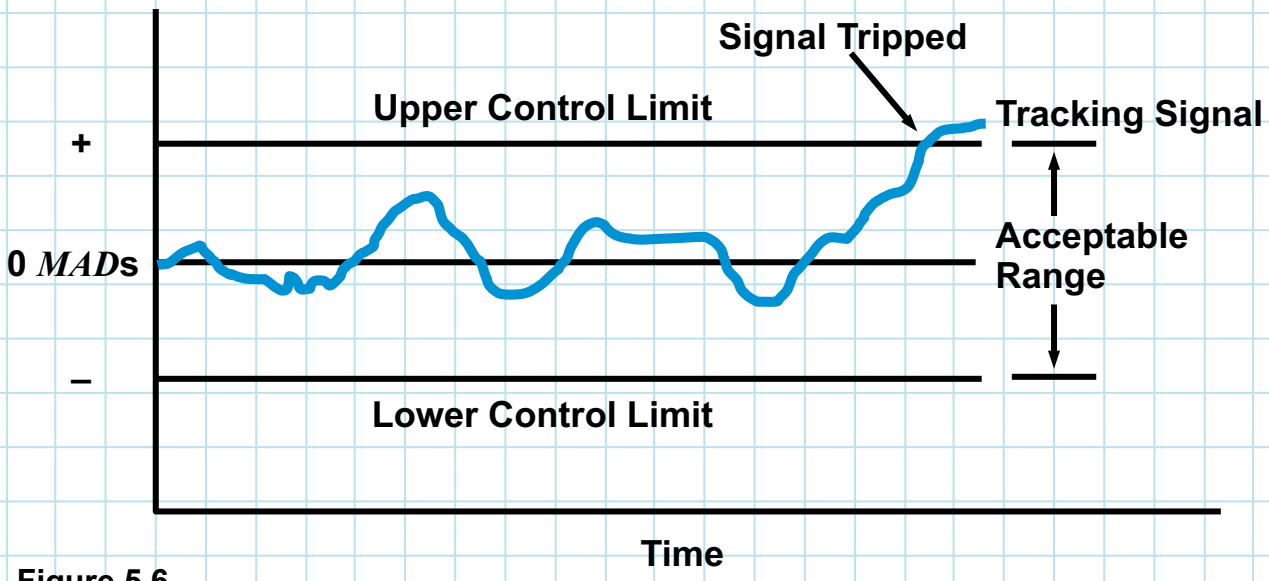


Figure 5.6

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# Monitoring and Controlling Forecasts

- Positive tracking signals indicate demand is greater than forecast.
- Negative tracking signals indicate demand is less than forecast.
- Some variation is expected, but a good forecast will have about as much positive error as negative error.
- Problems are indicated when the signal trips either the upper or lower predetermined limits.
- This indicates there has been an unacceptable amount of variation.
- Limits should be reasonable and may vary from item to item.

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# Kimball's Bakery

Quarterly sales of croissants (in thousands):

| TIME PERIOD | FORECAST DEMAND | ACTUAL DEMAND | ERROR | RSFE | \|FORECAST ERROR\| | CUMULATIVE ERROR | MAD | TRACKING SIGNAL |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | –10 | –10 | 10 | 10 | 10.0 | –1 |
| 2 | 100 | 95 | –5 | –15 | 5 | 15 | 7.5 | –2 |
| 3 | 100 | 115 | +15 | 0 | 15 | 30 | 10.0 | 0 |
| 4 | 110 | 100 | –10 | –10 | 10 | 40 | 10.0 | –1 |
| 5 | 110 | 125 | +15 | +5 | 15 | 55 | 11.0 | +0.5 |
| 6 | 110 | 140 | +30 | +35 | 35 | 85 | 14.2 | +2.5 |

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n} = \frac{85}{6} = 14.2$$

$$\text{Tracking signal} = \frac{\text{RSFE}}{\text{MAD}} = \frac{35}{14.2} = 2.5 MADs$$

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# Adaptive Smoothing

- *Adaptive smoothing* is the computer monitoring of tracking signals and self-adjustment if a limit is tripped.
- In exponential smoothing, the values of $\alpha$ and $\beta$ are adjusted when the computer detects an excessive amount of variation.

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# Chapter 4

# Regression Models

To accompany
*Quantitative Analysis for Management, Eleventh Edition*,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

## Learning Objectives

**After completing this chapter, students will be able to:**

1. Identify variables and use them in a regression model.
2. Develop simple linear regression equations. from sample data and interpret the slope and intercept.
3. Compute the coefficient of determination and the coefficient of correlation and interpret their meanings.
4. Interpret the $F$-test in a linear regression model.
5. List the assumptions used in regression and use residual plots to identify problems.

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# Learning Objectives

**After completing this chapter, students will be able to:**

6. Develop a multiple regression model and use it for prediction purposes.
7. Use dummy variables to model categorical data.
8. Determine which variables should be included in a multiple regression model.
9. Transform a nonlinear function into a linear one for use in regression.
10. Understand and avoid common mistakes made in the use of regression analysis.

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# Chapter Outline

4.1 Introduction
4.2 Scatter Diagrams
4.3 Simple Linear Regression
4.4 Measuring the Fit of the Regression Model
4.5 Using Computer Software for Regression
4.6 Assumptions of the Regression Model

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# Chapter Outline


4.7 Testing the Model for Significance
4.8 Multiple Regression Analysis
4.9 Binary or Dummy Variables
4.10 Model Building
4.11 Nonlinear Regression
4.12 Cautions and Pitfalls in Regression Analysis


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# Introduction

- *Regression analysis* is a very valuable tool for a manager.
- Regression can be used to:
  - Understand the relationship between variables.
  - Predict the value of one variable based on another variable.
- Simple linear regression models have only two variables.
- Multiple regression models have more variables.

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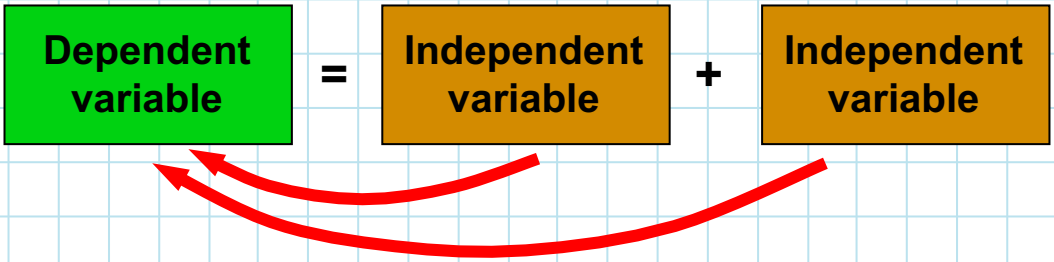
# Introduction

- The variable to be predicted is called the *dependent variable*.
  - This is sometimes called the *response variable.*
- The value of this variable depends on the value of the *independent variable.*
  - This is sometimes called the *explanatory* or *predictor variable.*

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# Scatter Diagram

- A *scatter diagram* or *scatter plot* is often used to investigate the relationship between variables.
- The independent variable is normally plotted on the $X$ axis.
- The dependent variable is normally plotted on the $Y$ axis.

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# Triple A Construction

- Triple A Construction renovates old homes.
- Managers have found that the dollar volume of renovation work is dependent on the area payroll.

| TRIPLE A'S SALES ($100,000s) | LOCAL PAYROLL ($100,000,000s) |
|---|---|
| 6 | 3 |
| 8 | 4 |
| 9 | 6 |
| 5 | 4 |
| 4.5 | 2 |
| 9.5 | 5 |

Table 4.1

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# Triple A Construction

## Scatter Diagram of Triple A Construction Company Data

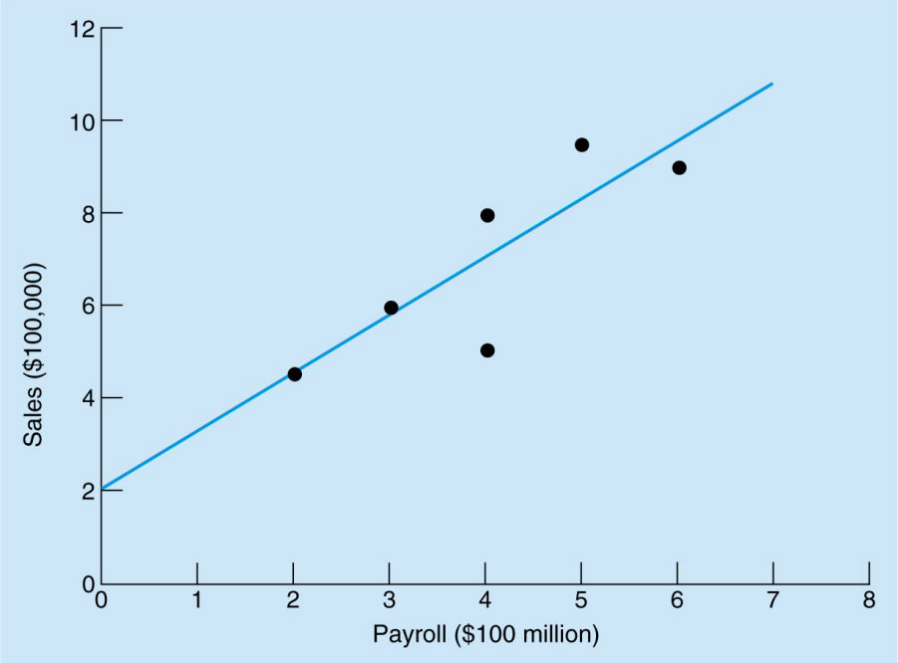


Figure 4.1

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# Simple Linear Regression

- Regression models are used to test if there is a relationship between variables.
- There is some random error that cannot be predicted.

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

where

$Y$ = dependent variable (response)

$X$ = independent variable (predictor or explanatory)

$\beta_0$ = intercept (value of $Y$ when $X$ = 0)

$\beta_1$ = slope of the regression line

$\varepsilon$ = random error

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# Simple Linear Regression

- True values for the slope and intercept are not known so they are estimated using sample data.

$$\hat{Y} = b_0 + b_1 X$$

where

$\hat{Y}$ = predicted value of $Y$

$b_0$ = estimate of $\beta_0$, based on sample results

$b_1$ = estimate of $\beta_1$, based on sample results

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# Triple A Construction

Triple A Construction is trying to predict sales based on area payroll.

$Y$ = Sales
$X$ = Area payroll

The line chosen in Figure 4.1 is the one that minimizes the errors.

Error = (Actual value) – (Predicted value)

$$e = Y - \hat{Y}$$

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# Triple A Construction

For the simple linear regression model, the values of the intercept and slope can be calculated using the formulas below.

$$\hat{Y} = b_0 + b_1 X$$

$$\overline{X} = \frac{\sum X}{n} = \text{average (mean) of } X \text{ values}$$

$$\overline{Y} = \frac{\sum Y}{n} = \text{average (mean) of } Y \text{ values}$$

$$b_1 = \frac{\sum (X - \overline{X})(Y - \overline{Y})}{\sum (X - \overline{X})^2}$$

$$b_0 = \overline{Y} - b_1 \overline{X}$$

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# Triple A Construction

## Regression calculations for Triple A Construction

| $Y$ | $X$ | $(X-\bar{X})^2$ | $(X-\bar{X})(Y-\bar{Y})$ |
|---|---|---|---|
| 6 | 3 | $(3-4)^2 = 1$ | $(3-4)(6-7) = 1$ |
| 8 | 4 | $(4-4)^2 = 0$ | $(4-4)(8-7) = 0$ |
| 9 | 6 | $(6-4)^2 = 4$ | $(6-4)(9-7) = 4$ |
| 5 | 4 | $(4-4)^2 = 0$ | $(4-4)(5-7) = 0$ |
| 4.5 | 2 | $(2-4)^2 = 4$ | $(2-4)(4.5-7) = 5$ |
| 9.5 | 5 | $(5-4)^2 = 1$ | $(5-4)(9.5-7) = 2.5$ |
| $\Sigma Y = 42$ | $\Sigma X = 24$ | $\Sigma(X-\bar{X})^2 = 10$ | $\Sigma(X-\bar{X})(Y-\bar{Y}) = 12.5$ |
| $\bar{Y} = 42/6 = 7$ | $\bar{X} = 24/6 = 4$ | | |

Table 4.2

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# Triple A Construction

## Regression calculations

$$\bar{X} = \frac{\sum X}{6} = \frac{24}{6} = 4$$

$$\bar{Y} = \frac{\sum Y}{6} = \frac{42}{6} = 7$$

$$b_1 = \frac{\sum (X-\bar{X})(Y-\bar{Y})}{\sum (X-\bar{X})^2} = \frac{12.5}{10} = 1.25$$

$$b_0 = \bar{Y} - b_1\bar{X} = 7 - (1.25)(4) = 2$$

Therefore $\hat{Y} = 2 + 1.25X$

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# Triple A Construction

Regression calculations

$$\overline{X} = \frac{\sum X}{6} = \frac{24}{6}$$

$$\overline{Y} = \frac{\sum Y}{6} = \frac{42}{6}$$

$$b_1 = \frac{\sum (X - \overline{X})}{\sum (X -}$$

$$b_0 = \overline{Y} - b_1 \overline{X} = 7 - (1.25)(4) = 2$$

Therefore $\hat{Y} = 2 + 1.25X$

sales = 2 + 1.25(payroll)

If the payroll next year is $600 million

$$\hat{Y} = 2 + 1.25(6) = 9.5 \text{ or } \$\,950{,}000$$

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# Measuring the Fit of the Regression Model

- Regression models can be developed for any variables $X$ and $Y$.
- How do we know the model is actually helpful in predicting $Y$ based on $X$?
  - We could just take the average error, but the positive and negative errors would cancel each other out.
- Three measures of variability are:
  - $SST$ – Total variability about the mean.
  - $SSE$ – Variability about the regression line.
  - $SSR$ – Total variability that is explained by the model.

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# Measuring the Fit of the Regression Model

- **Sum of the squares total:**

$$SST = \sum (Y - \bar{Y})^2$$

- **Sum of the squared error:**

$$SSE = \sum e^2 = \sum (Y - \hat{Y})^2$$

- **Sum of squares due to regression:**

$$SSR = \sum (\hat{Y} - \bar{Y})^2$$

- **An important relationship:**

$$SST = SSR + SSE$$

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# Measuring the Fit of the Regression Model

## Sum of Squares for Triple A Construction

| $Y$ | $X$ | $(Y - \bar{Y})^2$ | $\hat{Y}$ | $(Y - \hat{Y})^2$ | $(\hat{Y} - \bar{Y})^2$ |
|---|---|---|---|---|---|
| 6 | 3 | $(6 - 7)^2 = 1$ | 2 + 1.25(3) = 5.75 | 0.0625 | 1.563 |
| 8 | 4 | $(8 - 7)^2 = 1$ | 2 + 1.25(4) = 7.00 | 1 | 0 |
| 9 | 6 | $(9 - 7)^2 = 4$ | 2 + 1.25(6) = 9.50 | 0.25 | 6.25 |
| 5 | 4 | $(5 - 7)^2 = 4$ | 2 + 1.25(4) = 7.00 | 4 | 0 |
| 4.5 | 2 | $(4.5 - 7)^2 = 6.25$ | 2 + 1.25(2) = 4.50 | 0 | 6.25 |
| 9.5 | 5 | $(9.5 - 7)^2 = 6.25$ | 2 + 1.25(5) = 8.25 | 1.5625 | 1.563 |
| | | $\sum(Y - \bar{Y})^2 = 22.5$ | | $\sum(Y - \hat{Y})^2 = 6.875$ | $\sum(\hat{Y} - \bar{Y})^2 = 15.625$ |
| $\bar{Y} = 7$ | | $SST = 22.5$ | | $SSE = 6.875$ | $SSR = 15.625$ |

Table 4.3

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# Measuring the Fit of the Regression Model

- Sum of the square

$$SST$$

- Sum of the square

$$SSE = \sum$$

**For Triple A Construction**

$SST$ = 22.5
$SSE$ = 6.875
$SSR$ = 15.625

- Sum of squares due to regression

$$SSR = \sum (\hat{Y} - \bar{Y})^2$$

- An important relationship

$$SST = SSR + SSE$$

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# Measuring the Fit of the Regression Model

## Deviations from the Regression Line and from the Mean

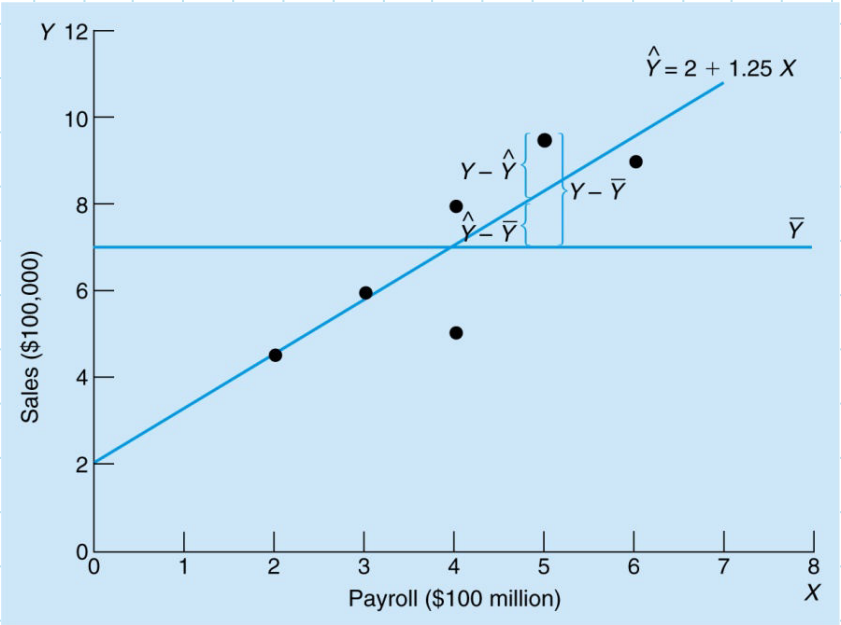


Figure 4.2

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# Coefficient of Determination

- The proportion of the variability in $Y$ explained by the regression equation is called the ***coefficient of determination.***
- The coefficient of determination is $r^2$.

$$r^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

- For Triple A Construction:

$$r^2 = \frac{15.625}{22.5} = 0.6944$$

- About 69% of the variability in $Y$ is explained by the equation based on payroll ($X$).

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# Correlation Coefficient

- The ***correlation coefficient*** is an expression of the strength of the linear relationship.
- It will always be between +1 and –1.
- The correlation coefficient is $r$.

$$r = \sqrt{r^2}$$

- For Triple A Construction:

$$r = \sqrt{0.6944} = 0.8333$$

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# Four Values of the Correlation Coefficient

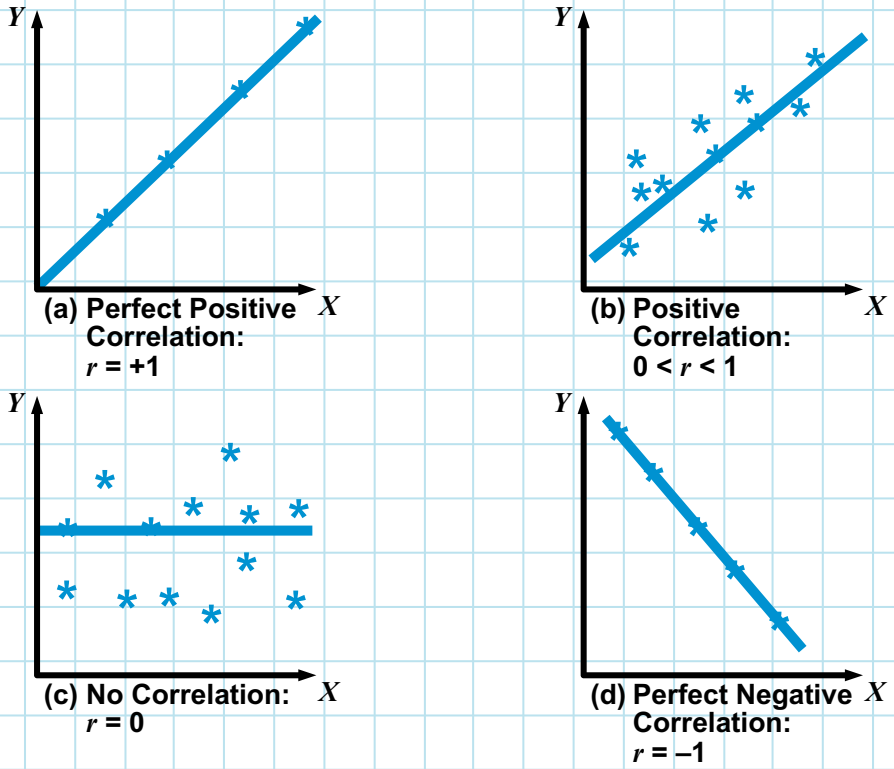


Figure 4.3

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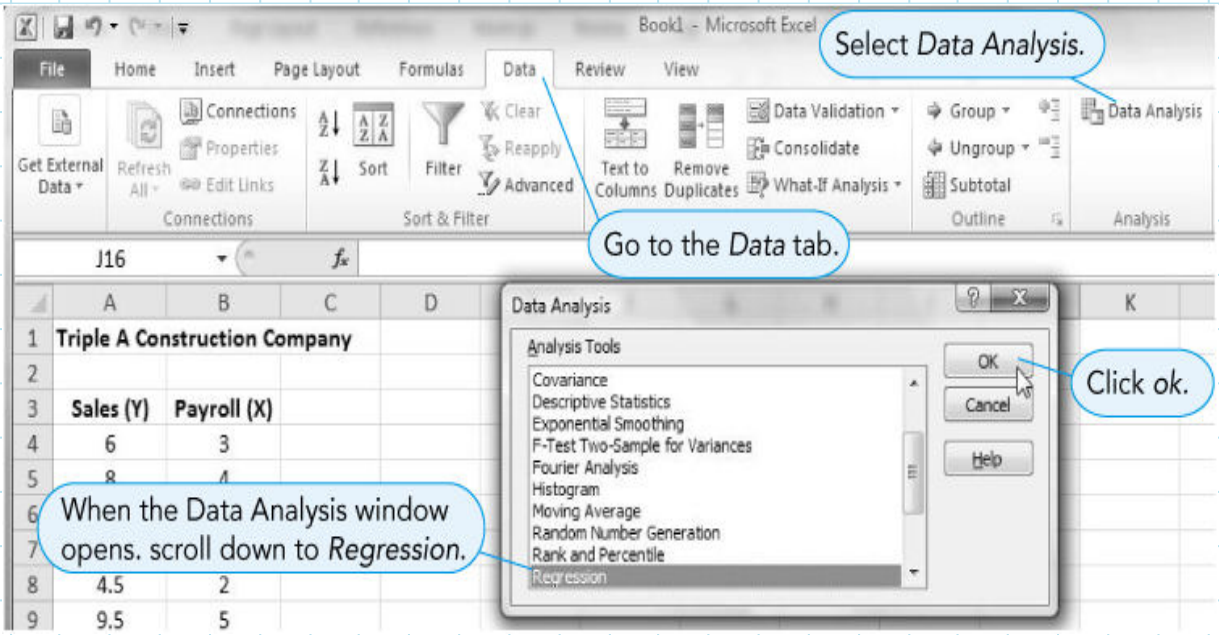
# Using Computer Software for Regression

## Accessing the Regression Option in Excel 2010




Program 4.1A

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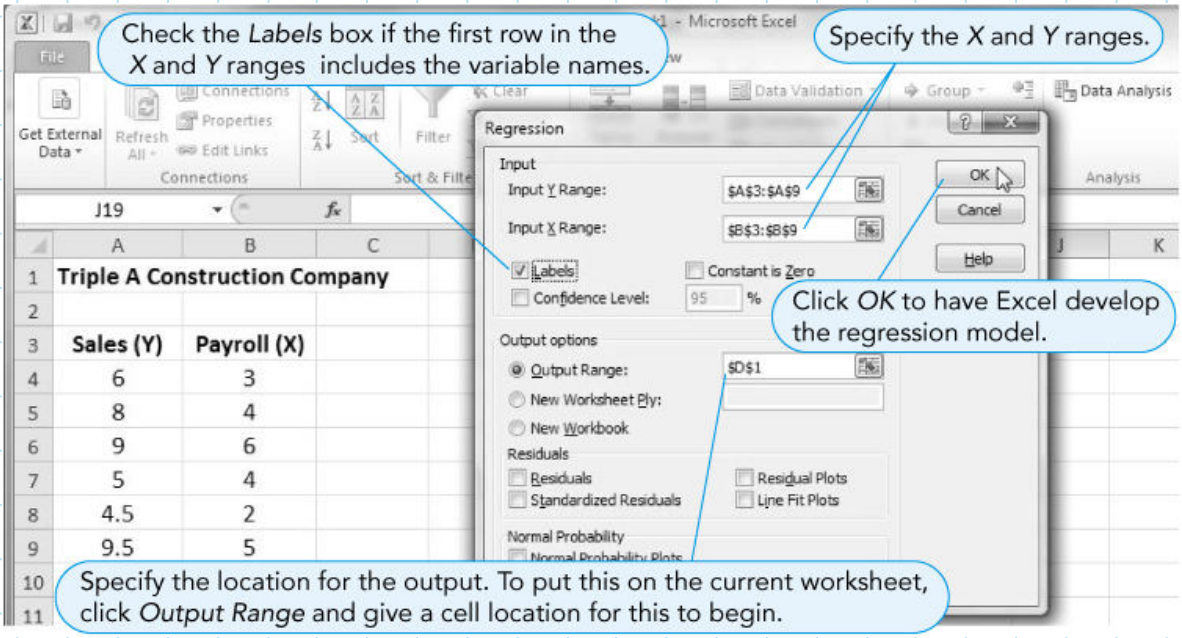
# Using Computer Software for Regression

## Data Input for Regression in Excel




**Program 4.1B**

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# Using Computer Software for Regression

## Excel Output for the Triple A Construction Example

A high $r^2$ (close to 1) is desirable.

The SSR (regression), SSE (residual or error), and SST (total) are shown in the SS column of the ANOVA table.

A low (e.g., less than 0.05) *Significance F* (*p*-value for overall model) indicates a significant relationship between *X* and *Y*.

The regression coefficients are given here.

| SUMMARY OUTPUT | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *Regression Statistics* | | | | | | | | |
| Multiple R | 0.8333 | | | | | | | |
| R Square | 0.6944 | | | | | | | |
| Adjusted R Square | 0.6181 | | | | | | | |
| Standard Error | 1.3110 | | | | | | | |
| Observations | 6 | | | | | | | |
| ANOVA | | | | | | | | |
| | *df* | *SS* | *MS* | *F* | *Significance F* | | | |
| Regression | 1 | 15.6250 | 15.6250 | 9.0909 | 0.0394 | | | |
| Residual | 4 | 6.8750 | 1.7188 | | | | | |
| Total | 5 | 22.5 | | | | | | |
| | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *Lower 95.0%* | *Upper 95.0%* |
| Intercept | 2 | 1.7425 | 1.1477 | 0.3150 | -2.8381 | 6.8381 | -2.8381 | 6.8381 |
| Payroll (X) | 1.25 | 0.4146 | 3.0151 | 0.0394 | 0.0989 | 2.4011 | 0.0989 | 2.4011 |

**Program 4.1C**

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# Assumptions of the Regression Model

- If we make certain assumptions about the errors in a regression model, we can perform statistical tests to determine if the model is useful.
  1. Errors are independent.
  2. Errors are normally distributed.
  3. Errors have a mean of zero.
  4. Errors have a constant variance.
- A plot of the residuals (errors) will often highlight any glaring violations of the assumption.

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# Residual Plots

## Pattern of Errors Indicating Randomness

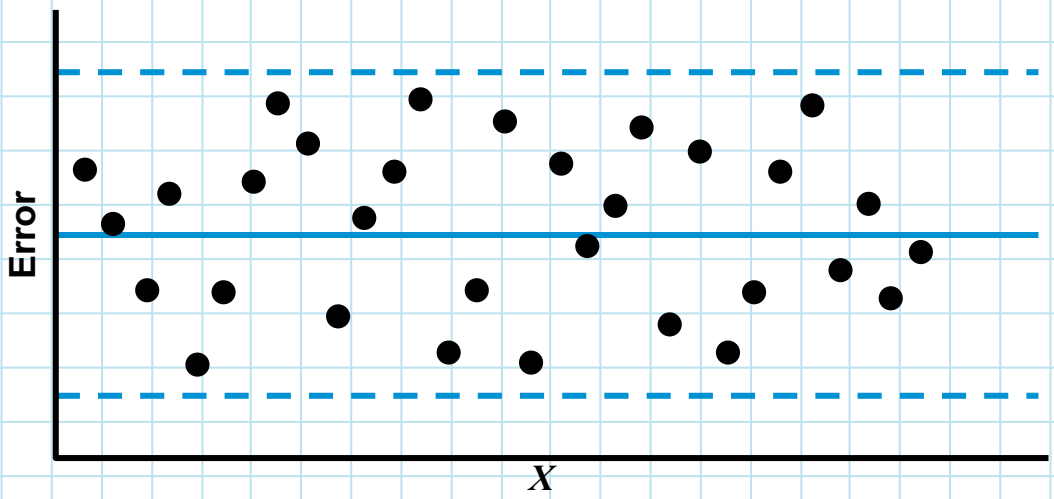


Figure 4.4A

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# Residual Plots

## Nonconstant error variance

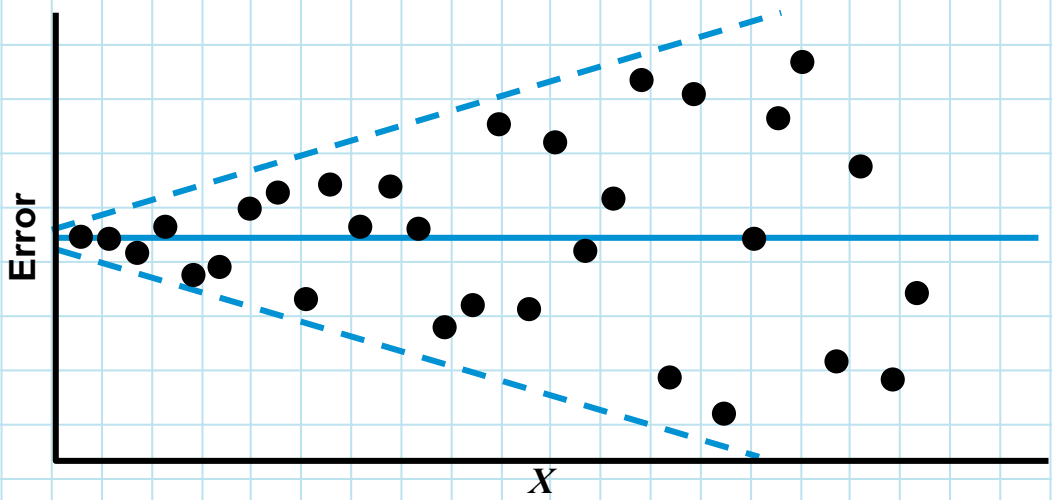


Figure 4.4B

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# Residual Plots

## Errors Indicate Relationship is not Linear

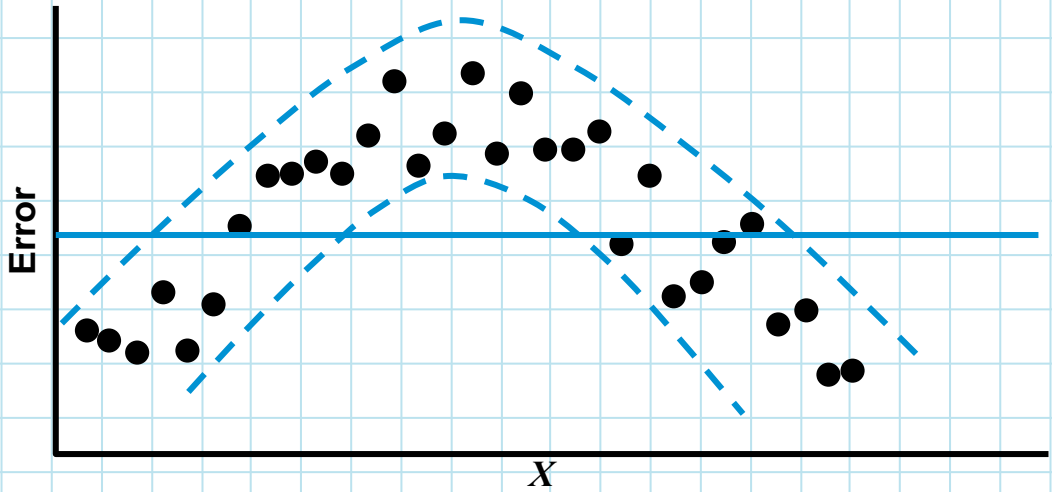


Figure 4.4C

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# Estimating the Variance

- Errors are assumed to have a constant variance ($\sigma^2$), but we usually don't know this.
- It can be estimated using the ***mean squared error*** (*MSE*), $s^2$.

$$s^2 = MSE = \frac{SSE}{n-k-1}$$

where

$n$ = number of observations in the sample
$k$ = number of independent variables

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# Estimating the Variance

- For Triple A Construction:

$$s^2 = MSE = \frac{SSE}{n-k-1} = \frac{6.8750}{6-1-1} = \frac{6.8750}{4} = 1.7188$$

- We can estimate the standard deviation, $s$.
- This is also called the ***standard error of the estimate*** or the ***standard deviation of the regression.***

$$s = \sqrt{MSE} = \sqrt{1.7188} = 1.31$$

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# Testing the Model for Significance

- When the sample size is too small, you can get good values for $MSE$ and $r^2$ even if there is no relationship between the variables.
- Testing the model for significance helps determine if the values are meaningful.
- We do this by performing a statistical hypothesis test.

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# Testing the Model for Significance

- We start with the general linear model

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

- If $\beta_1 = 0$, the null hypothesis is that there is *no* relationship between $X$ and $Y$.
- The alternate hypothesis is that there *is* a linear relationship ($\beta_1 \neq 0$).
- If the null hypothesis can be rejected, we have proven there is a relationship.
- We use the $F$ statistic for this test.

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# Testing the Model for Significance

- The $F$ statistic is based on the $MSE$ and $MSR$:

$$MSR = \frac{SSR}{k}$$

where

$k$ = number of independent variables in the model

- The $F$ statistic is:

$$F = \frac{MSR}{MSE}$$

- This describes an $F$ distribution with:

degrees of freedom for the numerator = $df_1 = k$

degrees of freedom for the denominator = $df_2 = n - k - 1$

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# Testing the Model for Significance

- If there is very little error, the $MSE$ would be small and the $F$-statistic would be large indicating the model is useful.
- If the $F$-statistic is large, the significance level ($p$-value) will be low, indicating it is unlikely this would have occurred by chance.
- So when the $F$-value is large, we can reject the null hypothesis and accept that there is a linear relationship between $X$ and $Y$ and the values of the $MSE$ and $r^2$ are meaningful.

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# Steps in a Hypothesis Test

1. Specify null and alternative hypotheses:

$$H_0 : \beta_1 = 0$$
$$H_1 : \beta_1 \neq 0$$

2. Select the level of significance ($\alpha$). Common values are 0.01 and 0.05.
3. Calculate the value of the test statistic using the formula:

$$F = \frac{MSR}{MSE}$$

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# Steps in a Hypothesis Test

4. Make a decision using one of the following methods:
   a) Reject the null hypothesis if the test statistic is greater than the $F$-value from the table in Appendix D. Otherwise, do not reject the null hypothesis:

$$\text{Reject if } F_{calculated} > F_{\alpha, df_1, df_2}$$
$$df_1 = k$$
$$df_2 = n - k - 1$$

   b) Reject the null hypothesis if the observed significance level, or $p$-value, is less than the level of significance ($\alpha$). Otherwise, do not reject the null hypothesis:

$$p\text{-value} = P(F > \text{calculated test statistic})$$
$$\text{Reject if } p\text{-value} < \alpha$$

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# Triple A Construction

**Step 1.**

$H_0$: $\beta_1 = 0$ (no linear relationship between $X$ and $Y$)

$H_1$: $\beta_1 \neq 0$ (linear relationship exists between $X$ and $Y$)

**Step 2.**

Select $\alpha = 0.05$

**Step 3.**

Calculate the value of the test statistic.

$$MSR = \frac{SSR}{k} = \frac{15.6250}{1} = 15.6250$$

$$F = \frac{MSR}{MSE} = \frac{15.6250}{1.7188} = 9.09$$

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# Triple A Construction

**Step 4.**

Reject the null hypothesis if the test statistic is greater than the $F$-value in Appendix D.

$df_1 = k = 1$

$df_2 = n - k - 1 = 6 - 1 - 1 = 4$

The value of $F$ associated with a 5% level of significance and with degrees of freedom 1 and 4 is found in Appendix D.

$F_{0.05,1,4} = 7.71$

$F_{\text{calculated}} = 9.09$

Reject $H_0$ because 9.09 > 7.71

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# Triple A Construction

- We can conclude there is a statistically significant relationship between $X$ and $Y$.
- The $r^2$ value of 0.69 means about 69% of the variability in sales ($Y$) is explained by local payroll ($X$).

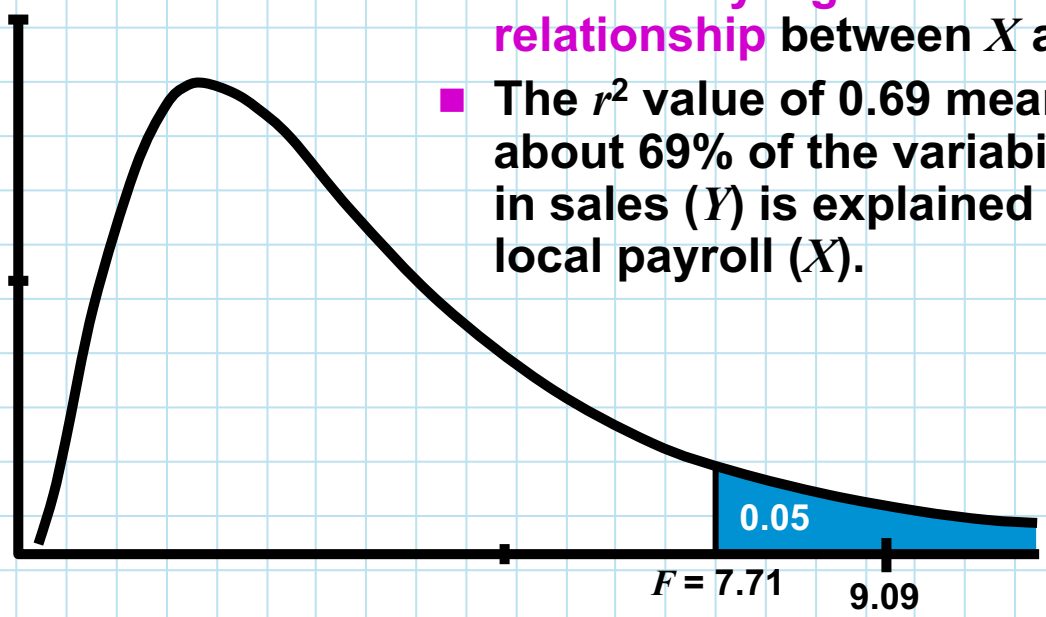


Figure 4.5

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# Analysis of Variance (ANOVA) Table

- When software is used to develop a regression model, an ANOVA table is typically created that shows the observed significance level ($p$-value) for the calculated $F$ value.
- This can be compared to the level of significance ($\alpha$) to make a decision.

| | *DF* | *SS* | *MS* | *F* | SIGNIFICANCE |
|---|---|---|---|---|---|
| Regression | $k$ | $SSR$ | $MSR = SSR/k$ | $MSR/MSE$ | $P(F > MSR/MSE)$ |
| Residual | $n - k - 1$ | $SSE$ | $MSE = SSE/(n - k - 1)$ | | |
| Total | $n - 1$ | $SST$ | | | |

Table 4.4

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# ANOVA for Triple A Construction

| | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|
| 1 | SUMMARY OUTPUT | | | | | | |
| 2 | | | | | | | |
| 3 | *Regression Statistics* | | | | | | |
| 4 | Multiple R | 0.8333 | | | | | |
| 5 | R Square | 0.6944 | | | | | |
| 6 | Adjusted R Square | 0.6181 | | | | | |
| 7 | Standard Error | 1.3110 | | | | | |
| 8 | Observations | 6 | | | | | |
| 9 | | | | | | | |
| 10 | ANOVA | | | | | | |
| 11 | | *df* | *SS* | *MS* | *F* | *Significance F* | |
| 12 | Regression | 1 | 15.6250 | 15.6250 | 9.0909 | 0.0394 | |
| 13 | Residual | 4 | 6.8750 | 1.7188 | | | |
| 14 | Total | 5 | 22.5 | | | | |
| 15 | | | | | | | |

**Program 4.1C (partial)**

$$P(F > 9.0909) = 0.0394$$

**Because this probability is less than 0.05, we reject the null hypothesis of no linear relationship and conclude there is a linear relationship between $X$ and $Y$.**

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# Multiple Regression Analysis

- ***Multiple regression models*** **are extensions to the simple linear model and allow the creation of models with more than one independent variable.**

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \varepsilon$$

**where**

$Y$ = dependent variable (response variable)

$X_i$ = $i^{th}$ independent variable (predictor or explanatory variable)

$\beta_0$ = intercept (value of $Y$ when all $X_i = 0$)

$\beta_i$ = coefficient of the $i^{th}$ independent variable

$k$ = number of independent variables

$\varepsilon$ = random error

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# Multiple Regression Analysis

To estimate these values, a sample is taken the following equation developed

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2 + \ldots + b_k X_k$$

where

$\hat{Y}$ = predicted value of $Y$

$b_0$ = sample intercept (and is an estimate of $\beta_0$)

$b_i$ = sample coefficient of the *ith* variable (and is an estimate of $\beta_i$)

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# Jenny Wilson Realty

Jenny Wilson wants to develop a model to determine the suggested listing price for houses based on the size and age of the house.

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2$$

where

$\hat{Y}$ = predicted value of dependent variable (selling price)

$b_0$ = $Y$ intercept

$X_1$ and $X_2$ = value of the two independent variables (square footage and age) respectively

$b_1$ and $b_2$ = slopes for $X_1$ and $X_2$ respectively

She selects a sample of houses that have sold recently and records the data shown in Table 4.5

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# Jenny Wilson Real Estate Data

| SELLING PRICE ($) | SQUARE FOOTAGE | AGE | CONDITION |
|---|---|---|---|
| 95,000 | 1,926 | 30 | Good |
| 119,000 | 2,069 | 40 | Excellent |
| 124,800 | 1,720 | 30 | Excellent |
| 135,000 | 1,396 | 15 | Good |
| 142,000 | 1,706 | 32 | Mint |
| 145,000 | 1,847 | 38 | Mint |
| 159,000 | 1,950 | 27 | Mint |
| 165,000 | 2,323 | 30 | Excellent |
| 182,000 | 2,285 | 26 | Mint |
| 183,000 | 3,752 | 35 | Good |
| 200,000 | 2,300 | 18 | Good |
| 211,000 | 2,525 | 17 | Good |
| 215,000 | 3,800 | 40 | Excellent |
| 219,000 | 1,740 | 12 | Mint |

Table 4.5

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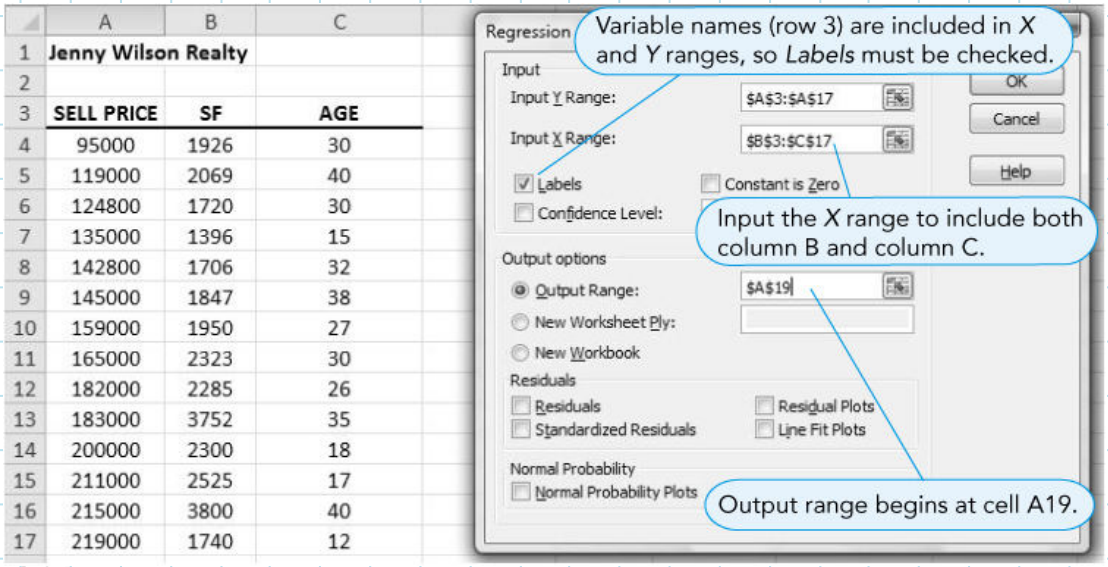
# Jenny Wilson Realty

## Input Screen for the Jenny Wilson Realty Multiple Regression Example




Program 4.2A

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# Jenny Wilson Realty

## Output for the Jenny Wilson Realty Multiple Regression Example

The coefficient of determination ($r^2$) is 0.67.

The regression coefficients are found here.

A low significance level for $F$ proves a relationship exists between $Y$ and at least one of the independent ($X$) variables.

The $p$-values are used to test the individual variables for significance.

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 19 | SUMMARY OUTPUT | | | | | | | | |
| 20 | | | | | | | | | |
| 21 | *Regression Statistics* | | | | | | | | |
| 22 | Multiple R | 0.8197 | | | | | | | |
| 23 | R Square | 0.6719 | | | | | | | |
| 24 | Adjusted R Square | 0.6122 | | | | | | | |
| 25 | Standard Error | 24312.607 | | | | | | | |
| 26 | Observations | 14 | | | | | | | |
| 27 | | | | | | | | | |
| 28 | ANOVA | | | | | | | | |
| 29 | | *df* | *SS* | *MS* | *F* | *Significance F* | | | |
| 30 | Regression | 2 | 13313936968 | 6656968484 | 11.2619 | 0.00217877 | | | |
| 31 | Residual | 11 | 6502131603 | 591102873 | | | | | |
| 32 | Total | 13 | 19816068571 | | | | | | |
| 33 | | | | | | | | | |
| 34 | | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *Lower 95.0%* | *Upper 95.0%* |
| 35 | Intercept | 146630.89 | 25482.0829 | 5.7543 | 0.0001 | 90545.2073 | 202716.5798 | 90545.2073 | 202716.5798 |
| 36 | SF | 43.82 | 10.2810 | 4.2622 | 0.0013 | 21.1911 | 66.4476 | 21.1911 | 66.4476 |
| 37 | AGE | -2898.69 | 796.5649 | -3.6390 | 0.0039 | -4651.9139 | -1145.4586 | -4651.9139 | -1145.4586 |

Program 4.2B

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# Evaluating Multiple Regression Models

- Evaluation is similar to simple linear regression models.
  - The $p$-value for the $F$-test and $r^2$ are interpreted the same.
- The hypothesis is different because there is more than one independent variable.
  - The $F$-test is investigating whether all the coefficients are equal to 0 at the same time.

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# Evaluating Multiple Regression Models

- To determine which independent variables are significant, tests are performed for each variable.

$$H_0 : \beta_1 = 0$$
$$H_1 : \beta_1 \neq 0$$

- The test statistic is calculated and if the $p$-value is lower than the level of significance ($\alpha$), the null hypothesis is rejected.

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# Jenny Wilson Realty

- The model is statistically significant
  - The $p$-value for the $F$-test is 0.002.
  - $r^2$ = 0.6719 so the model explains about 67% of the variation in selling price ($Y$).
- But the $F$-test is for the entire model and we can't tell if one or both of the independent variables are significant.
- By calculating the $p$-value of each variable, we can assess the significance of the individual variables.
- Since the p-value for $X_1$ (square footage) and $X_2$ (age) are both less than the significance level of 0.05, both null hypotheses can be rejected.

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# Binary or Dummy Variables

- *Binary* (or *dummy* or *indicator*) variables are special variables created for qualitative data.
- A dummy variable is assigned a value of 1 if a particular condition is met and a value of 0 otherwise.
- The number of dummy variables must equal one less than the number of categories of the qualitative variable.

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# Jenny Wilson Realty

- Jenny believes a better model can be developed if she includes information about the condition of the property.

$X_3$ = 1 if house is in excellent condition
= 0 otherwise
$X_4$ = 1 if house is in mint condition
= 0 otherwise

- Two dummy variables are used to describe the three categories of condition.
- No variable is needed for "good" condition since if both $X_3$ and $X_4$ = 0, the house must be in good condition.

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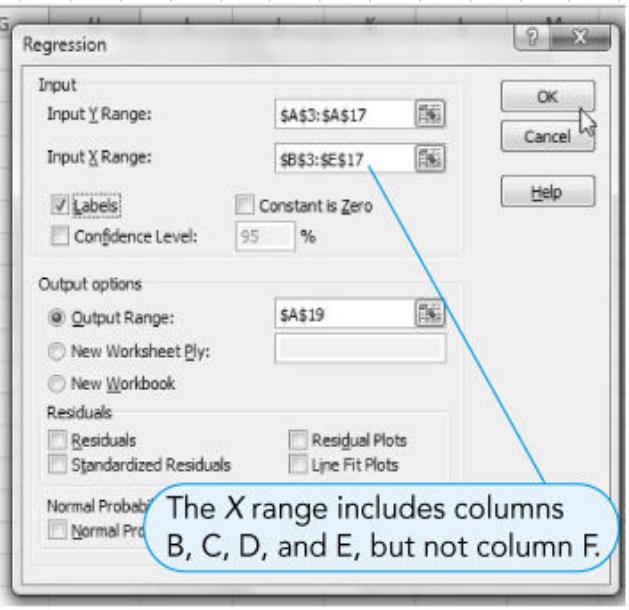
# Jenny Wilson Realty

## Input Screen for the Jenny Wilson Realty Example with Dummy Variables

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | Jenny Wilson Realty | | | | | |
| 2 | | | | | | |
| 3 | SELL PRICE | SF | AGE | X3 (Exc.) | X4 (Mint) | Condition |
| 4 | 95000 | 1926 | 30 | 0 | 0 | Good |
| 5 | 119000 | 2069 | 40 | 1 | 0 | Excellent |
| 6 | 124800 | 1720 | 30 | 1 | 0 | Excellent |
| 7 | 135000 | 1396 | 15 | 0 | 0 | Good |
| 8 | 142800 | 1706 | 32 | 0 | 1 | Mint |
| 9 | 145000 | 1847 | 38 | 0 | 1 | Mint |
| 10 | 159000 | 1950 | 27 | 0 | 1 | Mint |
| 11 | 165000 | 2323 | 30 | 1 | 0 | Excellent |
| 12 | 182000 | 2285 | 26 | 0 | 1 | Mint |
| 13 | 183000 | 3752 | 35 | 0 | 0 | Good |
| 14 | 200000 | 2300 | 18 | 0 | 0 | Good |
| 15 | 211000 | 2525 | 17 | 0 | 0 | Good |
| 16 | 215000 | 3800 | 40 | 1 | 0 | Excellent |
| 17 | 219000 | 1740 | 12 | 0 | 1 | Mint |

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**Program 4.3A**



# Jenny Wilson Realty

## Output for the Jenny Wilson Realty Example with Dummy Variables

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 19 | SUMMARY OUTPUT | | | | | | | | |
| 20 | | | | | | | | | |
| 21 | *Regression Statistics* | | | | | | | | |
| 22 | Multiple R | 0.9476 | | | | | | | |
| 23 | R Square | 0.8980 | | | | | | | |
| 24 | Adjusted R Squ | 0.8526 | | | | | | | |
| 25 | Standard Error | 14987.5545 | | | | | | | |
| 26 | Observations | 14 | | | | | | | |
| 27 | | | | | | | | | |
| 28 | ANOVA | | | | | | | | |
| 29 | | *df* | *SS* | *MS* | *F* | *Significance F* | | | |
| 30 | Regression | 4 | 17794427451 | 4.449E+09 | 19.804436 | 0.000174 | | | |
| 31 | Residual | 9 | 2021641120 | 224626791 | | | | | |
| 32 | Total | 13 | 19816068571 | | | | | | |
| 33 | | | | | | | | | |
| 34 | | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *ower 95.0%* | *pper 95.0%* |
| 35 | Intercept | 121658.45 | 17426.61 | 6.981 | 0.000 | 82236.71 | 161080.19 | 82236.71 | 161080.19 |
| 36 | SF | 56.43 | 6.95 | 8.122 | 0.000 | 40.71 | 72.14 | 40.71 | 72.14 |
| 37 | AGE | -3962.82 | 596.03 | -6.649 | 0.000 | -5311.13 | -2614.51 | -5311.13 | -2614.51 |
| 38 | X3 (Exc.) | 33162.65 | 12179.62 | 2.723 | 0.023 | 5610.43 | 60714.87 | 5610.43 | 60714.87 |
| 39 | X4 (Mint) | 47369.25 | 10649.27 | 4.448 | 0.002 | 23278.93 | 71459.57 | 23278.93 | 71459.57 |

The coefficient of age is negative, indicating that the price decreases as a house gets older.

The overall model is helpful because the significance *F* probability is low (much less than 5%).

Each of the variables individually is helpful because the *p*-values for each of them is low (much less than 5%).

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**Program 4.3B**

# Model Building

- The best model is a statistically significant model with a high $r^2$ and few variables.
- As more variables are added to the model, the $r^2$-value usually increases.
- For this reason, the *adjusted* $r^2$ value is often used to determine the usefulness of an additional variable.
- The adjusted $r^2$ takes into account the number of independent variables in the model.

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# Model Building

- The formula for $r^2$

$$r^2 = \frac{\text{SSR}}{\text{SST}} = 1 - \frac{\text{SSE}}{\text{SST}}$$

- The formula for adjusted $r^2$

$$\text{Adjusted } r^2 = 1 - \frac{\text{SSE}/(n-k-1)}{\text{SST}/(n-1)}$$

- As the number of variables increases, the adjusted $r^2$ gets smaller unless the increase due to the new variable is large enough to offset the change in $k$.

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# Model Building

- In general, if a new variable increases the adjusted $r^2$, it should probably be included in the model.
- In some cases, variables contain duplicate information.
- When two independent variables are correlated, they are said to be *collinear.*
- When more than two independent variables are correlated, *multicollinearity* exists.
- When multicollinearity is present, hypothesis tests for the individual coefficients are not valid but the model may still be useful.

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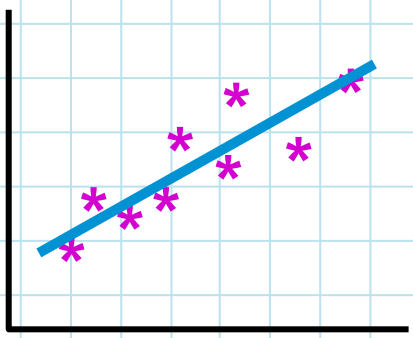
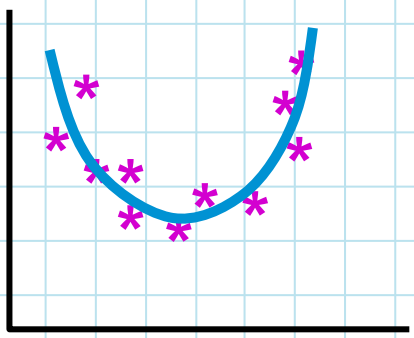
# Nonlinear Regression

- In some situations, variables are not linear.
- Transformations may be used to turn a nonlinear model into a linear model.



Linear relationship



Nonlinear relationship

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# Colonel Motors

- Engineers at Colonel Motors want to use regression analysis to improve fuel efficiency.
- They have been asked to study the impact of weight on miles per gallon (MPG).

| MPG | WEIGHT (1,000 LBS.) | MPG | WEIGHT (1,000 LBS.) |
|---|---|---|---|
| 12 | 4.58 | 20 | 3.18 |
| 13 | 4.66 | 23 | 2.68 |
| 15 | 4.02 | 24 | 2.65 |
| 18 | 2.53 | 33 | 1.70 |
| 19 | 3.09 | 36 | 1.95 |
| 19 | 3.11 | 42 | 1.92 |

Table 4.6

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# Colonel Motors

## Linear Model for MPG Data

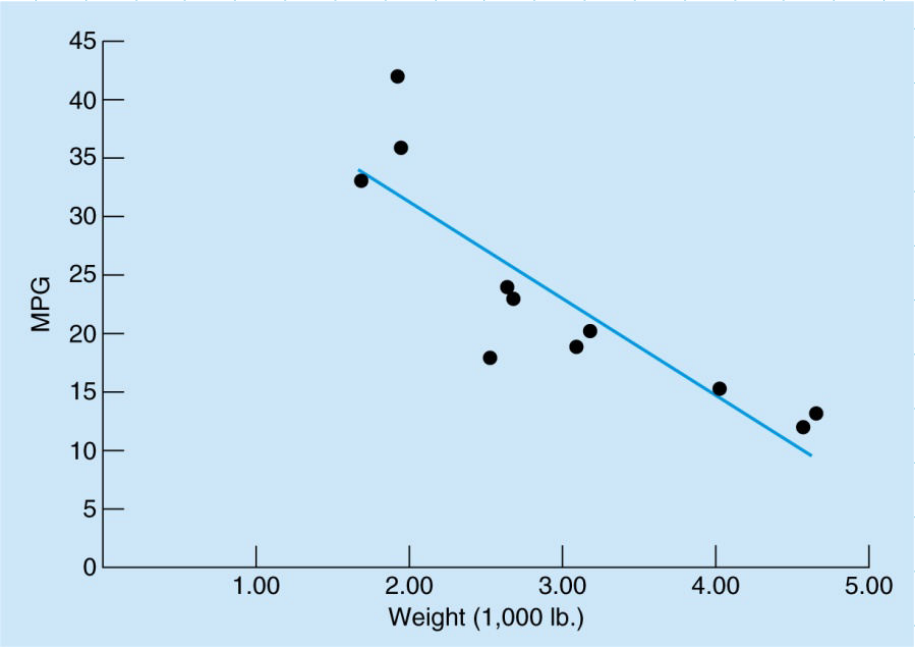


Figure 4.6A

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# Colonel Motors

## Excel Output for Linear Regression Model with MPG Data

| | A | B | C | D | E | F | G | H | I | J | K | L | M |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Automobile Weight vs. MPG | | | SUMMARY OUTPUT | | | | | | | | | |
| 2 | | | | | | | | | | | | | |
| 3 | MPG (Y) | Weight (X1) | | *Regression Statistics* | | | | | | | | | |
| 4 | 12 | 4.58 | | Multiple R | 0.8629 | | | | | | | | |
| 5 | 13 | 4.66 | | R Square | 0.7446 | | | | | | | | |
| 6 | 15 | 4.02 | | Adjusted R Sc | 0.7190 | | | | | | | | |
| 7 | 18 | 2.53 | | Standard Erro | 5.0076 | | | | | | | | |
| 8 | 19 | 3.09 | | Observations | 12 | | | | | | | | |
| 9 | 19 | 3.11 | | | | | | | | | | | |
| 10 | 20 | 3.18 | | ANOVA | | | | | | | | | |
| 11 | 23 | 2.68 | | | *df* | *SS* | *MS* | *F* | *Significance F* | | | | |
| 12 | 24 | 2.65 | | Regression | 1 | 730.9090 | 730.9090 | 29.1480 | 0.0003 | | | | |
| 13 | 33 | 1.70 | | Residual | 10 | 250.7577 | 25.0758 | | | | | | |
| 14 | 36 | 1.95 | | Total | 11 | 981.6667 | | | | | | | |
| 15 | 42 | 1.92 | | | | | | | | | | | |
| 16 | | | | | *Coefficients* | *Standard Err* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *Lower 95.0* | *Upper 95.0%* | |
| 17 | | | | Intercept | 47.6193 | 4.8132 | 9.8936 | 0.0000 | 36.8950 | 58.3437 | 36.8950 | 58.3437 | |
| 18 | | | | Weight | -8.2460 | 1.5273 | -5.3989 | 0.0003 | -11.6491 | -4.8428 | -11.6491 | -4.8428 | |

**Program 4.4**

**This is a useful model with a small $F$-test for significance and a good $r^2$ value.**

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# Colonel Motors

## Nonlinear Model for MPG Data

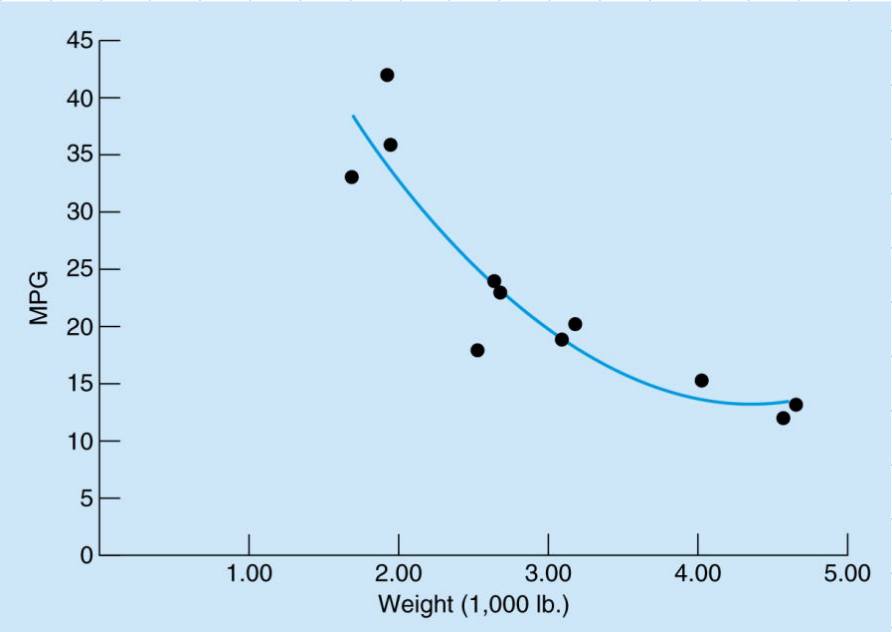


**Figure 4.6B**

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# Colonel Motors

- The nonlinear model is a quadratic model.
- The easiest way to work with this model is to develop a new variable.

$$X_2 = (\text{weight})^2$$

- This gives us a model that can be solved with linear regression software:

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2$$

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# Colonel Motors

| | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Automobile Weight vs. MPG | | | SUMMARY OUTPUT | | | | | | | | |
| 2 | | | | | | | | | | | | |
| 3 | MPG (Y) | Weight (X1) | WeightSq. (X2) | *Regression Statistics* | | | | | | | | |
| 4 | 12 | 4.58 | 20.98 | Multiple R | 0.9208 | | | | | | | |
| 5 | 13 | 4.66 | 21.72 | R Square | 0.8478 | | | | | | | |
| 6 | 15 | 4.02 | 16.16 | Adjusted R Sc | 0.8140 | | | | | | | |
| 7 | 18 | 2.53 | 6.40 | Standard Erro | 4.0745 | | | | | | | |
| 8 | 19 | 3.09 | 9.55 | Observations | 12 | | | | | | | |
| 9 | 19 | 3.11 | 9.67 | | | | | | | | | |
| 10 | 20 | 3.18 | 10.11 | ANOVA | | | | | | | | |
| 11 | 23 | 2.68 | 7.18 | | *df* | *SS* | *MS* | *F* | *Significance F* | | | |
| 12 | 24 | 2.65 | 7.02 | Regression | 2 | 832.2557 | 416.1278 | 25.0661 | 0.0002 | | | |
| 13 | 33 | 1.70 | 2.89 | Residual | 9 | 149.4110 | 16.6012 | | | | | |
| 14 | 36 | 1.95 | 3.80 | Total | 11 | 981.6667 | | | | | | |
| 15 | 42 | 1.92 | 3.69 | | | | | | | | | |
| 16 | | | | | *Coefficient* | *Standard Er* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *Lower 95.0* | *Upper 95.0%* |
| 17 | | | | Intercept | 79.7888 | 13.5962 | 5.8685 | 0.0002 | 49.0321 | 110.5454 | 49.0321 | 110.5454 |
| 18 | | | | Weight | -30.2224 | 8.9809 | -3.3652 | 0.0083 | -50.5386 | -9.9061 | -50.5386 | -9.9061 |
| 19 | | | | Weight2 | 3.4124 | 1.3811 | 2.4708 | 0.0355 | 0.2881 | 6.5367 | 0.2881 | 6.5367 |

$$\hat{Y} = 79.8 - 30.2X_1 + 3.4X_2$$

**Program 4.5**

**A better model with a smaller $F$-test for significance and a larger adjusted $r^2$ value**

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# Cautions and Pitfalls

- If the assumptions are not met, the statistical test may not be valid.
- Correlation does not necessarily mean causation.
- Multicollinearity makes interpreting coefficients problematic, but the model may still be good.
- Using a regression model beyond the range of $X$ is questionable, as the relationship may not hold outside the sample data.

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# Cautions and Pitfalls

- *A* $t$-test for the intercept ($b_0$) may be ignored as this point is often outside the range of the model.
- A linear relationship may not be the best relationship, even if the $F$-test returns an acceptable value.
- A nonlinear relationship can exist even if a linear relationship does not.
- Even though a relationship is statistically significant it may not have any practical value.

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# Chapter 13

# Waiting Lines and Queuing Theory Models

To accompany
*Quantitative Analysis for Management, Eleventh Edition,*
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

# Learning Objectives

**After completing this chapter, students will be able to:**

1. Describe the trade-off curves for cost-of-waiting time and cost of service.
2. Understand the three parts of a queuing system: the calling population, the queue itself, and the service facility.
3. Describe the basic queuing system configurations.
4. Understand the assumptions of the common models dealt with in this chapter.
5. Analyze a variety of operating characteristics of waiting lines.

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# Chapter Outline

13.1 Introduction
13.2 Waiting Line Costs
13.3 Characteristics of a Queuing System
13.4 Single-Channel Queuing Model with Poisson Arrivals and Exponential Service Times (*M/M/1*)
13.5 Multichannel Queuing Model with Poisson Arrivals and Exponential Service Times (*M/M/m*)

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# Chapter Outline


13.6 Constant Service Time Model (*M/D/1*)

13.7 Finite Population Model (*M/M/1* with Finite Source)

13.8 Some General Operating Characteristic Relationships

13.9 More Complex Queuing Models and the Use of Simulation


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# Introduction

- ***Queuing theory*** is the study of ***waiting lines.***
- It is one of the oldest and most widely used quantitative analysis techniques.
- The three basic components of a queuing process are arrivals, service facilities, and the actual waiting line.
- Analytical models of waiting lines can help managers evaluate the cost and effectiveness of service systems.

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# Waiting Line Costs

- Most waiting line problems are focused on finding the ideal level of service a firm should provide.
- In most cases, this service level is something management can control.
- When an organization *does* have control, they often try to find the balance between two extremes.

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# Waiting Line Costs

- There is generally a trade-off between cost of providing service and cost of waiting time.
  - A *large staff* and *many* service facilities generally results in high levels of service but have high costs.
  - Having the *minimum* number of service facilities keeps *service cost* down but may result in dissatisfied customers.
- Service facilities are evaluated on their *total expected cost* which is the sum of *service costs* and *waiting costs.*
- Organizations typically want to find the service level that minimizes the total expected cost.

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# Queuing Costs and Service Levels

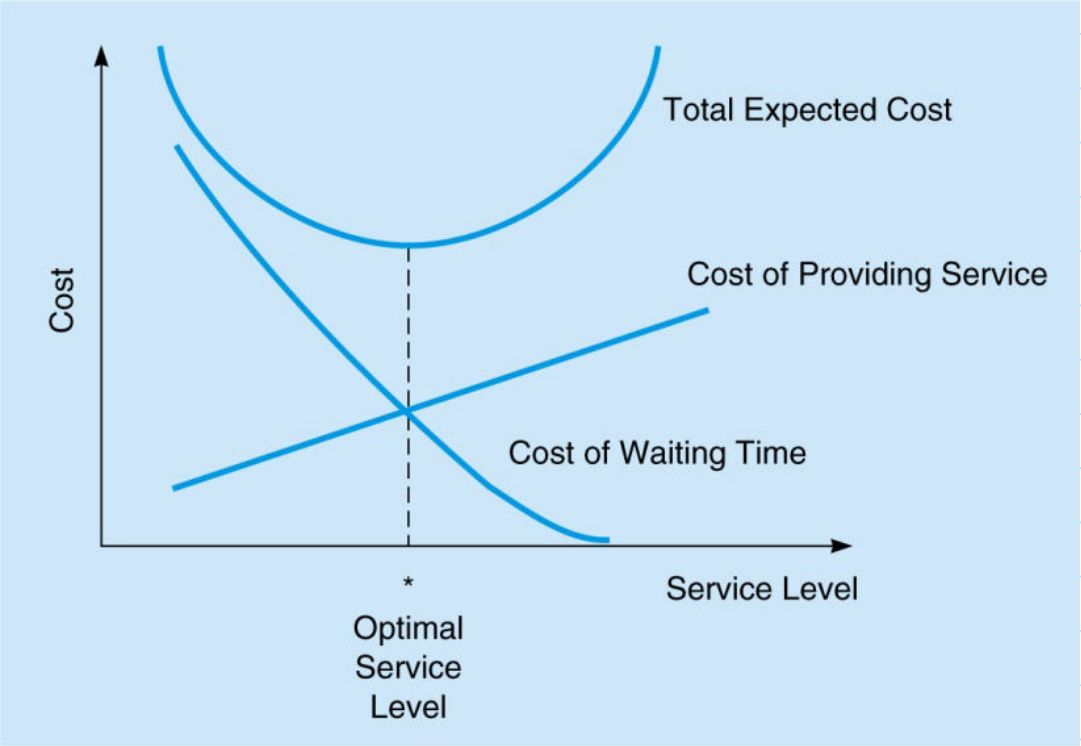


**Figure 13.1**

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# Three Rivers Shipping Company

- **Three Rivers Shipping operates a docking facility on the Ohio River.**
- **An average of 5 ships arrive to unload their cargos each shift.**
- **Idle ships are expensive.**
- **More staff can be hired to unload the ships, but that is expensive as well.**
- **Three Rivers Shipping Company wants to determine the optimal number of teams of stevedores to employ each shift to obtain the minimum total expected cost.**

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# Three Rivers Shipping Company Waiting Line Cost Analysis

| | | NUMBER OF TEAMS OF STEVEDORES WORKING | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| (a) | Average number of ships arriving per shift | 5 | 5 | 5 | 5 |
| (b) | Average time each ship waits to be unloaded (hours) | 7 | 4 | 3 | 2 |
| (c) | Total ship hours lost per shift (a x b) | 35 | 20 | 15 | 10 |
| (d) | Estimated cost per hour of idle ship time | $1,000 | $1,000 | $1,000 | $1,000 |
| (e) | Value of ship's lost time or waiting cost (c x d) | $35,000 | $20,000 | $15,000 | $10,000 |
| (f) | Stevedore team salary or service cost | $6,000 | $12,000 | $18,000 | $24,000 |
| (g) | Total expected cost (e + f) | $41,000 | $32,000 | $33,000 | $34,000 |

*Optimal cost*

Table 13.1

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# Characteristics of a Queuing System

- There are three parts to a queuing system:
  1. The arrivals or inputs to the system (sometimes referred to as the *calling population*).
  2. The queue or waiting line itself.
  3. The service facility.
- These components have their own characteristics that must be examined before mathematical models can be developed.

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## Characteristics of a Queuing System

**Arrival Characteristics have three major characteristics: *size*, *pattern*, and *behavior.***

- **The size of the calling population can be either unlimited (essentially *infinite*) or limited (*finite*).**
- **The pattern of arrivals can arrive according to a known pattern or can arrive *randomly.***
  - **Random arrivals generally follow a *Poisson distribution.***

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## Characteristics of a Queuing System

**Behavior of arrivals**

- **Most queuing models assume customers are patient and will wait in the queue until they are served and do not switch lines.**
- ***Balking* refers to customers who refuse to join the queue.**
- ***Reneging* customers enter the queue but become impatient and leave without receiving their service.**
- **That these behaviors exist is a strong argument for the use of queuing theory to managing waiting lines.**

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# Characteristics of a Queuing System

## Waiting Line Characteristics

- Waiting lines can be either *limited* or *unlimited.*
- Queue discipline refers to the rule by which customers in the line receive service.
  - The most common rule is *first-in, first-out* (*FIFO*).
  - Other rules are possible and may be based on other important characteristics.
- Other rules can be applied to select which customers enter which queue, but may apply FIFO once they are in the queue.

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# Characteristics of a Queuing System

## Service Facility Characteristics

- Basic queuing system configurations:
  - Service systems are classified in terms of the number of channels, or servers, and the number of phases, or service stops.
  - A *single-channel system* with one server is quite common.
  - *Multichannel systems* exist when multiple servers are fed by one common waiting line.
  - In a *single-phase system,* the customer receives service form just one server.
  - In a *multiphase system,* the customer has to go through more than one server.

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# Four basic queuing system configurations

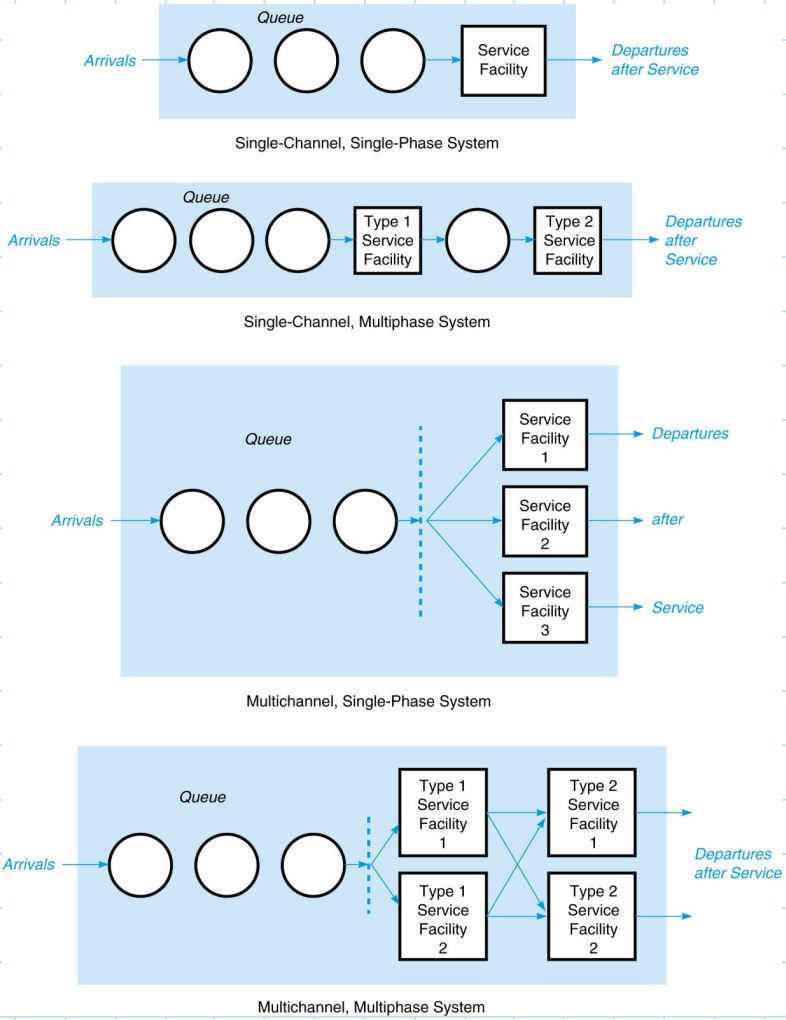


**Figure 13.2**

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# Characteristics of a Queuing System

**Service time distribution**

- **Service patterns can be either constant or random.**
- **Constant service times are often machine controlled.**
- **More often, service times are randomly distributed according to a *negative exponential probability distribution.***
- **Analysts should observe, collect, and plot service time data to ensure that the observations fit the assumed distributions when applying these models.**

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# Identifying Models Using Kendall Notation

- D. G. Kendall developed a notation for queuing models that specifies the pattern of arrival, the service time distribution, and the number of channels.
- Notation takes the form:

Arrival distribution / Service time distribution / Number of service channels open

- Specific letters are used to represent probability distributions.

$M$ = Poisson distribution for number of occurrences

$D$ = constant (deterministic) rate

$G$ = general distribution with known mean and variance

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# Identifying Models Using Kendall Notation

- A single-channel model with Poisson arrivals and exponential service times would be represented by:

$M/M/1$

- If a second channel is added the notation would read:

$M/M/2$

- A three-channel system with Poisson arrivals and constant service time would be

$M/D/3$

- A four-channel system with Poisson arrivals and normally distributed service times would be

$M/G/4$

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## *Single-Channel Model, Poisson Arrivals, Exponential Service Times (M/M/1)*

**Assumptions of the model:**

- **Arrivals are served on a FIFO basis.**
- **There is no balking or reneging.**
- **Arrivals are independent of each other but the arrival rate is constant over time.**
- **Arrivals follow a Poisson distribution.**
- **Service times are variable and independent but the average is known.**
- **Service times follow a negative exponential distribution.**
- **Average service rate is greater than the average arrival rate.**

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## *Single-Channel Model, Poisson Arrivals, Exponential Service Times (M/M/1)*

- **When these assumptions are met, we can develop a series of equations that define the queue's *operating characteristics.***
- **Queuing Equations:**

**Let**

$\lambda$ = **mean number of arrivals per time period**

$\mu$ = **mean number of customers or units served per time period**

**The arrival rate and the service rate must be defined for the same time period.**

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# Single-Channel Model, Poisson Arrivals, Exponential Service Times (M/M/1)

1. The average number of customers or units in the system, $L$:

$$L = \frac{\lambda}{\mu - \lambda}$$

2. The average time a customer spends in the system, $W$:

$$W = \frac{1}{\mu - \lambda}$$

3. The average number of customers in the queue, $L_q$:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

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# Single-Channel Model, Poisson Arrivals, Exponential Service Times (M/M/1)

4. The average time a customer spends waiting in the queue, $W_q$:

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

5. The *utilization factor* for the system, $\rho$, the probability the service facility is being used:

$$\rho = \frac{\lambda}{\mu}$$

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# Single-Channel Model, Poisson Arrivals, Exponential Service Times (M/M/1)

6. The percent idle time, $P_0$, or the probability no one is in the system:

$$P_0 = 1 - \frac{\lambda}{\mu}$$

7. The probability that the number of customers in the system is greater than $k$, $P_{n>k}$:

$$P_{n>k} = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

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# Arnold's Muffler Shop

- Arnold's mechanic can install mufflers at a rate of 3 per hour.
- Customers arrive at a rate of 2 per hour.
- So:

$\lambda$ = 2 cars arriving per hour

$\mu$ = 3 cars serviced per hour

$$L = \frac{\lambda}{\mu - \lambda} = \frac{2}{3-2} = \frac{2}{1} = 2$$ cars in the system on average

$$W = \frac{1}{\mu - \lambda} = \frac{1}{3-2} = 1$$ hour that an average car spends in the system

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# Arnold's Muffler Shop

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{2^2}{3(3-2)} = \frac{4}{3(1)} = 1.33$$ cars waiting in line on average

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{2}{3}\text{ hour}$$ = 40 minutes average waiting time per car

$$\rho = \frac{\lambda}{\mu} = \frac{2}{3} = 0.67$$ = percentage of time mechanic is busy

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{2}{3} = 0.33$$ = probability that there are 0 cars in the system

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# Arnold's Muffler Shop

Probability of more than $k$ cars in the system

| k | $P_{n>k} = (2/3)^{k+1}$ |
|---|---|
| 0 | 0.667 |
| 1 | 0.444 |
| 2 | 0.296 |
| 3 | 0.198 |
| 4 | 0.132 |
| 5 | 0.088 |
| 6 | 0.058 |
| 7 | 0.039 |

Note that this is equal to $1 - P_0 = 1 - 0.33 = 0.667$

Implies that there is a 19.8% chance that more than 3 cars are in the system

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# Excel QM Solution to Arnold's Muffler Example

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Arnold's Muffler Shop | | | | |
| 2 | | | | | |
| 3 | Waiting Lines | | M/M/1 (Single Server Model) | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | Data | | | Results | |
| 7 | Arrival rate ($\lambda$) | 2 | | Average server utilization($\rho$) | 0.6666667 |
| 8 | Service rate ($\mu$) | 3 | | Average number of customers in the queue($L_q$) | 1.3333333 |
| 9 | | | | Average number of customers in the system($L_s$) | 2 |
| 10 | | | | Average waiting time in the queue($W_q$) | 0.6666667 |
| 11 | | | | Average time in the system($W_s$) | 1 |
| 12 | | | | Probability (% of time) system is empty ($P_0$) | 0.3333333 |
| 13 | | | | | |
| 14 | | | | | |
| 15 | Probabilities | | | | |
| 16 | Number | Probability | Cumulative Probability | | |
| 17 | 0 | 0.333333 | 0.333333 | | |
| 18 | 1 | 0.222222 | 0.555556 | | |
| 19 | 2 | 0.148148 | 0.703704 | | |
| 20 | 3 | 0.098765 | 0.802469 | | |
| 21 | 4 | 0.065844 | 0.868313 | | |
| 22 | 5 | 0.043896 | 0.912209 | | |

The arrival RATE and service RATE both must be rates and use the same time unit. Given a time such as 10 minutes, convert it to a rate such as 6 per hour.

**Program 13.1**

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# Arnold's Muffler Shop

- **Introducing costs into the model:**
  - **Arnold wants to do an economic analysis of the queuing system and determine the waiting cost and service cost.**
  - **The total service cost is:**

$$\text{Total service cost} = (\text{Number of channels}) \times (\text{Cost per channel})$$

$$\text{Total service cost} = mC_s$$

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# Arnold's Muffler Shop

Waiting cost when the cost is based on time in the system:

Total waiting cost = (Total time spent waiting by all arrivals) x (Cost of waiting)

= (Number of arrivals) x (Average wait per arrival)$C_w$

Total waiting cost = $(\lambda W)C_w$

If waiting time cost is based on time in the queue:

Total waiting cost = $(\lambda W_q)C_w$

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# Arnold's Muffler Shop

So the total cost of the queuing system when based on time in the system is:

Total cost = Total service cost + Total waiting cost

Total cost = $mC_s + \lambda W C_w$

And when based on time in the queue:

Total cost = $mC_s + \lambda W_q C_w$

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# Arnold's Muffler Shop

- Arnold estimates the cost of customer *waiting* time in line is $50 per hour.

  Total daily waiting cost = (8 hours per day)$\lambda W_q C_w$

  = (8)(2)($^2/_3$)($50) = $533.33

- Arnold has identified the mechanics wage $7 per hour as the *service* cost.

  Total daily service cost = (8 hours per day)$mC_s$

  = (8)(1)($15) = $120

- So the total cost of the system is:

  Total daily cost of the queuing system = $533.33 + $120 = $653.33

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# Arnold's Muffler Shop

- Arnold is thinking about hiring a different mechanic who can install mufflers at a faster rate.
- The new operating characteristics would be:

  $\lambda$ = 2 cars arriving per hour

  $\mu$ = 4 cars serviced per hour

$$L = \frac{\lambda}{\mu - \lambda} = \frac{2}{4-2} = \frac{2}{2} = 1$$ car in the system on the average

$$W = \frac{1}{\mu - \lambda} = \frac{1}{4-2} = 1/2$$ hour that an average car spends in the system

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# Arnold's Muffler Shop

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{2^2}{4(4-2)} = \frac{4}{8(1)} = 1/2$$ car waiting in line on the average

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{1}{4} \text{ hour}$$ = 15 minutes average waiting time per car

$$\rho = \frac{\lambda}{\mu} = \frac{2}{4} = 0.5$$ = percentage of time mechanic is busy

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{2}{4} = 0.5$$ = probability that there are 0 cars in the system

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# Arnold's Muffler Shop

Probability of more than $k$ cars in the system

| k | $P_{n>k} = (2/4)^{k+1}$ |
|---|---|
| 0 | 0.500 |
| 1 | 0.250 |
| 2 | 0.125 |
| 3 | 0.062 |
| 4 | 0.031 |
| 5 | 0.016 |
| 6 | 0.008 |
| 7 | 0.004 |

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# Arnold's Muffler Shop Case

- The customer waiting cost is the same $50 per hour:

  Total daily waiting cost = (8 hours per day)$\lambda W_q C_w$

  = (8)(2)($^1/_4$)($50) = $200.00

- The new mechanic is more expensive at $20 per hour:

  Total daily service cost = (8 hours per day)$mC_s$

  = (8)(1)($20) = $160

- So the total cost of the system is:

  Total daily cost of the queuing system = $200 + $160 = $360

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# Arnold's Muffler Shop

- The total time spent waiting for the 16 customers per day was formerly:

  (16 cars per day) x ($^2/_3$ hour per car) = 10.67 hours

- It is now is now:

  (16 cars per day) x ($^1/_4$ hour per car) = 4 hours

- The total daily system costs are less with the new mechanic resulting in significant savings:

  $653.33 – $360 = $293.33

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## Enhancing the Queuing Environment

- Reducing waiting time is not the only way to reduce waiting cost.
- Reducing the unit waiting cost ($C_w$) will also reduce total waiting cost.
- This might be less expensive to achieve than reducing either $W$ or $W_q$.

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## Multichannel Queuing Model with Poisson Arrivals and Exponential Service Times (M/M/m)

**Assumptions of the model:**

- Arrivals are served on a FIFO basis.
- There is no balking or reneging.
- Arrivals are independent of each other but the arrival rate is constant over time.
- Arrivals follow a Poisson distribution.
- Service times are variable and independent but the average is known.
- Service times follow a negative exponential distribution.
- The average service rate is greater than the average arrival rate.

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## Multichannel Queuing Model with Poisson Arrivals and Exponential Service Times (M/M/m)

- Equations for the multichannel queuing model:
- Let

$m$ = number of channels open
$\lambda$ = average arrival rate
$\mu$ = average service rate at each channel

1. The probability that there are zero customers in the system is:

$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=m-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{m\mu}{m\mu-\lambda}} \quad \text{for } m\mu > \lambda$$

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## Multichannel Model, Poisson Arrivals, Exponential Service Times (M/M/m)

2. The average number of customers or units in the system

$$L = \frac{\lambda\mu(\lambda/\mu)^m}{(m-1)!(m\mu-\lambda)^2} P_0 + \frac{\lambda}{\mu}$$

3. The average time a unit spends in the waiting line or being served, in the system

$$W = \frac{\mu(\lambda/\mu)^m}{(m-1)!(m\mu-\lambda)^2} P_0 + \frac{1}{\mu} = \frac{L}{\lambda}$$

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# Multichannel Model, Poisson Arrivals, Exponential Service Times (M/M/m)

4. The average number of customers or units in line waiting for service

$$L_q = L - \frac{\lambda}{\mu}$$

5. The average number of customers or units in line waiting for service

$$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

6. The average number of customers or units in line waiting for service

$$\rho = \frac{\lambda}{m\mu}$$

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# Arnold's Muffler Shop Revisited

- Arnold wants to investigate opening a second garage bay.
- He would hire a second worker who works at the same rate as his first worker.
- The customer arrival rate remains the same.

$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=m-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{m\mu}{m\mu - \lambda}} \text{ for } m\mu > \lambda$$

$$P_0 = \frac{1}{\left[\sum_{n=0}^{1} \frac{1}{n!}\left(\frac{2}{3}\right)^n\right] + \frac{1}{2!}\left(\frac{2}{3}\right)^2 \frac{2(3)}{2(3)-2}} = 0.5$$

= probability of 0 cars in the system

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# Arnold's Muffler Shop Revisited

- **Average number of cars in the system**

$$L = \frac{\lambda\mu(\lambda/\mu)^m}{(m-1)!(m\mu-\lambda)^2}P_0 + \frac{\lambda}{\mu}$$

$$L = \frac{2(3)(2/3)^2}{(1)![2(3)-2]^2}(\frac{1}{2}) + \frac{2}{3} = 0.75$$

- **Average time a car spends in the system**

$$W = \frac{L}{\lambda} = \frac{3}{8}\text{ hour} = 22\,{}^{1}\!/_{2}\text{ minutes}$$

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# Arnold's Muffler Shop Revisited

- **Average number of cars in the queue**

$$L_q = L - \frac{\lambda}{\mu} = \frac{3}{4} - \frac{2}{3} = \frac{1}{12} = 0.083$$

- **Average time a car spends in the queue**

$$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda} = \frac{0.083}{2} = 0.0415\text{ hour} = 2\,{}^{1}\!/_{2}\text{ minutes}$$

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# Arnold's Muffler Shop Revisited

- Adding the second service bay reduces the waiting time in line but will increase the service cost as a second mechanic needs to be hired.

$$\text{Total daily waiting cost} = (8 \text{ hours per day})\lambda W_q C_w$$
$$= (8)(2)(0.0415)(\$50) = \$33.20$$

$$\text{Total daily service cost} = (8 \text{ hours per day}) m C_s$$
$$= (8)(2)(\$15) = \$240$$

- So the total cost of the system is

$$\text{Total system cost} = \$33.20 + \$240 = \$273.20$$

- This is the cheapest option: open the second bay and hire a second worker at the same $15 rate.

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# Effect of Service Level on Arnold's Operating Characteristics

| OPERATING CHARACTERISTIC | LEVEL OF SERVICE | | |
|---|---|---|---|
| | ONE MECHANIC $\mu = 3$ | TWO MECHANICS $\mu = 3$ FOR BOTH | ONE FAST MECHANIC $\mu = 4$ |
| Probability that the system is empty ($P_0$) | 0.33 | 0.50 | 0.50 |
| Average number of cars in the system ($L$) | 2 cars | 0.75 cars | 1 car |
| Average time spent in the system ($W$) | 60 minutes | 22.5 minutes | 30 minutes |
| Average number of cars in the queue ($L_q$) | 1.33 cars | 0.083 car | 0.50 car |
| Average time spent in the queue ($W_q$) | 40 minutes | 2.5 minutes | 15 minutes |

Table 13.2

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# Excel QM Solution to Arnold's Muffler Multichannel Example

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Arnold's Muffler Shop Multichannel | | | | |
| 2 | | | | | |
| 3 | Waiting Lines | | M/M/s | | |
| 4 | The arrival RATE and service RATE both must be rates and use the same time unit. Given a | | | | |
| 5 | | | | | |
| 6 | Data | | | Results | |
| 7 | Arrival rate (λ) | 2 | | Average server utilization(ρ) | 0.33333 |
| 8 | Service rate (μ) | 3 | | Average number of customers in the queue( | 0.08333 |
| 9 | Number of servers(s) | 2 | | Average number of customers in the system | 0.75 |
| 10 | | | | Average waiting time in the queue($W_q$) | 0.04167 |
| 11 | | | | Average time in the system($W_s$) | 0.375 |
| 12 | Enter the arrival rate, service rate, and number of servers (channels). | | | Probability (% of time) system is empty ($P_0$) | 0.5 |
| 13 | | | | | |
| 14 | Probabilities | | | | |
| 15 | Number | Probability | Cumulative Probability | | |
| 16 | 0 | 0.500000 | 0.500000 | | |
| 17 | 1 | 0.333333 | 0.833333 | | |
| 18 | 2 | 0.111111 | 0.944444 | | |
| 19 | 3 | 0.037037 | 0.981481 | | |

**Program 13.2**

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# Constant Service Time Model (M/D/1)

- Constant service times are used when customers or units are processed according to a fixed cycle.
- The values for $L_q$, $W_q$, $L$, and $W$ are always less than they would be for models with variable service time.
- In fact both average queue length and average waiting time are ***halved*** in constant service rate models.

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# Constant Service Time Model (M/D/1)

1. Average length of the queue

$$L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)}$$

2. Average waiting time in the queue

$$W_q = \frac{\lambda}{2\mu(\mu - \lambda)}$$

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# Constant Service Time Model (M/D/1)

3. Average number of customers in the system

$$L = L_q + \frac{\lambda}{\mu}$$

4. Average time in the system

$$W = W_q + \frac{1}{\mu}$$

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# Garcia-Golding Recycling, Inc.

- The company collects and compacts aluminum cans and glass bottles.
- Trucks arrive at an average rate of 8 per hour (Poisson distribution).
- Truck drivers wait about 15 minutes before they empty their load.
- Drivers and trucks cost $60 per hour.
- A new automated machine can process truckloads at a constant rate of 12 per hour.
- A new compactor would be amortized at $3 per truck unloaded.

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# Constant Service Time Model (M/D/1)

**Analysis of cost versus benefit of the purchase**

*Current* waiting cost/trip = ($^1/_4$ hour waiting time)($60/hour cost)
= $15/trip

*New* system: $\lambda$ = 8 trucks/hour arriving
$\mu$ = 12 trucks/hour served

Average waiting time in queue = $W_q = {}^1/_{12}$ hour

Waiting cost/trip with new compactor = ($^1/_{12}$ hour wait)($60/hour cost) = $5/trip

Savings with new equipment = $15 (current system) – $5 (new system)
= $10 per trip

Cost of new equipment amortized = $3/trip

Net savings = $7/trip

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# Excel QM Solution for Constant Service Time Model with Garcia-Golding Recycling Example

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Garcia-Golding Recycling | | | | |
| 2 | | | | | |
| 3 | Waiting Lines | | M/D/1 (Constant Service Times) | | |
| 4 | The arrival RATE and service RATE both must be rates and use the same time unit. Given a time | | | | |
| 5 | | | | | |
| 6 | Data | | | Results | |
| 7 | Arrival rate ($\lambda$) | 8 | | Average server utilization($\rho$) | 0.66667 |
| 8 | Service rate ($\mu$) | 12 | | Average number of customers in the queue($L_q$) | 0.66667 |
| 9 | | | | Average number of customers in the system($L_s$) | 1.33333 |
| 10 | | | | Average waiting time in the queue($W_q$) | 0.08333 |
| 11 | | | | Average time in the system($W_s$) | 0.16667 |
| 12 | | | | Probability (% of time) system is empty ($P_0$) | 0.33333 |

**Program 13.3**

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# Finite Population Model (M/M/1 with Finite Source)

- When the population of potential customers is limited, the models are different.
- There is now a dependent relationship between the length of the queue and the arrival rate.
- The model has the following assumptions:
  1. There is only one server.
  2. The population of units seeking service is finite.
  3. Arrivals follow a Poisson distribution and service times are exponentially distributed.
  4. Customers are served on a first-come, first-served basis.

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# Finite Population Model (M/M/1 with Finite Source)

## Equations for the finite population model:

Using $\lambda$ = mean arrival rate, $\mu$ = mean service rate, and $N$ = size of the population, the operating characteristics are:

1. Probability that the system is empty:

$$P_0 = \frac{1}{\sum_{n=0}^{N} \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n}$$

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# Finite Population Model (M/M/1 with Finite Source)

2. Average length of the queue:

$$L_q = N - \left(\frac{\lambda + \mu}{\lambda}\right)(1 - P_0)$$

3. Average number of customers (units) in the system:

$$L = L_q + (1 - P_0)$$

4. Average waiting time in the queue:

$$W_q = \frac{L_q}{(N - L)\lambda}$$

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# Finite Population Model (M/M/1 with Finite Source)

5. Average time in the system:

$$W = W_q + \frac{1}{\mu}$$

6. Probability of $n$ units in the system:

$$P_n = \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n P_0 \text{ for } n = 0,1,\ldots,N$$

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# Department of Commerce

- The Department of Commerce has five printers that each need repair after about 20 hours of work.
- Breakdowns will follow a Poisson distribution.
- The technician can service a printer in an average of about 2 hours, following an exponential distribution.
- Therefore:

$$\lambda = {}^1/_{20} = 0.05 \text{ printer/hour}$$
$$\mu = {}^1/_2 = 0.50 \text{ printer/hour}$$

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# Department of Commerce Example

1.
$$P_0 = \frac{1}{\sum_{n=0}^{5} \frac{5!}{(5-n)!}\left(\frac{0.05}{0.5}\right)^n} = 0.564$$

2.
$$L_q = 5 - \left(\frac{0.05+0.5}{0.05}\right)(1-P_0) = 0.2 \text{ printer}$$

3.
$$L = 0.2 + (1-0.564) = 0.64 \text{ printer}$$

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# Department of Commerce Example

4.
$$W_q = \frac{0.2}{(5-0.64)(0.05)} = \frac{0.2}{0.22} = 0.91 \text{ hour}$$

5.
$$W = 0.91 + \frac{1}{0.50} = 2.91 \text{ hours}$$

**If printer downtime costs $120 per hour and the technician is paid $25 per hour, the total cost is:**

Total hourly cost = (Average number of printers down) (Cost per downtime hour) + Cost per technician hour

= (0.64)($120) + $25 = $101.80

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# Excel QM For Finite Population Model with Department of Commerce Example

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Department of Commerce | | | | |
| 2 | | | | | |
| 3 | Waiting Lines | | M/M/s with a finite population | | |
| 4 | The arrival rate is for each member of the population. If they go for service every 20 | | | | |
| 5 | | | | | |
| 6 | Data | | | Results | |
| 7 | Arrival rate (λ) per customer | 0.05 | | Average server utilization(ρ) | 0.43605 |
| 8 | Service rate (μ) | 0.5 | | Average number of customers in the queue($L_q$) | 0.20347 |
| 9 | Number of servers | 1 | | Average number of customers in the system($L_s$) | 0.63952 |
| 10 | Population size (N) | 5 | | Average waiting time in the queue($W_q$) | 0.93326 |
| 11 | | | | Average time in the system($W_s$) | 2.93326 |
| 12 | | | | Probability (% of time) system is empty ($P_0$) | 0.56395 |
| 13 | | | | Effective arrival rate | 0.21802 |

**Program 13.4**

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# Some General Operating Characteristic Relationships

- Certain relationships exist among specific operating characteristics for any queuing system in a *steady state.*
- A steady state condition exists when a system is in its normal stabilized condition, usually after an initial *transient state.*
- The first of these are referred to as Little's Flow Equations:

$$L = \lambda W \qquad (\text{or } W = L/\lambda)$$
$$L_q = \lambda W_q \qquad (\text{or } W_q = L_q/\lambda)$$

- And

$$W = W_q + 1/\mu$$

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# More Complex Queuing Models and the Use of Simulation

- In the real world there are often *variations* from basic queuing models.
- *Computer simulation* can be used to solve these more complex problems.
- Simulation allows the analysis of controllable factors.
- Simulation should be used when standard queuing models provide only a poor approximation of the actual service system.

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# Chapter 7

# Linear Programming Models: Graphical and Computer Methods

To accompany
*Quantitative Analysis for Management, Eleventh Edition*,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

## Learning Objectives

**After completing this chapter, students will be able to:**

1. Understand the basic assumptions and properties of linear programming (LP).
2. Graphically solve any LP problem that has only two variables by both the corner point and isoprofit line methods.
3. Understand special issues in LP such as infeasibility, unboundedness, redundancy, and alternative optimal solutions.
4. Understand the role of sensitivity analysis.
5. Use Excel spreadsheets to solve LP problems.

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# Chapter Outline


7.1 Introduction
7.2 Requirements of a Linear Programming Problem
7.3 Formulating LP Problems
7.4 Graphical Solution to an LP Problem
7.5 Solving Flair Furniture's LP Problem using QM for Windows and Excel
7.6 Solving Minimization Problems
7.7 Four Special Cases in LP
7.8 Sensitivity Analysis


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# Introduction

- Many management decisions involve trying to make the most effective use of limited resources.
- *Linear programming* (*LP*) is a widely used mathematical modeling technique designed to help managers in planning and decision making relative to resource allocation.
  - This belongs to the broader field of *mathematical programming.*
  - In this sense, *programming* refers to modeling and solving a problem mathematically.

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# Requirements of a Linear Programming Problem

- All LP problems have 4 properties in common:
  1. All problems seek to *maximize* or *minimize* some quantity (the *objective function*).
  2. Restrictions or *constraints* that limit the degree to which we can pursue our objective are present.
  3. There must be alternative courses of action from which to choose.
  4. The objective and constraints in problems must be expressed in terms of *linear* equations or *inequalities.*

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# Basic Assumptions of LP

- We assume conditions of *certainty* exist and numbers in the objective and constraints are known with certainty and do not change during the period being studied.
- We assume *proportionality* exists in the objective and constraints.
- We assume *additivity* in that the total of all activities equals the sum of the individual activities.
- We assume *divisibility* in that solutions need not be whole numbers.
- All answers or variables are *nonnegative.*

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# LP Properties and Assumptions

| PROPERTIES OF LINEAR PROGRAMS |
|---|
| 1. One objective function |
| 2. One or more constraints |
| 3. Alternative courses of action |
| 4. Objective function and constraints are linear – proportionality and divisibility |
| 5. Certainty |
| 6. Divisibility |
| 7. Nonnegative variables |

Table 7.1

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# Formulating LP Problems

- Formulating a linear program involves developing a mathematical model to represent the managerial problem.
- The steps in formulating a linear program are:
  1. Completely understand the managerial problem being faced.
  2. Identify the objective and the constraints.
  3. Define the decision variables.
  4. Use the decision variables to write mathematical expressions for the objective function and the constraints.

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# Formulating LP Problems

- One of the most common LP applications is the *product mix problem.*
- Two or more products are produced using limited resources such as personnel, machines, and raw materials.
- The profit that the firm seeks to maximize is based on the profit contribution per unit of each product.
- The company would like to determine how many units of each product it should produce so as to maximize overall profit given its limited resources.

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# Flair Furniture Company

- The Flair Furniture Company produces inexpensive tables and chairs.
- Processes are similar in that both require a certain amount of hours of carpentry work and in the painting and varnishing department.
- Each table takes 4 hours of carpentry and 2 hours of painting and varnishing.
- Each chair requires 3 of carpentry and 1 hour of painting and varnishing.
- There are 240 hours of carpentry time available and 100 hours of painting and varnishing.
- Each table yields a profit of $70 and each chair a profit of $50.

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# Flair Furniture Company Data

The company wants to determine the best combination of tables and chairs to produce to reach the maximum profit.

| DEPARTMENT | HOURS REQUIRED TO PRODUCE 1 UNIT (*T*) TABLES | (*C*) CHAIRS | AVAILABLE HOURS THIS WEEK |
|---|---|---|---|
| Carpentry | 4 | 3 | 240 |
| Painting and varnishing | 2 | 1 | 100 |
| Profit per unit | $70 | $50 | |

Table 7.2

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# Flair Furniture Company

- The objective is to:

  Maximize profit
- The constraints are:
  1. The hours of carpentry time used cannot exceed 240 hours per week.
  2. The hours of painting and varnishing time used cannot exceed 100 hours per week.
- The decision variables representing the actual decisions we will make are:

  $T$ = number of tables to be produced per week.

  $C$ = number of chairs to be produced per week.

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# Flair Furniture Company

- We create the LP objective function in terms of $T$ and $C$:

$$\text{Maximize profit} = \$70T + \$50C$$

- Develop mathematical relationships for the two constraints:
  - For carpentry, total time used is:

    (4 hours per table)(Number of tables produced)
    + (3 hours per chair)(Number of chairs produced).
  - We know that:

Carpentry time used ≤ Carpentry time available.

$$4T + 3C \leq 240 \text{ (hours of carpentry time)}$$

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# Flair Furniture Company

- Similarly,

Painting and varnishing time used
≤ Painting and varnishing time available.

$$2T + 1C \leq 100 \text{ (hours of painting and varnishing time)}$$

This means that each table produced requires two hours of painting and varnishing time.

- Both of these constraints restrict production capacity and affect total profit.

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# Flair Furniture Company

The values for $T$ and $C$ must be nonnegative.

$T \geq 0$ (number of tables produced is greater than or equal to 0)

$C \geq 0$ (number of chairs produced is greater than or equal to 0)

The complete problem stated mathematically:

$$\text{Maximize profit} = \$70T + \$50C$$

subject to

$4T + 3C \leq 240$ (carpentry constraint)

$2T + 1C \leq 100$ (painting and varnishing constraint)

$T, C \geq 0$ (nonnegativity constraint)

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# Graphical Solution to an LP Problem

- The easiest way to solve a small LP problems is graphically.
- The graphical method only works when there are just two decision variables.
- When there are more than two variables, a more complex approach is needed as it is not possible to plot the solution on a two-dimensional graph.
- The graphical method provides valuable insight into how other approaches work.

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# Graphical Representation of a Constraint

## Quadrant Containing All Positive Values

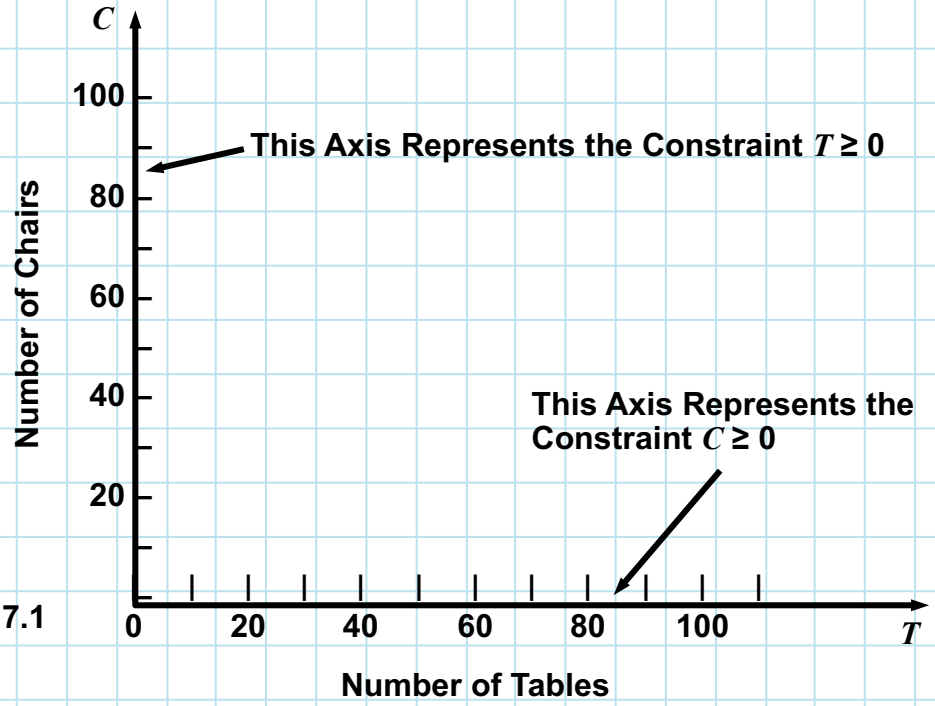


Figure 7.1

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# Graphical Representation of a Constraint

- The first step in solving the problem is to identify a set or region of feasible solutions.
- To do this we plot each constraint equation on a graph.
- We start by graphing the equality portion of the constraint equations:

$$4T + 3C = 240$$

- We solve for the axis intercepts and draw the line.

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# Graphical Representation of a Constraint

- When Flair produces no tables, the carpentry constraint is:

$$4(0) + 3C = 240$$
$$3C = 240$$
$$C = 80$$

- Similarly for no chairs:

$$4T + 3(0) = 240$$
$$4T = 240$$
$$T = 60$$

- This line is shown on the following graph:

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# Graphical Representation of a Constraint

Graph of carpentry constraint equation

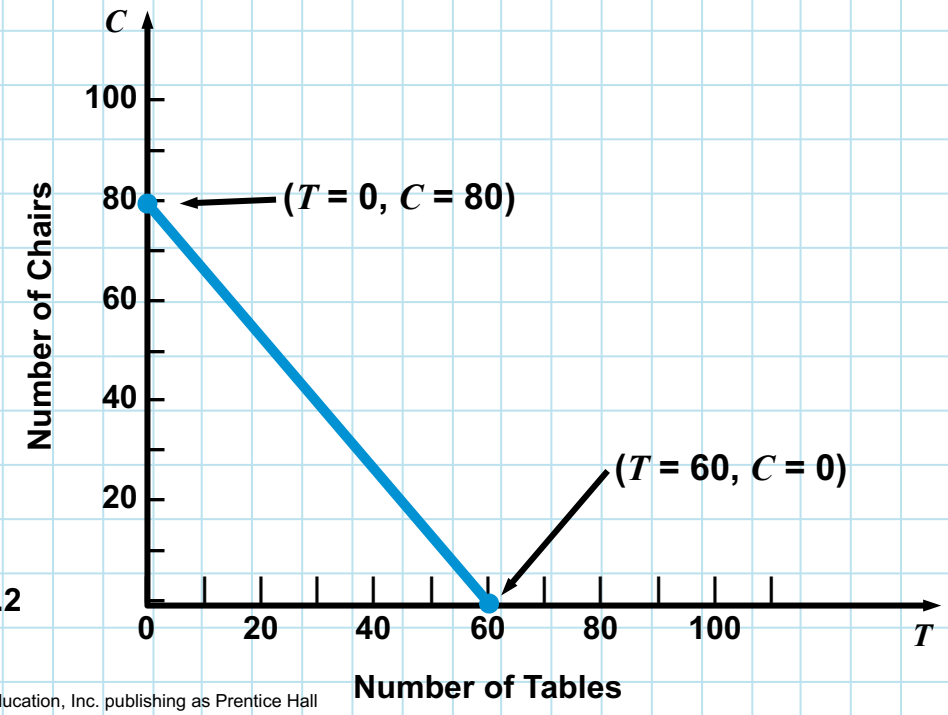


Figure 7.2

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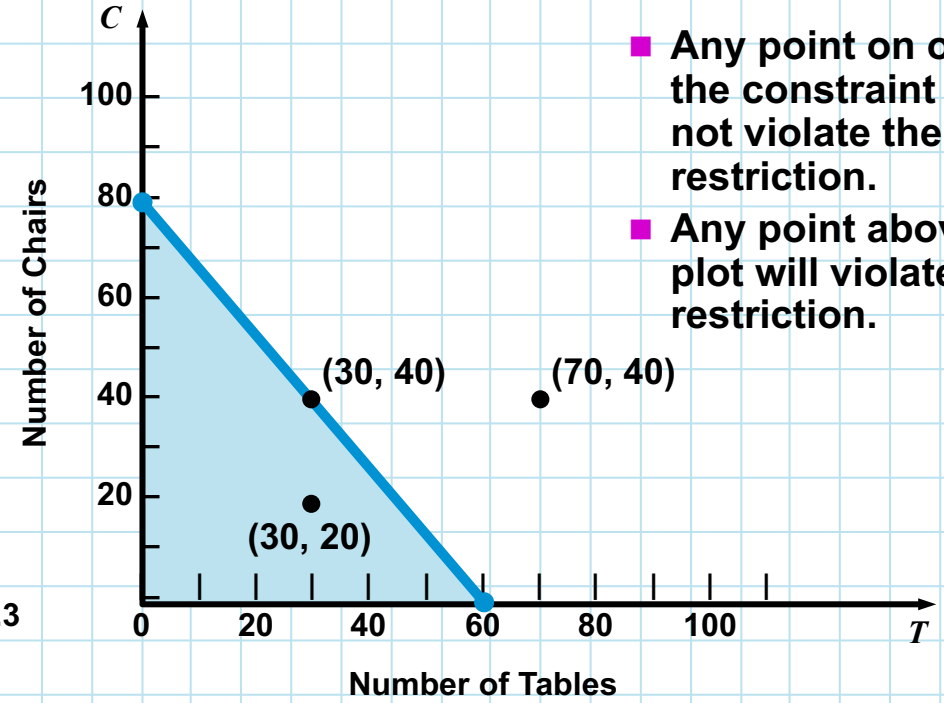
# Graphical Representation of a Constraint

## Region that Satisfies the Carpentry Constraint




- Any point on or below the constraint plot will not violate the restriction.
- Any point above the plot will violate the restriction.

Figure 7.3

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# Graphical Representation of a Constraint

- The point (30, 40) lies on the plot and exactly satisfies the constraint

$$4(30) + 3(40) = 240.$$

- The point (30, 20) lies below the plot and satisfies the constraint

$$4(30) + 3(20) = 180.$$

- The point (70, 40) lies above the plot and does not satisfy the constraint

$$4(70) + 3(40) = 400.$$

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# Graphical Representation of a Constraint

**Region that Satisfies the Painting and Varnishing Constraint**

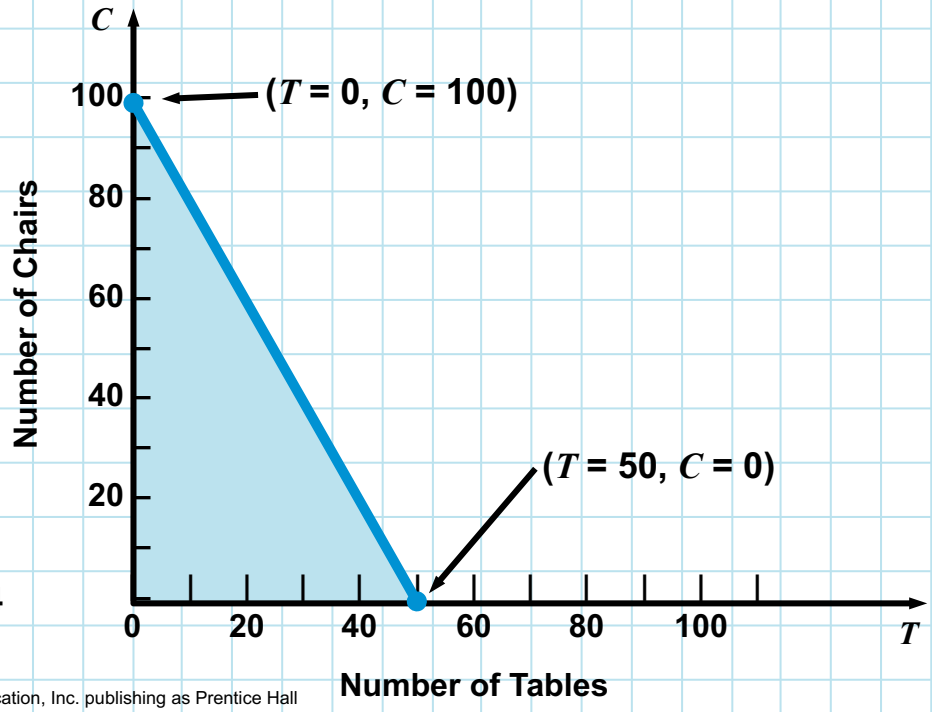


Figure 7.4

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# Graphical Representation of a Constraint

- To produce tables and chairs, both departments must be used.
- We need to find a solution that satisfies both constraints *simultaneously.*
- A new graph shows both constraint plots.
- The *feasible region* (or *area of feasible solutions*) is where all constraints are satisfied.
- Any point inside this region is a *feasible* solution.
- Any point outside the region is an *infeasible* solution.

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# Graphical Representation of a Constraint

**Feasible Solution Region for the Flair Furniture Company Problem**

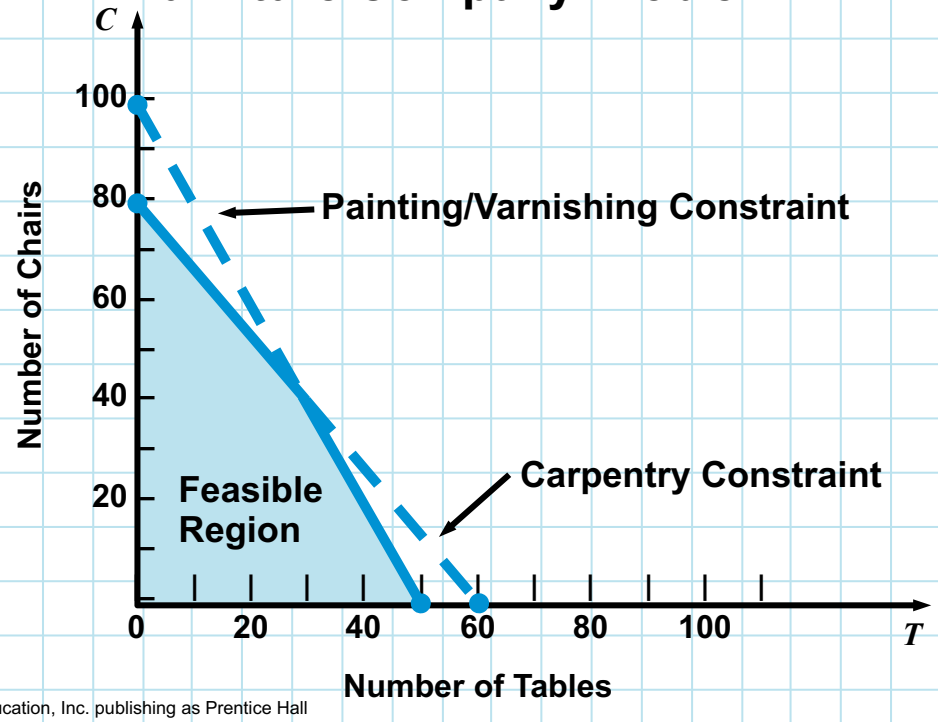


Figure 7.5

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# Graphical Representation of a Constraint

- **For the point (30, 20)**

| | | |
|---|---|---|
| *Carpentry constraint* | $4T + 3C \leq 240$ hours available<br>$(4)(30) + (3)(20) = 180$ hours used | ✓ |
| *Painting constraint* | $2T + 1C \leq 100$ hours available<br>$(2)(30) + (1)(20) = 80$ hours used | ✓ |

- **For the point (70, 40)**

| | | |
|---|---|---|
| *Carpentry constraint* | $4T + 3C \leq 240$ hours available<br>$(4)(70) + (3)(40) = 400$ hours used | ✗ |
| *Painting constraint* | $2T + 1C \leq 100$ hours available<br>$(2)(70) + (1)(40) = 180$ hours used | ✗ |

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# Graphical Representation of a Constraint

- For the point (50, 5)

| | | |
|---|---|---|
| *Carpentry constraint* | $4T + 3C \leq 240$ hours available<br>(4)(50) + (3)(5) = 215 hours used | ✓ |
| *Painting constraint* | $2T + 1C \leq 100$ hours available<br>(2)(50) + (1)(5) = 105 hours used | ✗ |

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# Isoprofit Line Solution Method

- Once the feasible region has been graphed, we need to find the optimal solution from the many possible solutions.
- The speediest way to do this is to use the isoprofit line method.
- Starting with a small but possible profit value, we graph the objective function.
- We move the objective function line in the direction of increasing profit while maintaining the slope.
- The last point it touches in the feasible region is the optimal solution.

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# Isoprofit Line Solution Method

- For Flair Furniture, choose a profit of $2,100.
- The objective function is then

$$\$2{,}100 = 70T + 50C$$

- Solving for the axis intercepts, we can draw the graph.
- This is obviously not the best possible solution.
- Further graphs can be created using larger profits.
- The further we move from the origin, the larger the profit will be.
- The highest profit ($4,100) will be generated when the isoprofit line passes through the point (30, 40).

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# Isoprofit Line Solution Method

**Profit line of $2,100 Plotted for the Flair Furniture Company**

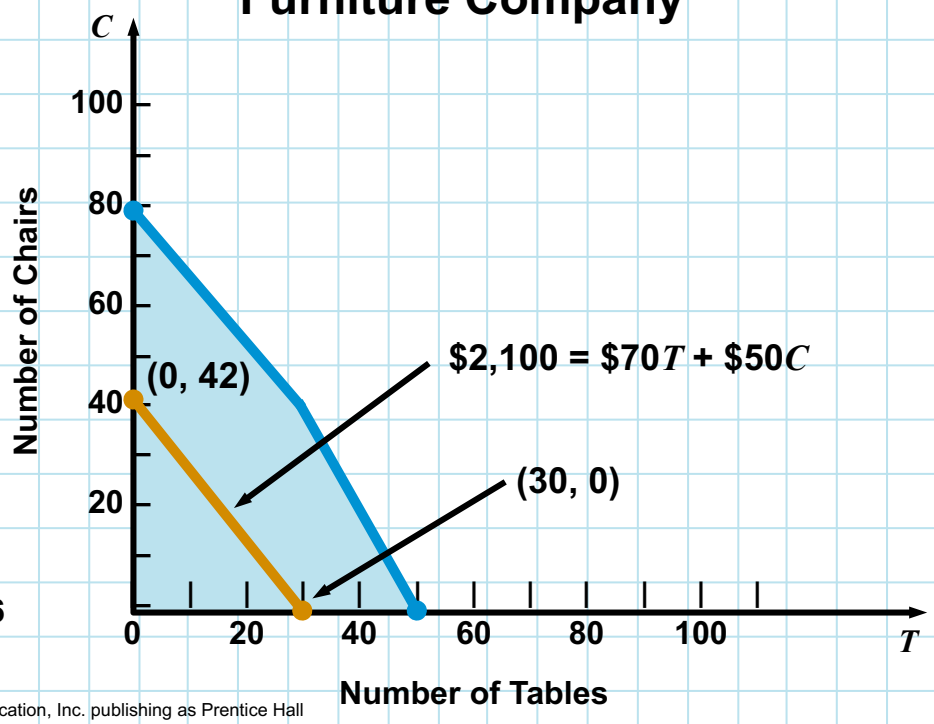


Figure 7.6

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# Isoprofit Line Solution Method

## Four Isoprofit Lines Plotted for the Flair Furniture Company

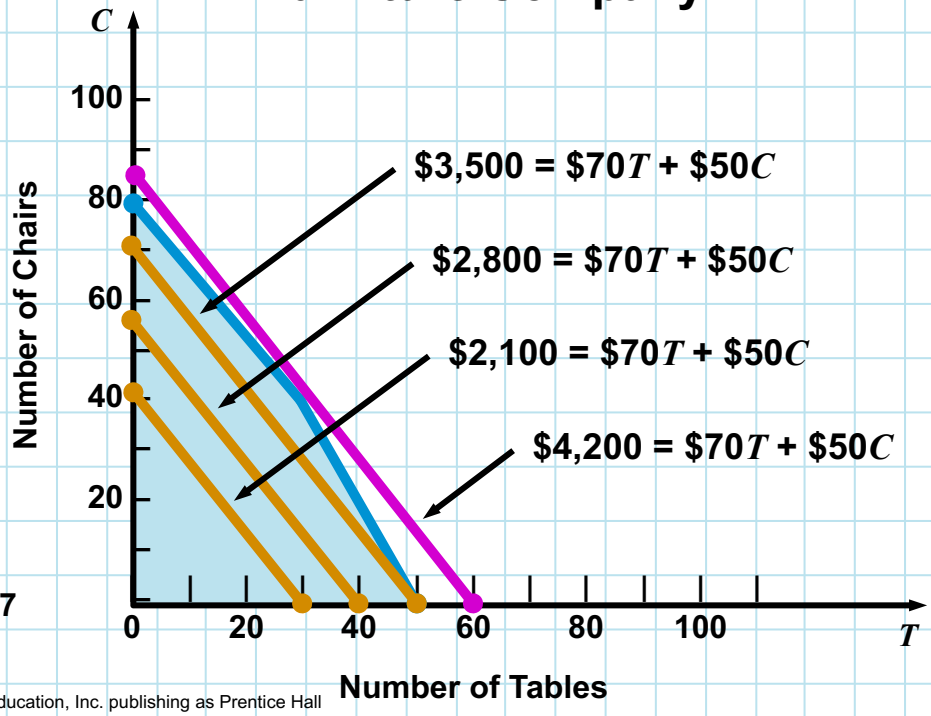


Figure 7.7

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# Isoprofit Line Solution Method

## Optimal Solution to the Flair Furniture problem

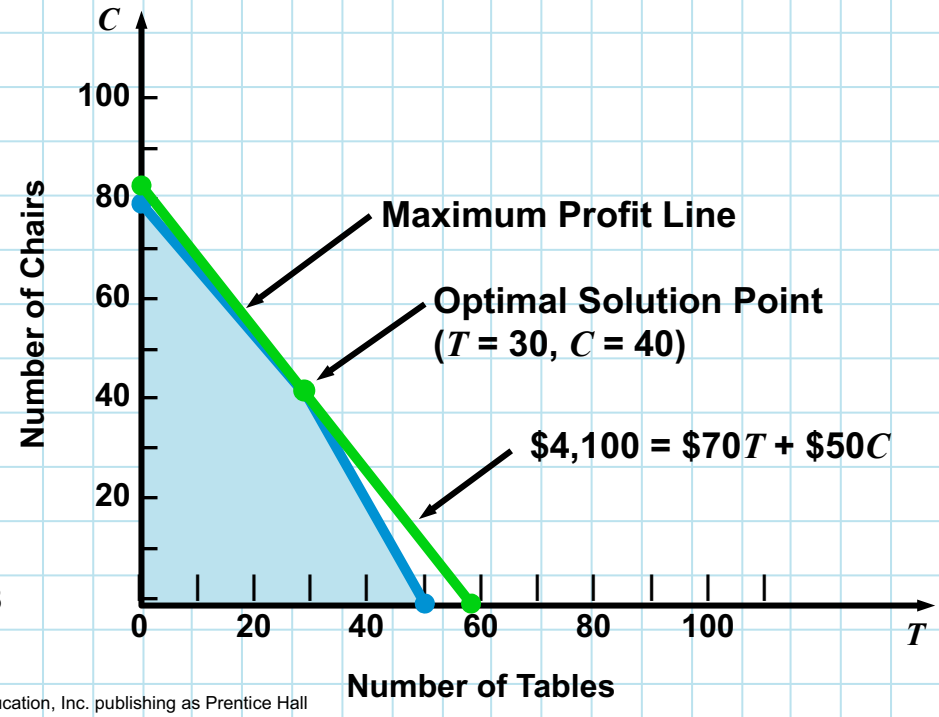


Figure 7.8

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# Corner Point Solution Method

- A second approach to solving LP problems employs the *corner point method.*
- It involves looking at the profit at every corner point of the feasible region.
- The mathematical theory behind LP is that the optimal solution must lie at one of the *corner points*, or *extreme point*, in the feasible region.
- For Flair Furniture, the feasible region is a four-sided polygon with four corner points labeled 1, 2, 3, and 4 on the graph.

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# Corner Point Solution Method

**Four Corner Points of the Feasible Region**

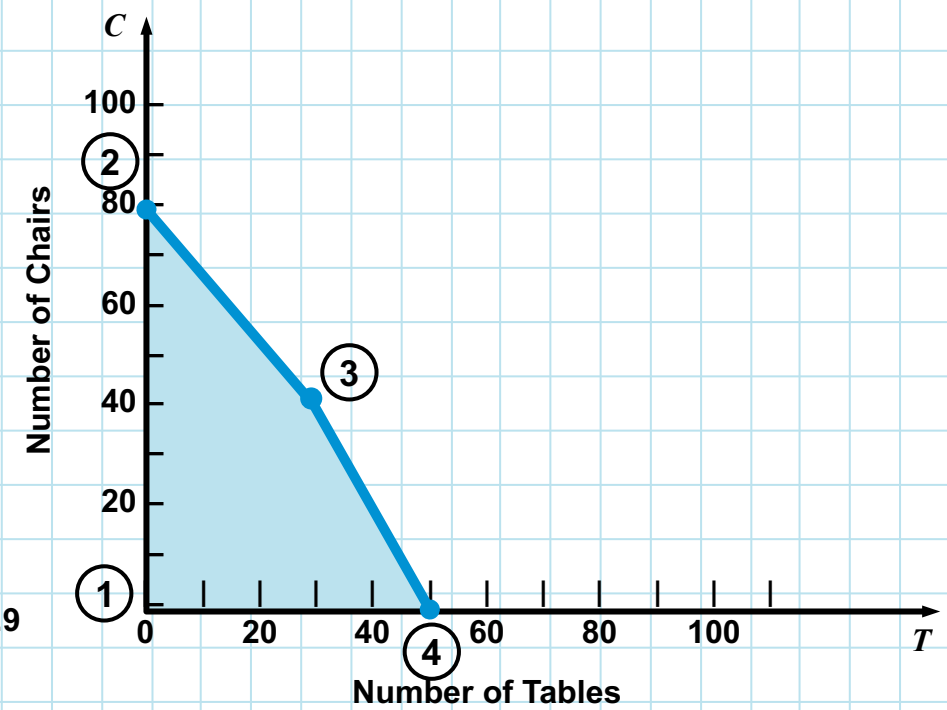


Figure 7.9

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# Corner Point Solution Method

- To find the coordinates for Point 3 accurately we have to solve for the intersection of the two constraint lines.
- Using the *simultaneous equations method*, we multiply the painting equation by –2 and add it to the carpentry equation

$$\begin{aligned} 4T + 3C &= 240 && \text{(carpentry line)} \\ -4T - 2C &= -200 && \text{(painting line)} \\ C &= 40 \end{aligned}$$

- Substituting 40 for $C$ in either of the original equations allows us to determine the value of $T$.

$$\begin{aligned} 4T + (3)(40) &= 240 && \text{(carpentry line)} \\ 4T + 120 &= 240 \\ T &= 30 \end{aligned}$$

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# Corner Point Solution Method

| | |
|---|---|
| Point 1: ($T$ = 0, $C$ = 0) | Profit = \$70(0) + \$50(0) = \$0 |
| Point 2: ($T$ = 0, $C$ = 80) | Profit = \$70(0) + \$50(80) = \$4,000 |
| Point 4: ($T$ = 50, $C$ = 0) | Profit = \$70(50) + \$50(0) = \$3,500 |
| Point 3: ($T$ = 30, $C$ = 40) | Profit = \$70(30) + \$50(40) = \$4,100 |

**Because Point 3 returns the highest profit, this is the optimal solution.**

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# Slack and Surplus

- **Slack** is the amount of a resource that is not used. For a less-than-or-equal constraint:
  - **Slack** = Amount of resource available – amount of resource used.
- Surplus is used with a greater-than-or-equal constraint to indicate the amount by which the right hand side of the constraint is exceeded.
  - **Surplus** = Actual amount – minimum amount.

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# Summary of Graphical Solution Methods

| ISOPROFIT METHOD |
|---|
| 1. Graph all constraints and find the feasible region. |
| 2. Select a specific profit (or cost) line and graph it to find the slope. |
| 3. Move the objective function line in the direction of increasing profit (or decreasing cost) while maintaining the slope. The last point it touches in the feasible region is the optimal solution. |
| 4. Find the values of the decision variables at this last point and compute the profit (or cost). |
| **CORNER POINT METHOD** |
| 1. Graph all constraints and find the feasible region. |
| 2. Find the corner points of the feasible reason. |
| 3. Compute the profit (or cost) at each of the feasible corner points. |
| 4. Select the corner point with the best value of the objective function found in Step 3. This is the optimal solution. |

Table 7.4

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## *Solving Flair Furniture's LP Problem Using QM for Windows and Excel*

- Most organizations have access to software to solve big LP problems.
- While there are differences between software implementations, the approach each takes towards handling LP is basically the same.
- Once you are experienced in dealing with computerized LP algorithms, you can easily adjust to minor changes.

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## *Using QM for Windows*

- First select the Linear Programming module.
- Specify the number of constraints (non-negativity is assumed).
- Specify the number of decision variables.
- Specify whether the objective is to be maximized or minimized.
- For the Flair Furniture problem there are two constraints, two decision variables, and the objective is to maximize profit.

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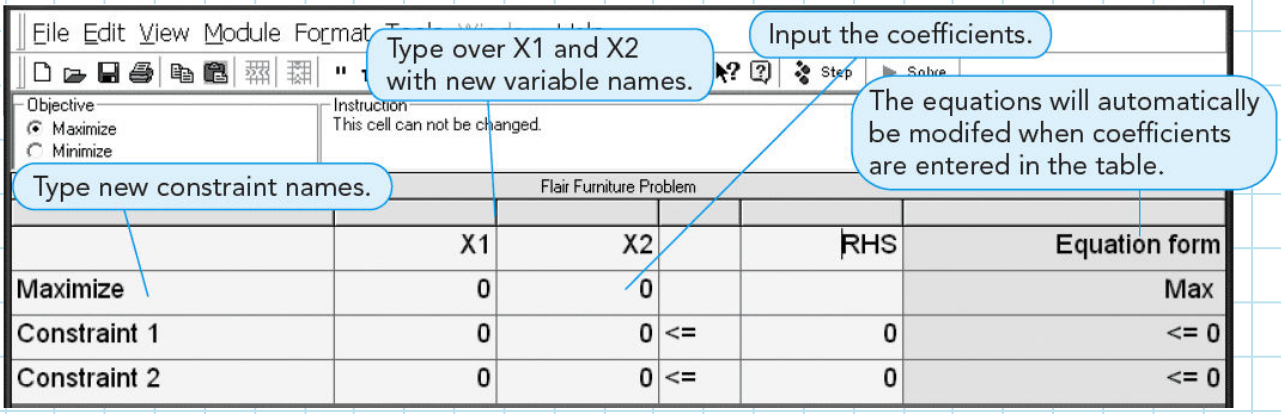
# Using QM for Windows

## QM for Windows Linear Programming Computer screen for Input of Data




| | X1 | X2 | | RHS | Equation form |
|---|---|---|---|---|---|
| Maximize | 0 | 0 | | | Max |
| Constraint 1 | 0 | 0 | <= | 0 | <= 0 |
| Constraint 2 | 0 | 0 | <= | 0 | <= 0 |

**Program 7.1A**

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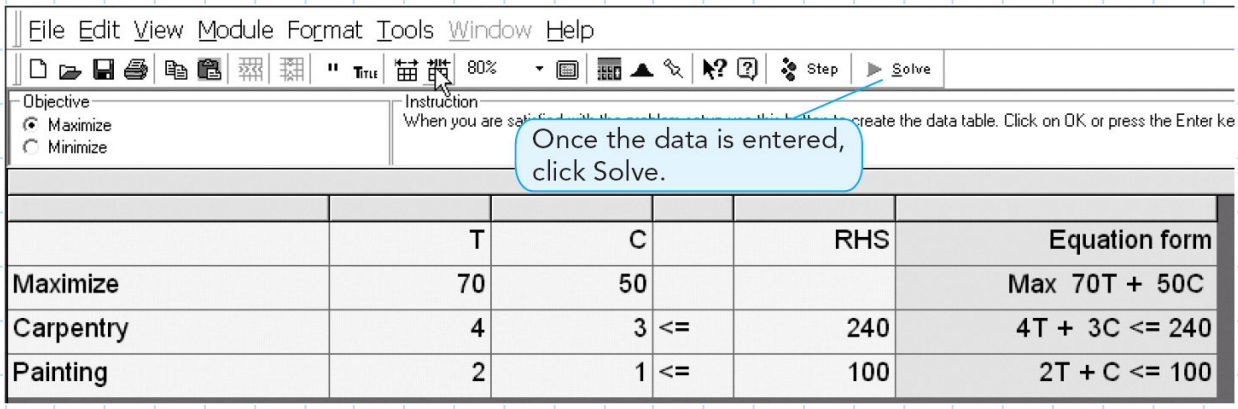
# Using QM for Windows

## QM for Windows Data Input for Flair Furniture Problem




| | T | C | | RHS | Equation form |
|---|---|---|---|---|---|
| Maximize | 70 | 50 | | | Max 70T + 50C |
| Carpentry | 4 | 3 | <= | 240 | 4T + 3C <= 240 |
| Painting | 2 | 1 | <= | 100 | 2T + C <= 100 |

**Program 7.1B**

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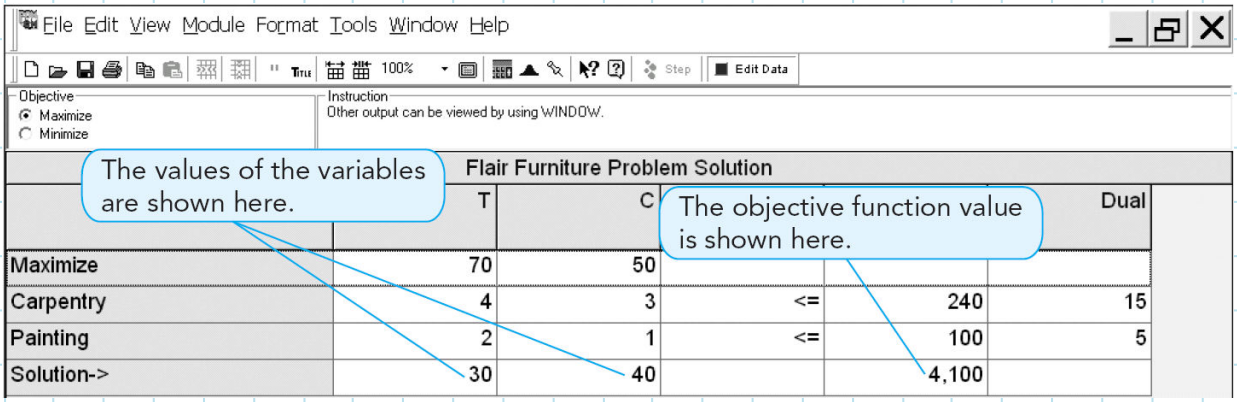
# Using QM for Windows

## QM for Windows Output for Flair Furniture Problem




Flair Furniture Problem Solution

| | T | C | | | Dual |
|---|---|---|---|---|---|
| Maximize | 70 | 50 | | | |
| Carpentry | 4 | 3 | <= | 240 | 15 |
| Painting | 2 | 1 | <= | 100 | 5 |
| Solution-> | 30 | 40 | | 4,100 | |

**Program 7.1C**

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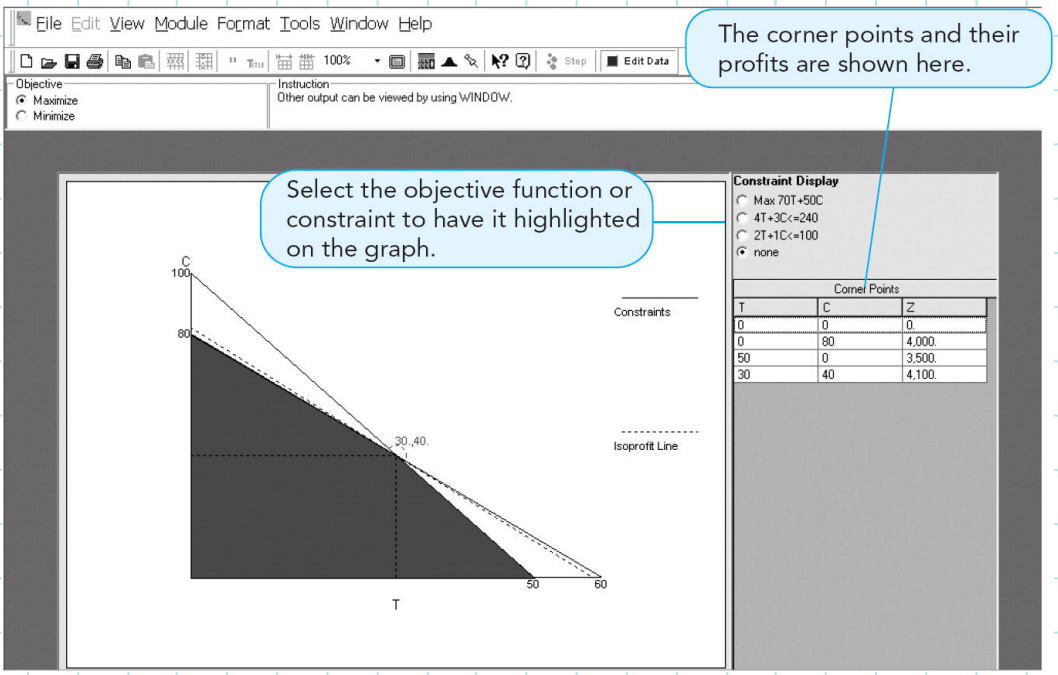
# Using QM for Windows

## QM for Windows Graphical Output for Flair Furniture Problem




**Program 7.1D**

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# Using Excel's Solver Command to Solve LP Problems

- The Solver tool in Excel can be used to find solutions to:
  - LP problems.
  - Integer programming problems.
  - Noninteger programming problems.
- Solver is limited to 200 variables and 100 constraints.

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# Using Solver to Solve the Flair Furniture Problem

- Recall the model for Flair Furniture is:

$$\text{Maximize profit} = \$70T + \$50C$$

$$\text{Subject to} \quad 4T + 3C \leq 240$$

$$2T + 1C \leq 100$$

- To use Solver, it is necessary to enter formulas based on the initial model.

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# Using Solver to Solve the Flair Furniture Problem

1. Enter the variable names, the coefficients for the objective function and constraints, and the right-hand-side values for each of the constraints.
2. Designate specific cells for the values of the decision variables.
3. Write a formula to calculate the value of the objective function.
4. Write a formula to compute the left-hand sides of each of the constraints.

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# Using Solver to Solve the Flair Furniture Problem

## Excel Data Input for the Flair Furniture Example

These cells are selected to contain the values of the decision variables. Solver will enter the optimal solution here, but you may enter numbers here also.

The signs for the constraints are entered here for reference only.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 2 | | | | | | |
| 3 | Variables | T (Tables) | C (Chairs) | | | |
| 4 | Units Produced | | | Profit | | |
| 5 | Objective function | 70 | 50 | | | |
| 6 | | | | | | |
| 7 | Constraints | | | LHS (Hours used) | | RHS |
| 8 | Carpentry | 4 | 3 | | < | 240 |
| 9 | Painting | | | | | |

The text in column A is combined with the text above the calculated values and above the cells with the values of the variables in some of the Solver output.

**Program 7.2A**

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# Using Solver to Solve the Flair Furniture Problem

## Formulas for the Flair Furniture Example

H16

A 1 was entered as the value of *T* and value of *C* to help find obvious errors in the formulas.

The values of the variables are in B4 and C4, and the profits for these are in cells B5 and C5. This formula will calculate B4*B5+C4*C5, or 1(70)+1(50), and return a value of 120.

| | A | | | | | |
|---|---|---|---|---|---|---|
| 1 | Flair Furniture | | | | | |
| 2 | | | | | | |
| 3 | Variables | T (Tables) | C (Chairs) | | | |
| 4 | Units Produced | 1 | 1 | Profit | | |
| 5 | Objective function | 70 | 50 | =SUMPRODUCT($B$4:$C$4,B5:C5) | | |
| 6 | | | | | | |
| 7 | Constraints | | | LHS (Hours used) | | RHS |
| 8 | Carpentry | 4 | 3 | =SUMPRODUCT($B$4:$C$4,B8:C8) | < | 240 |
| 9 | Painting | 2 | 1 | =SUMPRODUCT($B$4:$C$4,B9:C9) | < | 100 |

The formula for the LHS of each constraint can be copied from cell D5. The $ signs cause the cell addresses to remain unchanged when the cell (D5) is copied.

**Program 7.2B**

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# Using Solver to Solve the Flair Furniture Problem

## Excel Spreadsheet for the Flair Furniture Example

You can change these values to see how the profit and resource utilization change.

Because there is a 1 in each of these cells, the LHS values can be calculated very easily to see if a mistake hase been made.

| | A | | | | | |
|---|---|---|---|---|---|---|
| 1 | Flair Furniture | | | | | |
| 2 | | | | | | |
| 3 | Variables | T (Tables) | C (Chairs) | | | |
| 4 | Units Produced | 1 | 1 | | | |
| 5 | Objective function | 70 | 50 | | | |
| 6 | | | | | | |
| 7 | Constraints | | | LHS (Hours used) | | RHS |
| 8 | Carpentry | 4 | 3 | 7 | < | 240 |
| 9 | Painting | 2 | 1 | 3 | < | 100 |

The problem is ready to use the Solver add-in.

**Program 7.2C**

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## Using Solver to Solve the Flair Furniture Problem

- Once the model has been entered, the following steps can be used to solve the problem.

  In Excel 2010, select Data – Solver.

  *If Solver does not appear in the indicated place, see Appendix F for instructions on how to activate this add-in.*

1. In the Set Objective box, enter the cell address for the total profit.
2. In the By Changing Cells box, enter the cell addresses for the variable values.
3. Click *Max* for a maximization problem and *Min* for a minimization problem.

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## Using Solver to Solve the Flair Furniture Problem

4. Check the box for *Make Unconstrained Variables Non-negative*.
5. Click the *Select Solving Method* button and select *Simplex LP* from the menu that appears.
6. Click *Add* to add the constraints.
7. In the dialog box that appears, enter the cell references for the left-hand-side values, the type of equation, and the right-hand-side values.
8. Click *Solve*.

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# Using Solver to Solve the Flair Furniture Problem

## Starting Solver

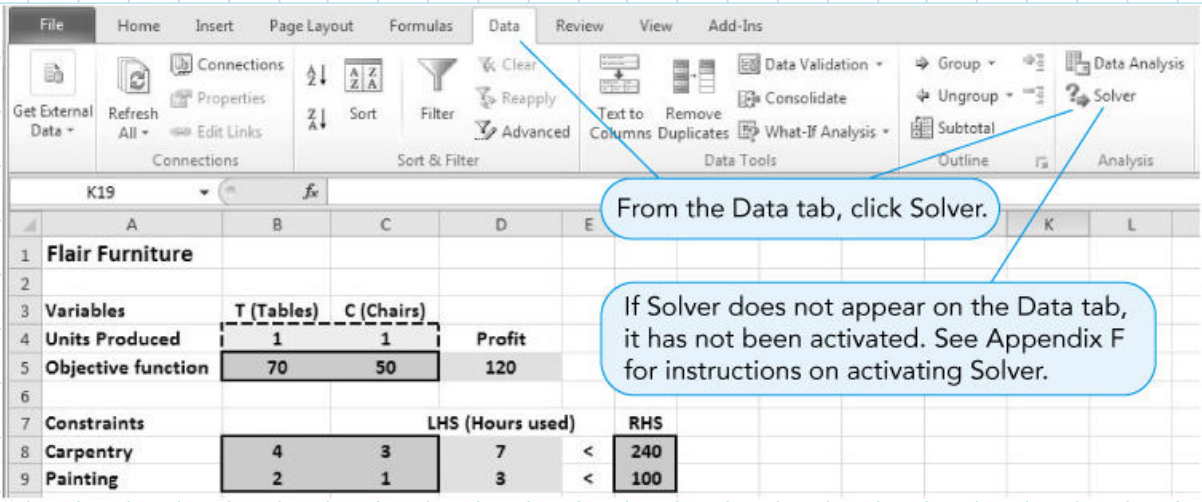


Figure 7.2D

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# Using Solver to Solve the Flair Furniture Problem

## Solver Parameters Dialog Box

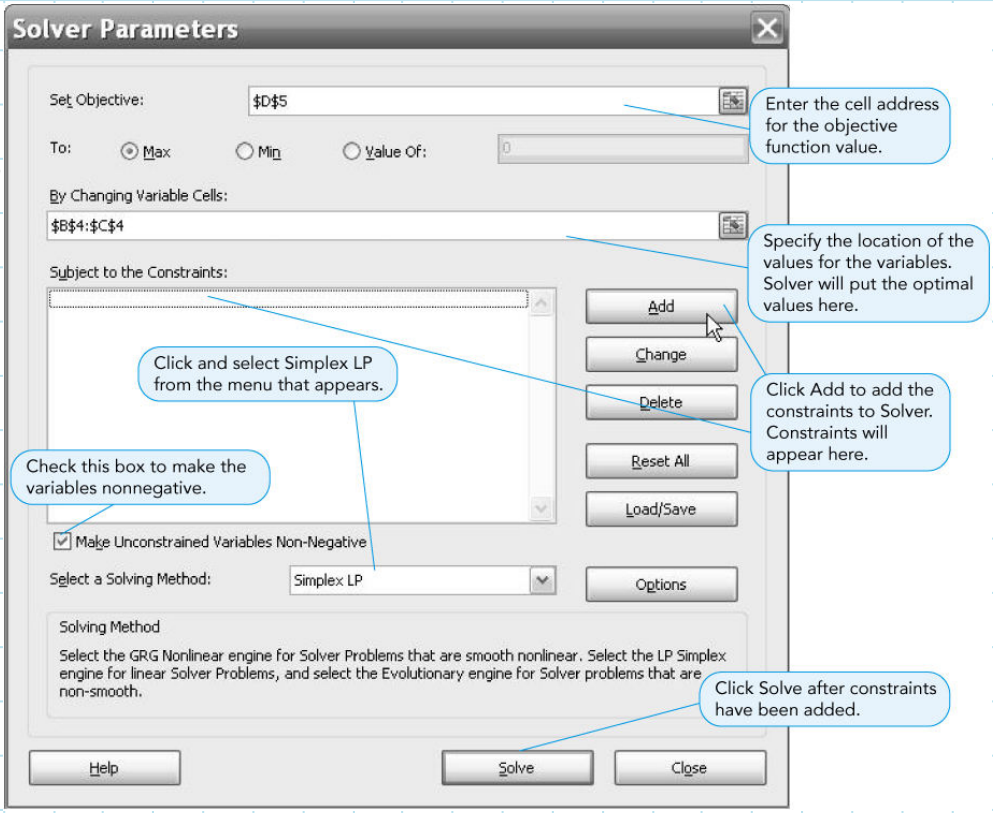


Figure 7.2E

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# Using Solver to Solve the Flair Furniture Problem

## Solver Add Constraint Dialog Box

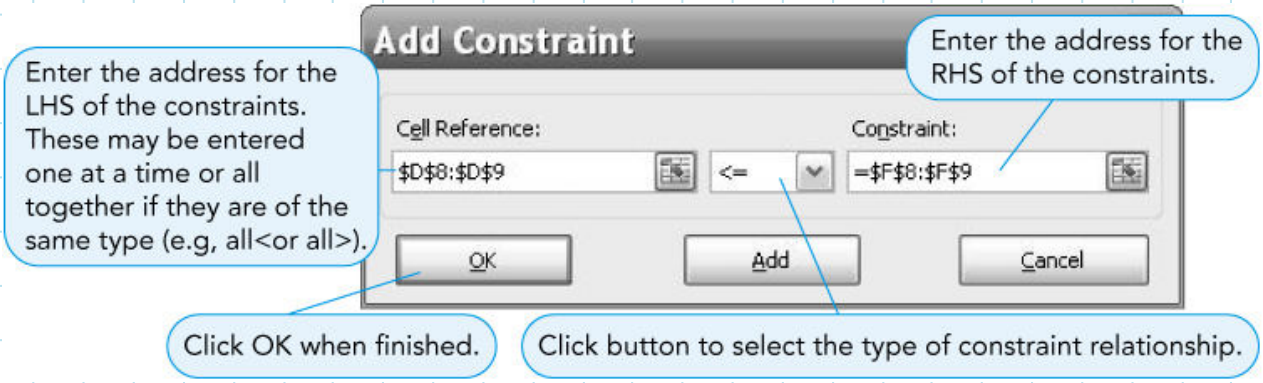


**Figure 7.2F**

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# Using Solver to Solve the Flair Furniture Problem

## Solver Results Dialog Box

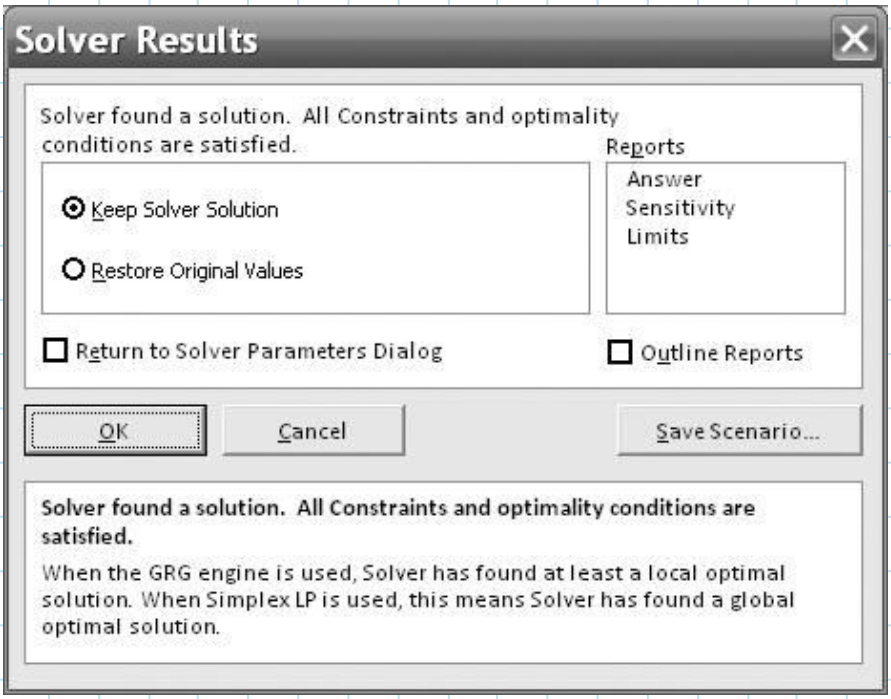


**Figure 7.2G**

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# Using Solver to Solve the Flair Furniture Problem

## Solution Found by Solver

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | Flair Furniture | | | | | |
| 2 | | | | | | |
| 3 | Variables | T (Tables) | C (Chairs) | | | |
| 4 | Units Produced | 30 | 40 | Profit | | |
| 5 | Objective function | 70 | 50 | 4100 | | |
| 6 | | | | | | |
| 7 | Constraints | | | LHS (Hours used) | | RHS |
| 8 | Carpentry | 4 | 3 | 240 | < | 240 |
| 9 | Painting | 2 | 1 | 100 | < | 100 |

The optimal solution is $T$=30, $C$=40, profit=4100.

The hours used are given here.

Figure 7.2H

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# Solving Minimization Problems

- Many LP problems involve minimizing an objective such as cost instead of maximizing a profit function.
- Minimization problems can be solved graphically by first setting up the feasible solution region and then using either the corner point method or an isocost line approach (which is analogous to the isoprofit approach in maximization problems) to find the values of the decision variables (e.g., $X_1$ and $X_2$) that yield the minimum cost.

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# Holiday Meal Turkey Ranch

**The Holiday Meal Turkey Ranch is considering buying two different brands of turkey feed and blending them to provide a good, low-cost diet for its turkeys**

**Let**

$X_1$ = number of pounds of brand 1 feed purchased

$X_2$ = number of pounds of brand 2 feed purchased

Minimize cost (in cents) = $2X_1 + 3X_2$

subject to:

$5X_1 + 10X_2 \geq 90$ ounces (ingredient constraint A)

$4X_1 + 3X_2 \geq 48$ ounces (ingredient constraint B)

$0.5X_1 \geq 1.5$ ounces (ingredient constraint C)

$X_1 \geq 0$ (nonnegativity constraint)

$X_2 \geq 0$ (nonnegativity constraint)

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# Holiday Meal Turkey Ranch

## Holiday Meal Turkey Ranch data

| INGREDIENT | COMPOSITION OF EACH POUND OF FEED (OZ.) | | MINIMUM MONTHLY REQUIREMENT PER TURKEY (OZ.) |
|---|---|---|---|
| | BRAND 1 FEED | BRAND 2 FEED | |
| A | 5 | 10 | 90 |
| B | 4 | 3 | 48 |
| C | 0.5 | 0 | 1.5 |
| Cost per pound | 2 cents | 3 cents | |

Table 7.5

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# Holiday Meal Turkey Ranch

- Use the corner point method.
- First construct the feasible solution region.
- The optimal solution will lie at one of the corners as it would in a maximization problem.

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# Feasible Region for the Holiday Meal Turkey Ranch Problem

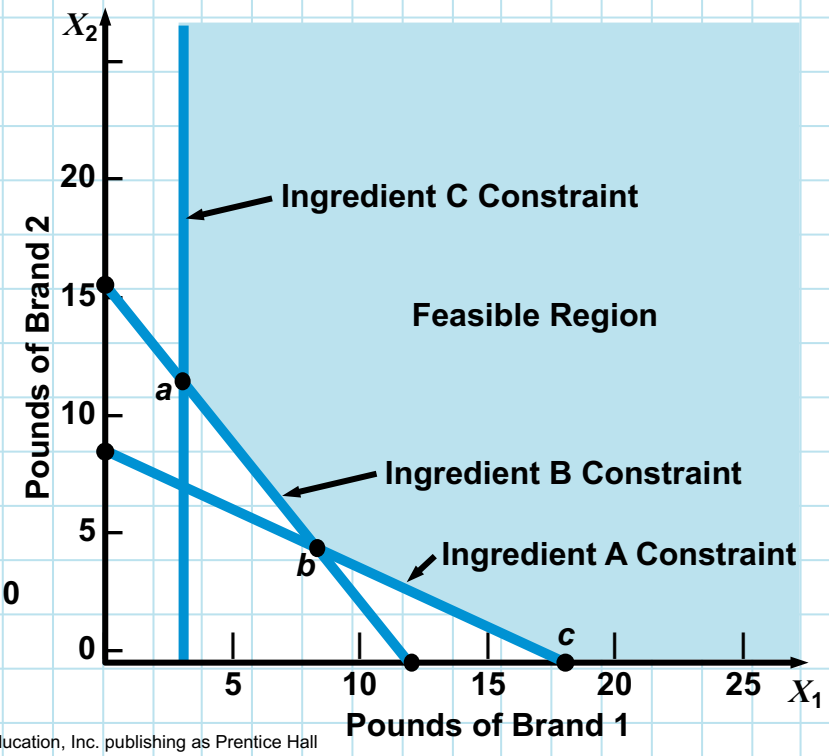


Figure 7.10

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# Holiday Meal Turkey Ranch

- Solve for the values of the three corner points.
- Point *a* is the intersection of ingredient constraints C and B.

$$4X_1 + 3X_2 = 48$$
$$X_1 = 3$$

- Substituting 3 in the first equation, we find $X_2 = 12$.
- Solving for point *b* with basic algebra we find $X_1 = 8.4$ and $X_2 = 4.8$.
- Solving for point *c* we find $X_1 = 18$ and $X_2 = 0$.

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# Holiday Meal Turkey Ranch

Substituting these value back into the objective function we find

$$\text{Cost} = 2X_1 + 3X_2$$
$$\text{Cost at point } a = 2(3) + 3(12) = 42$$
$$\boxed{\text{Cost at point } b = 2(8.4) + 3(4.8) = 31.2}$$
$$\text{Cost at point } c = 2(18) + 3(0) = 36$$

The lowest cost solution is to purchase 8.4 pounds of brand 1 feed and 4.8 pounds of brand 2 feed for a total cost of 31.2 cents per turkey.

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# Holiday Meal Turkey Ranch

## Graphical Solution to the Holiday Meal Turkey Ranch Problem Using the Isocost Approach

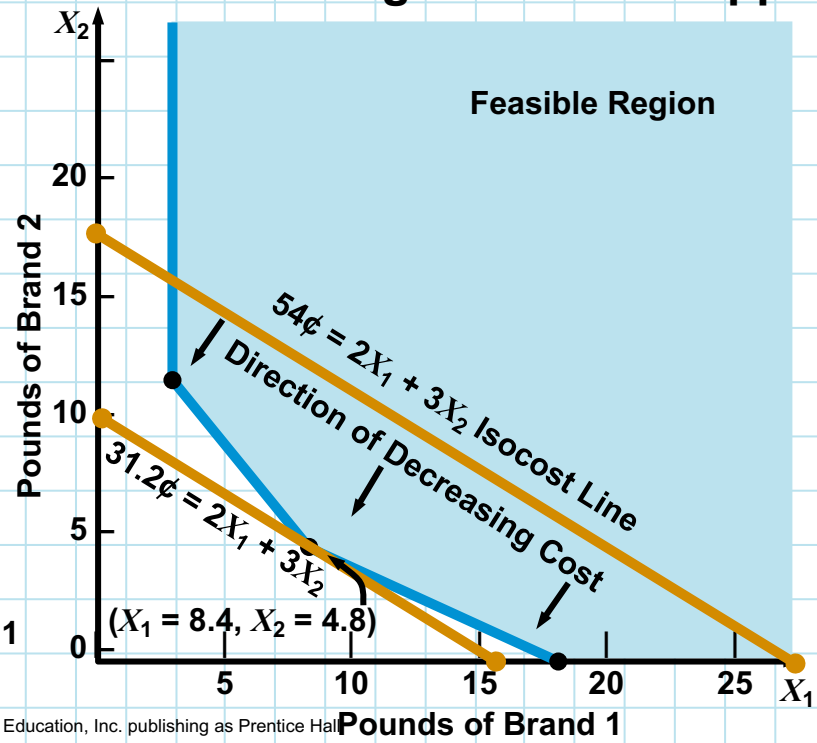


Figure 7.11

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# Holiday Meal Turkey Ranch

## Solving the Holiday Meal Turkey Ranch Problem Using QM for Windows

QM for Windows - C:\Prentice\Data\RenderStair7\HolidayMeal.LIN

Objective
- ○ Maximize
- ◉ Minimize

Linear Programming Results

Holiday Meal Turkey Ranch Solution

| | Brand 1 | Brand 2 | | RHS | Dual |
|---|---|---|---|---|---|
| Minimize | 2. | 3. | | | |
| Ingredient A | 5. | 10. | >= | 90. | -0.24 |
| Ingredient B | 4. | 3. | >= | 48. | -0.2 |
| Ingredient C | 0.5 | 0. | >= | 1.5 | 0. |
| Solution-> | 8.4 | 4.8 | | 31.2 | |

Program 7.3

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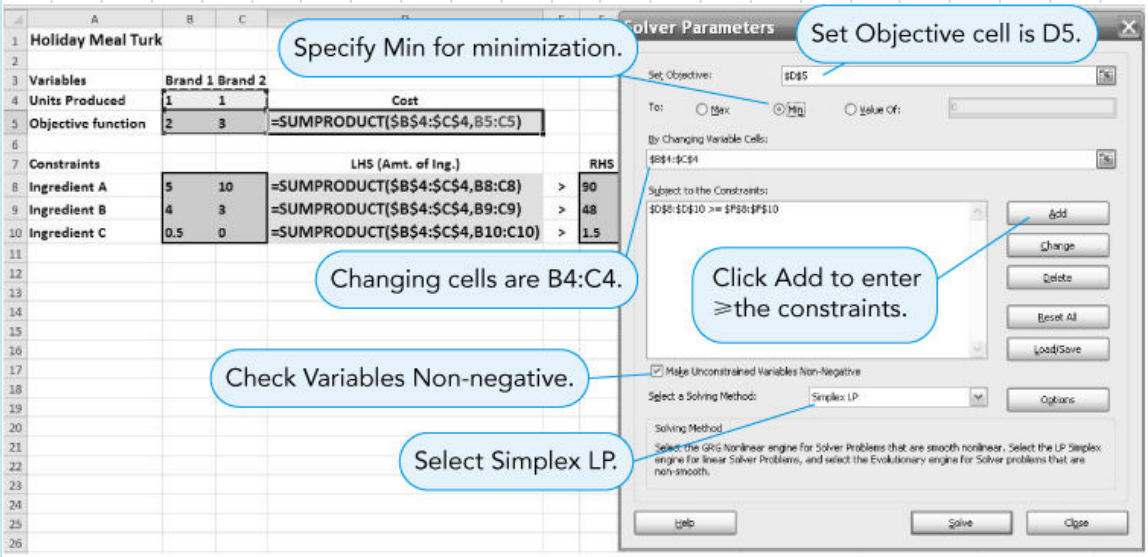
# Holiday Meal Turkey Ranch

## Excel 2010 Spreadsheet for the Holiday Meal Turkey Ranch problem




**Program 7.4A**

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# Holiday Meal Turkey Ranch

## Excel 2010 Solution to the Holiday Meal Turkey Ranch Problem

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | Holiday Meal Turkey Ranch | | | | | |
| 2 | | | | | | |
| 3 | Variables | Brand 1 | Brand 2 | | | |
| 4 | Units Produced | 8.4 | 4.8 | Cost | | |
| 5 | Objective function | 2 | 3 | 31.2 | | |
| 6 | | | | | | |
| 7 | Constraints | | | LHS (Amt. of Ing.) | | RHS |
| 8 | Ingredient A | 5 | 10 | 90 | > | 90 |
| 9 | Ingredient B | 4 | 3 | 48 | > | 48 |
| 10 | Ingredient C | 0.5 | 0 | 4.2 | > | 1.5 |

Notice that there is a surplus for ingredient C as LHS>RHS.

**Program 7.4B**

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# Four Special Cases in LP

- Four special cases and difficulties arise at times when using the graphical approach to solving LP problems.
  - No feasible solution
  - Unboundedness
  - Redundancy
  - Alternate Optimal Solutions

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# Four Special Cases in LP

## No feasible solution

- This exists when there is no solution to the problem that satisfies all the constraint equations.
- No feasible solution region exists.
- This is a common occurrence in the real world.
- Generally one or more constraints are relaxed until a solution is found.

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# Four Special Cases in LP

A problem with no feasible solution

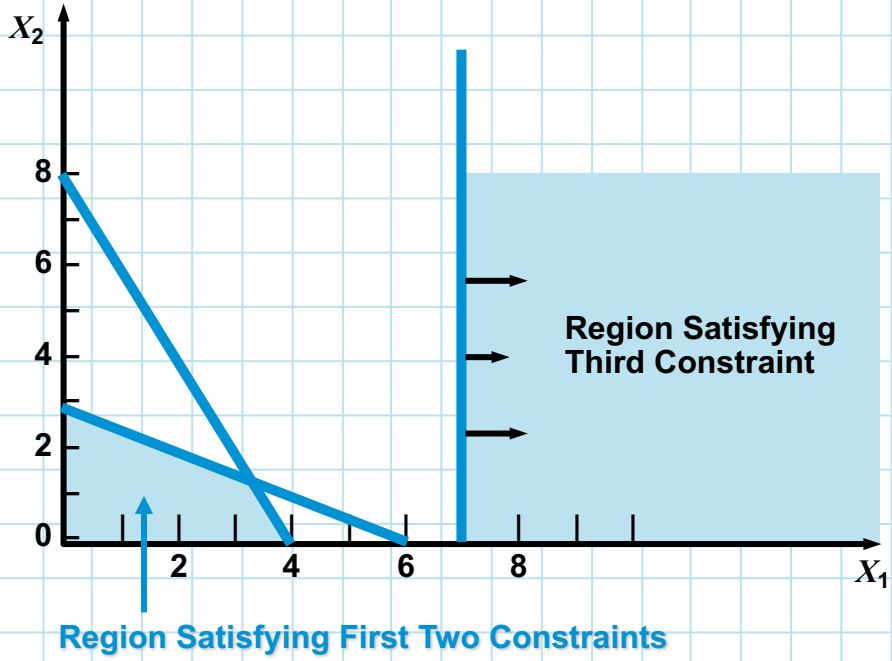


Figure 7.12

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# Four Special Cases in LP

## Unboundedness

- Sometimes a linear program will not have a finite solution.
- In a maximization problem, one or more solution variables, and the profit, can be made infinitely large without violating any constraints.
- In a graphical solution, the feasible region will be open ended.
- This usually means the problem has been formulated improperly.

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# Four Special Cases in LP

## A Feasible Region That is Unbounded to the Right

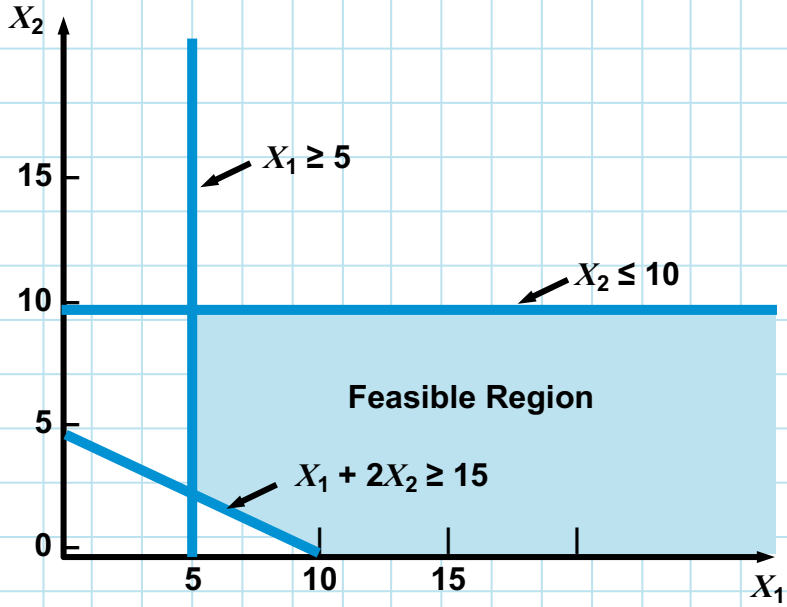


Figure 7.13

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# Four Special Cases in LP

## Redundancy

- A redundant constraint is one that does not affect the feasible solution region.
- One or more constraints may be binding.
- This is a very common occurrence in the real world.
- It causes no particular problems, but eliminating redundant constraints simplifies the model.

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# Four Special Cases in LP

## Problem with a Redundant Constraint

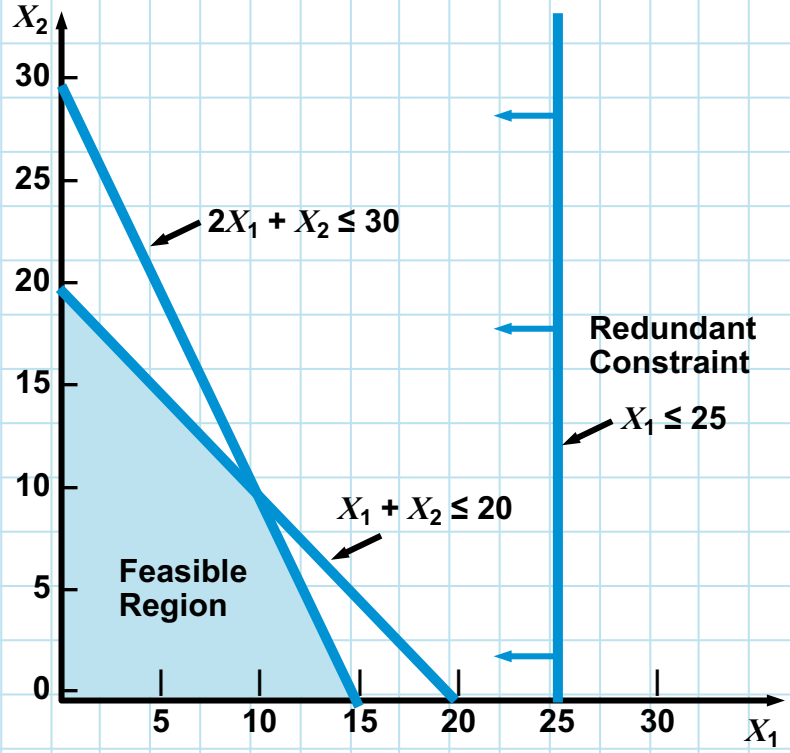


Figure 7.14

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# Four Special Cases in LP

## Alternate Optimal Solutions

- Occasionally two or more optimal solutions may exist.
- Graphically this occurs when the objective function's isoprofit or isocost line runs perfectly parallel to one of the constraints.
- This actually allows management great flexibility in deciding which combination to select as the profit is the same at each alternate solution.

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# Four Special Cases in LP

## Example of Alternate Optimal Solutions

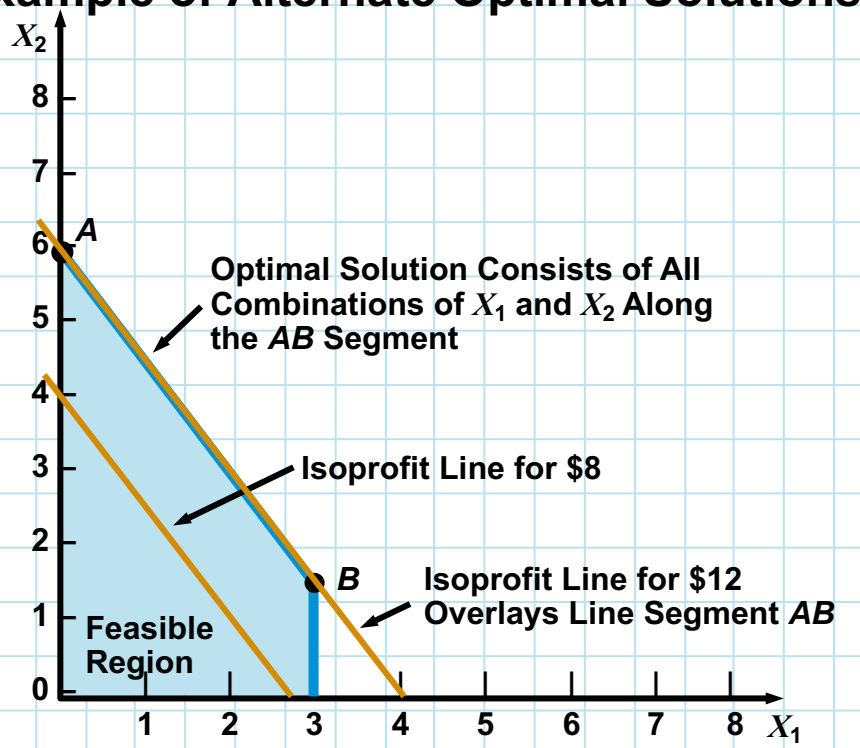


Figure 7.15

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# Sensitivity Analysis

- Optimal solutions to LP problems thus far have been found under what are called *deterministic assumptions.*
- This means that we assume complete certainty in the data and relationships of a problem.
- But in the real world, conditions are dynamic and changing.
- We can analyze how *sensitive* a deterministic solution is to changes in the assumptions of the model.
- This is called *sensitivity analysis*, *postoptimality analysis*, *parametric programming*, or *optimality analysis.*

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# Sensitivity Analysis

- Sensitivity analysis often involves a series of what-if? questions concerning constraints, variable coefficients, and the objective function.
- One way to do this is the trial-and-error method where values are changed and the entire model is resolved.
- The preferred way is to use an analytic postoptimality analysis.
- After a problem has been solved, we determine a range of changes in problem parameters that will not affect the optimal solution or change the variables in the solution.

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# High Note Sound Company

- The High Note Sound Company manufactures quality CD players and stereo receivers.
- Products require a certain amount of skilled artisanship which is in limited supply.
- The firm has formulated the following product mix LP model.

Maximize profit = $\$50X_1 + \$120X_2$

Subject to

$2X_1 + 4X_2 \leq 80$ (hours of electrician's time available)

$3X_1 + 1X_2 \leq 60$ (hours of audio technician's time available)

$X_1, X_2 \geq 0$

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# High Note Sound Company

## The High Note Sound Company Graphical Solution

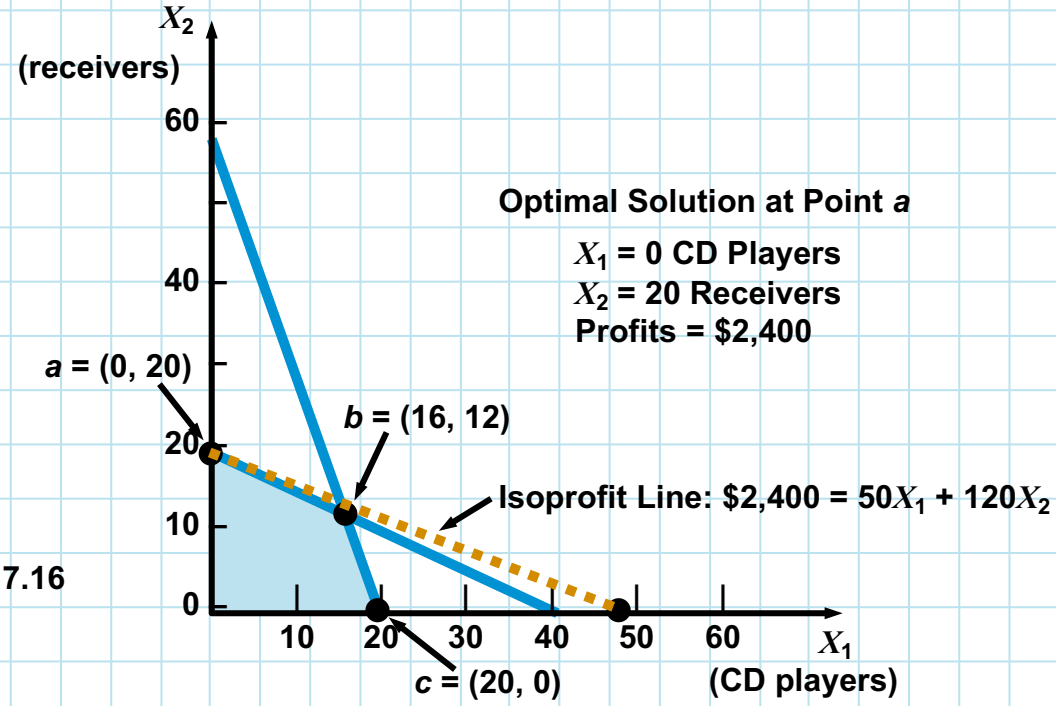


Figure 7.16

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# Changes in the Objective Function Coefficient

- In real-life problems, contribution rates in the objective functions fluctuate periodically.
- Graphically, this means that although the feasible solution region remains exactly the same, the slope of the isoprofit or isocost line will change.
- We can often make modest increases or decreases in the objective function coefficient of any variable without changing the current optimal corner point.
- We need to know how much an objective function coefficient can change before the optimal solution would be at a different corner point.

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# Changes in the Objective Function Coefficient

## Changes in the Receiver Contribution Coefficients

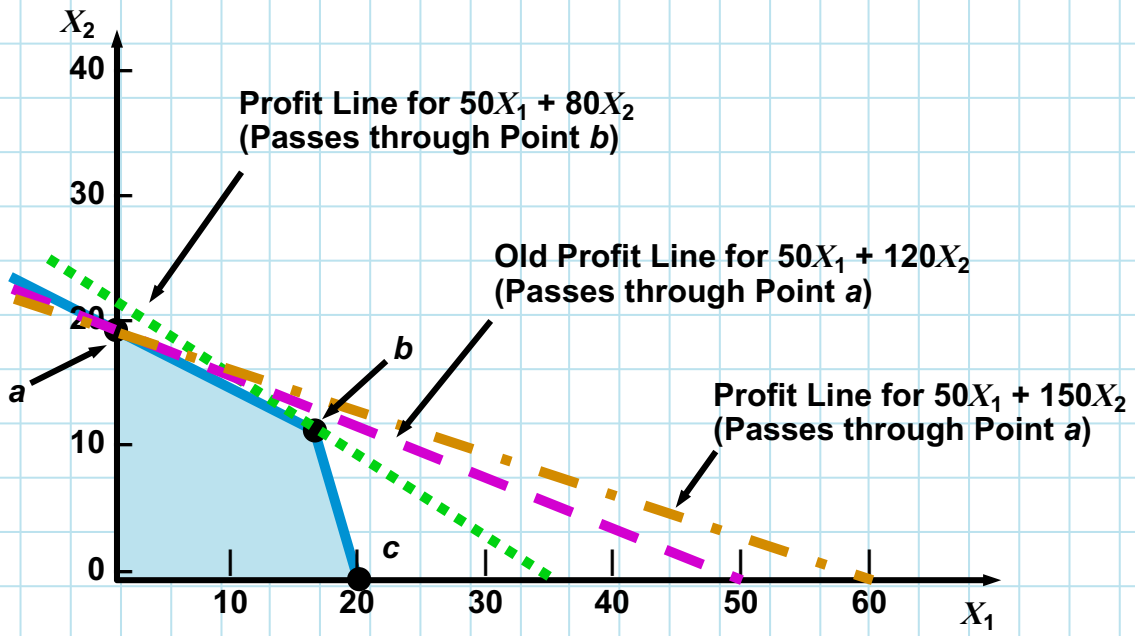


Figure 7.17

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# QM for Windows and Changes in Objective Function Coefficients

## Input and Sensitivity Analysis for High Note Sound Data Using QM For Windows

Objective
- (●) Maximize
- ( ) Minimize

High Note Sound

| | CD players | Receivers | | RHS |
|---|---|---|---|---|
| Maximize | 50 | 120 | | |
| Electrician hrs | 2 | 4 | <= | 80 |
| Audio tech hrs | 3 | 1 | <= | 60 |

Program 7.5A

Ranging

High Note Sound Solution

| Variable | Value | Reduced Cost | Original Val | Lower Bound | Upper Bound |
|---|---|---|---|---|---|
| CD players | 0. | 10. | 50. | -Infinity | 60. |
| Receivers | 20. | 0. | 120. | 100. | Infinity |
| Constraint | Dual Value | Slack/Surplus | Original Val | Lower Bound | Upper Bound |
| Electrician hrs | 30. | 0. | 80. | 0. | 240. |
| Audio tech hrs | 0. | 40. | 60. | 20. | Infinity |

Program 7.5B

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# Excel Solver and Changes in Objective Function Coefficients

## Excel 2010 Spreadsheet for High Note Sound Company

The Changing Variable cells in the Solver dialog box are B4:C4.

The Set Objective cell in the Solver dialog box is D5.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | High Note Sound C | | | | | |
| 2 | | | | | | |
| 3 | Variables | CD Player | Receivers | | | |
| 4 | Units Produced | 1 | 1 | Profit | | |
| 5 | Objective function | 50 | 120 | =SUMPRODUCT($B$4:$C$4,B5:C5) | | |
| 6 | | | | | | |
| 7 | Constraints | | | LHS (Hrs. Used) | | RHS |
| 8 | Electrician Hours | 2 | 4 | =SUMPRODUCT($B$4:$C$4,B8:C8) | < | 80 |
| 9 | Audio Tech Hours | 3 | 1 | =SUMPRODUCT($B$4:$C$4,B9:C9) | < | 60 |

The constraints added into Solver will be D8:D9 <=F8:F9.

**Program 7.6A**

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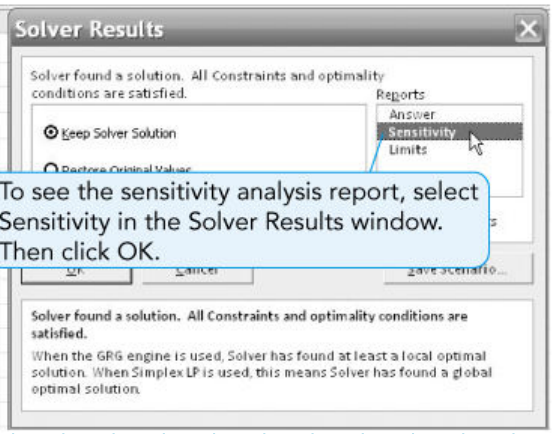
# Excel Solver and Changes in Objective Function Coefficients

## Excel 2010 Solution and Solver Results Window for High Note Sound Company

The solution found by Solver is here.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | High Note Sound Company | | | | | |
| 2 | | | | | | |
| 3 | Variables | CD Player | Receivers | | | |
| 4 | Units Produced | 0 | 20 | Profit | | |
| 5 | Objective function | 50 | 120 | 2400 | | |
| 6 | | | | | | |
| 7 | Constraints | | | LHS (Hrs. Used) | | RH |
| 8 | Electrician Hours | 2 | 4 | 80 | < | 8 |
| 9 | Audio Tech Hours | 3 | 1 | 20 | < | 60 |
| 10 | | | | | | |
| 11 | | | | | | |
| 12 | | | | | | |
| 13 | | | | | | |
| 14 | | | | | | |
| 15 | | | | | | |




To see the sensitivity analysis report, select Sensitivity in the Solver Results window. Then click OK.

**Figure 7.6B**

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# Excel Solver and Changes in Objective Function Coefficients

## Excel 2010 Sensitivity Report for High Note Sound Company

The names presented in the Sensitivity Report combine the text in column A and the text above the data, unless the cells have been named using the Name Manager from the Formulas tab.

Microsoft Excel 14.0 Sensitivity Repo

The profit on CD may change by these amounts, and the current corner point will remain optimal.

**Variable Cells**

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$4 | Units Produced CD Player | 0 | -10 | 50 | 10 | 1E+30 |
| $C$4 | Units Produced Receivers | 20 | 0 | 120 | 1E+30 | 20 |

**Constraints**

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $D$8 | Electrician Hours LHS (Hrs. Used) | 80 | 30 | 80 | 160 | 80 |
| $D$9 | Audio Tech Hours LHS (Hrs. Used | 20 | 0 | 60 | 1E+30 | 40 |

The resources used are here. The RHS can change by these amounts, and the shadow price will still be relevant.

**Program 7.6C**

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# Changes in the Technological Coefficients

- Changes in the *technological coefficients* often reflect changes in the state of technology.
- If the amount of resources needed to produce a product changes, coefficients in the constraint equations will change.
- This does not change the objective function, but it can produce a significant change in the shape of the feasible region.
- This may cause a change in the optimal solution.

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# Changes in the Technological Coefficients

## Change in the Technological Coefficients for the High Note Sound Company

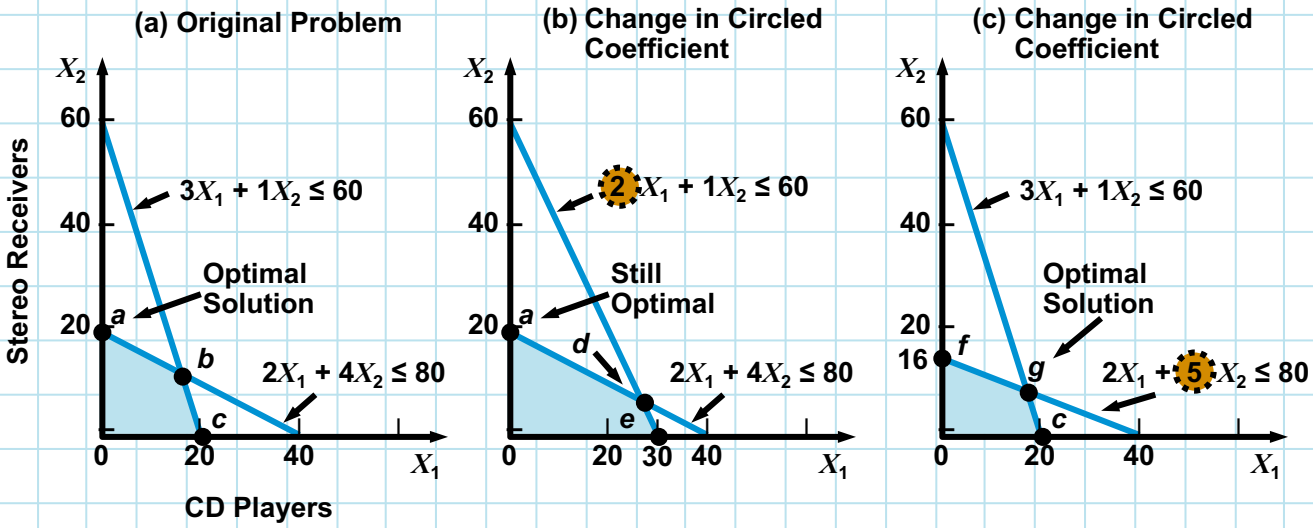


Figure 7.18

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# Changes in Resources or Right-Hand-Side Values

- The right-hand-side values of the constraints often represent resources available to the firm.
- If additional resources were available, a higher total profit could be realized.
- Sensitivity analysis about resources will help answer questions about how much should be paid for additional resources and how much more of a resource would be useful.

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## *Changes in Resources or Right-Hand-Side Values*

- **If the right-hand side of a constraint is changed, the feasible region will change (unless the constraint is redundant).**
- **Often the optimal solution will change.**
- **The amount of change in the objective function value that results from a unit change in one of the resources available is called the *dual price* or *dual value* .**
- **The dual price for a constraint is the improvement in the objective function value that results from a one-unit increase in the right-hand side of the constraint.**

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## *Changes in Resources or Right-Hand-Side Values*

- **However, the amount of possible increase in the right-hand side of a resource is limited.**
- **If the number of hours increased beyond the upper bound, then the objective function would no longer increase by the dual price.**
- **There would simply be excess (*slack*) hours of a resource or the objective function may change by an amount different from the dual price.**
- **The dual price is relevant only within limits.**

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# Changes in the Electricians' Time Resource for the High Note Sound Company

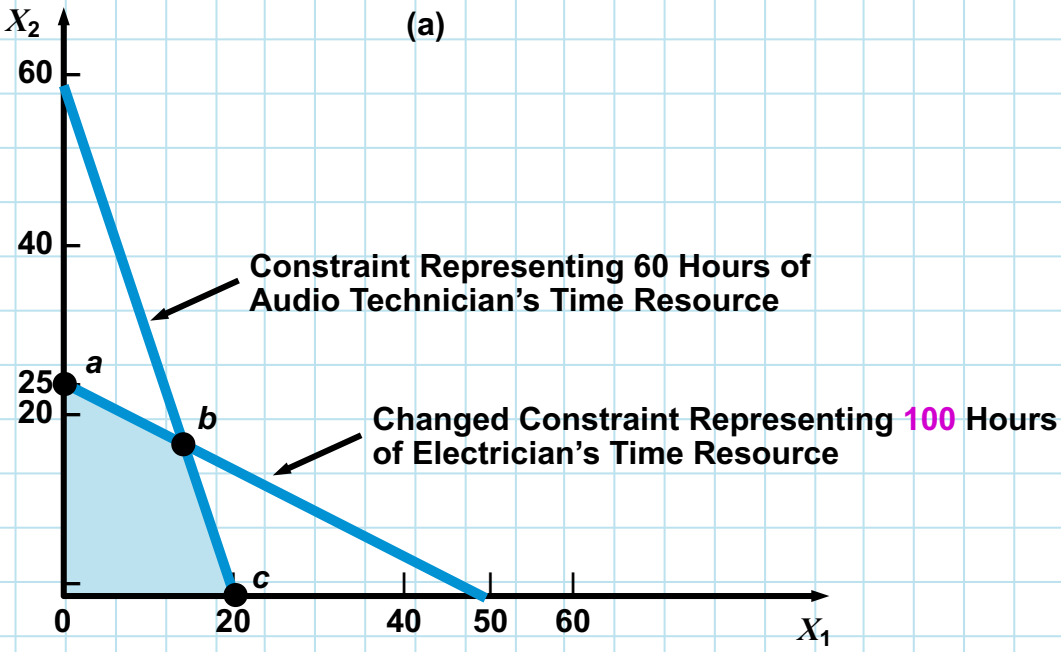


Figure 7.19

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# Changes in the Electricians' Time Resource for the High Note Sound Company

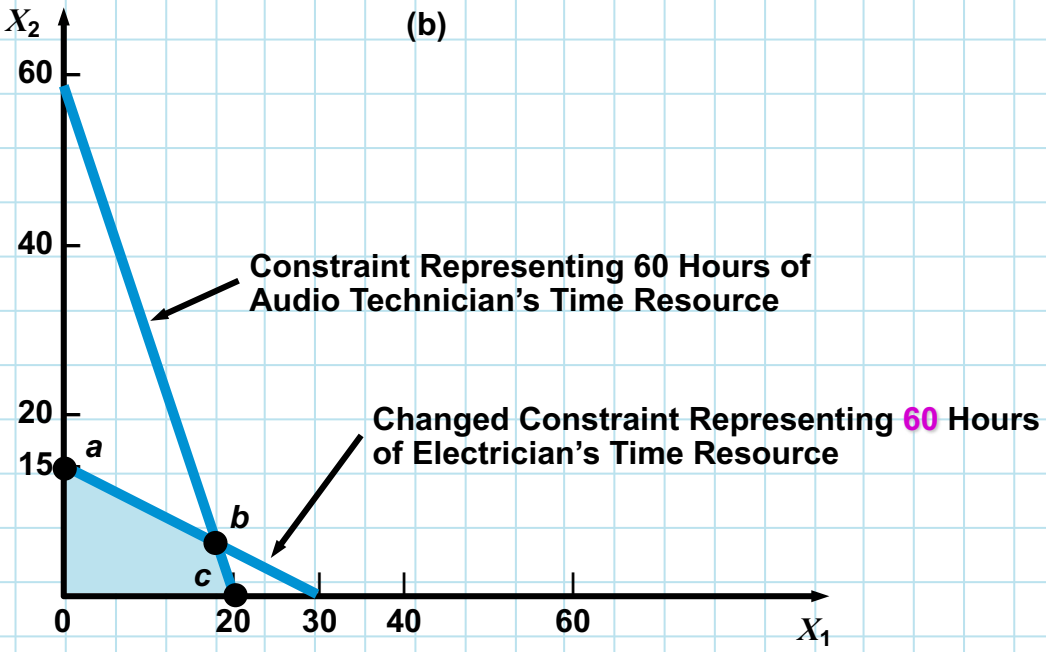


Figure 7.19

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# Changes in the Electricians' Time Resource for the High Note Sound Company

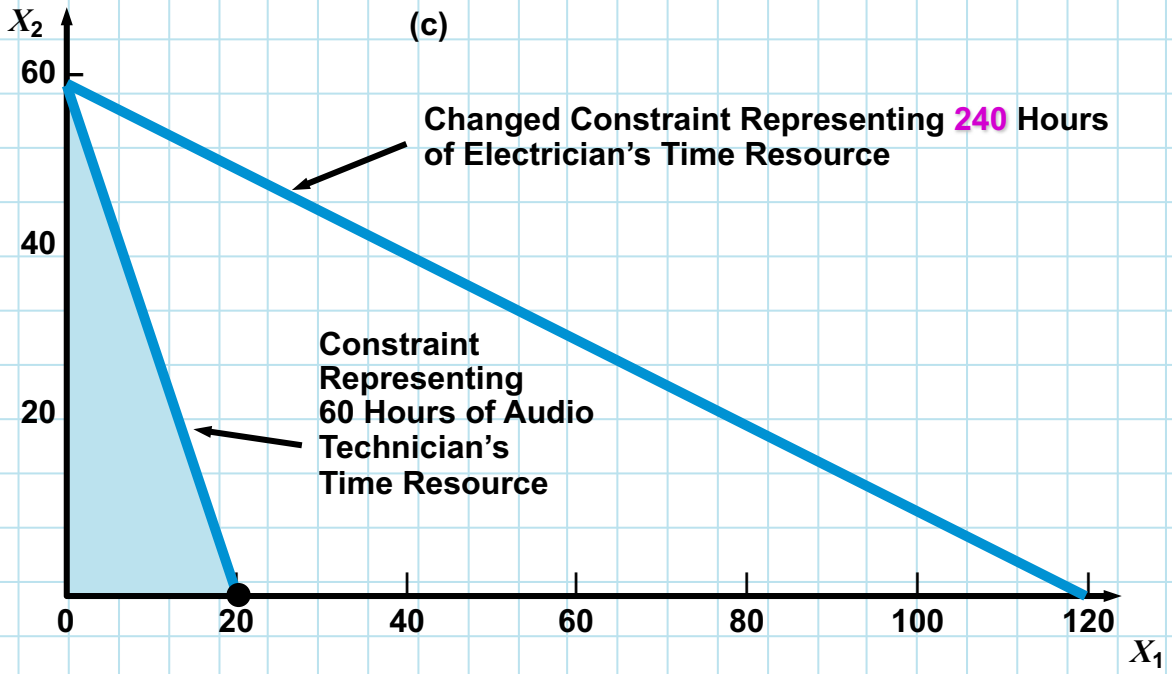


Figure 7.19

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# QM for Windows and Changes in Right-Hand-Side Values

## Sensitivity Analysis for High Note Sound Company Using QM for Windows

Ranging

| High Note Sound Solution | | | | | |
|---|---|---|---|---|---|
| Variable | Value | Reduced Cost | Original Val | Lower Bound | Upper Bound |
| CD players | 0. | 10. | 50. | -Infinity | 60. |
| Receivers | 20. | 0. | 120. | 100. | Infinity |
| Constraint | Dual Value | Slack/Surplus | Original Val | Lower Bound | Upper Bound |
| Electrician hrs | 30. | 0. | 80. | 0. | 240. |
| Audio tech hrs | 0. | 40. | 60. | 20. | Infinity |

Program 7.5B

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# Excel Solver and Changes in Right-Hand-Side Values

## Excel 2010 Sensitivity Analysis for High Note Sound Company

Microsoft Excel 8.0 Sensitivity Report
Worksheet: [captures.xls]7.8
Report Created: 8/13/98 10:54:59 AM

The solution values for the variables indicate that we should make 0 CD players and 20 receivers.

Adjustable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$4 | Value CD Players | 0 | -10 | 50 | 10 | 1E+30 |
| $C$4 | Value Receivers | 20 | 0 | 120 | 1E+30 | 20 |

If we produce a CD player, our profit will fall by $10.

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $D$9 | Electrician hours Used | 80 | 30 | 80 | 160 | 80 |
| $D$10 | Audio technician hours Used | 20 | 0 | 60 | 1E+30 | 40 |

We will use 80 hours and 20 hours of electrician and audio technician time, respectively.

If we use 1 more electrician hour, our profit will increase by $30. This is true for up to 160 more hours. The profit will fall by $30 for each electrician hour less than 80 hours.

**Program 7.6C**

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