Fundamentals of Corporate Finance, 2/e

ROBERT PARRINO, PH.D.
DAVID S. KIDWELL, PH.D.
THOMAS W. BATES, PH.D.

Chapter 6: Discounted Cash Flows and Valuation

Learning Objectives

1. EXPLAIN WHY CASH FLOWS OCCURRING AT DIFFERENT TIMES MUST BE ADJUSTED TO REFLECT THEIR VALUE AS OF A COMMON DATE BEFORE THEY CAN BE COMPARED, AND COMPUTE THE PRESENT VALUE AND FUTURE VALUE FOR MULTIPLE CASH FLOWS.
2. DESCRIBE HOW TO CALCULATE THE PRESENT VALUE AND THE FUTURE VALUE OF AN ORDINARY ANNUITY AND HOW AN ORDINARY ANNUITY DIFFERS FROM AN ANNUITY DUE.

Learning Objectives

3. EXPLAIN WHAT A PERPETUITY IS AND WHERE WE SEE THEM IN BUSINESS AND CALCULATE THE VALUE OF A PERPETUITY.
4. DISCUSS GROWING ANNUITIES AND PERPETUITIES, AS WELL AS THEIR APPLICATION IN BUSINESS, AND CALCULATE THEIR VALUES.
5. DISCUSS WHY THE EFFECTIVE ANNUAL INTEREST RATE (EAR) IS THE APPROPRIATE WAY TO ANNUALIZE INTERESTS RATES, AND CALCULATE THE EAR.

Multiple Cash Flows

- FUTURE VALUE OF MULTIPLE CASH FLOWS
  1. **Draw a timeline to determine the number of periods for which each cash flow will earn the rate-of-return**
  2. **Calculate the future value of each cash flow using Equation 5.1**
  3. **Add the future values**

Future Value of Two Cash Flows

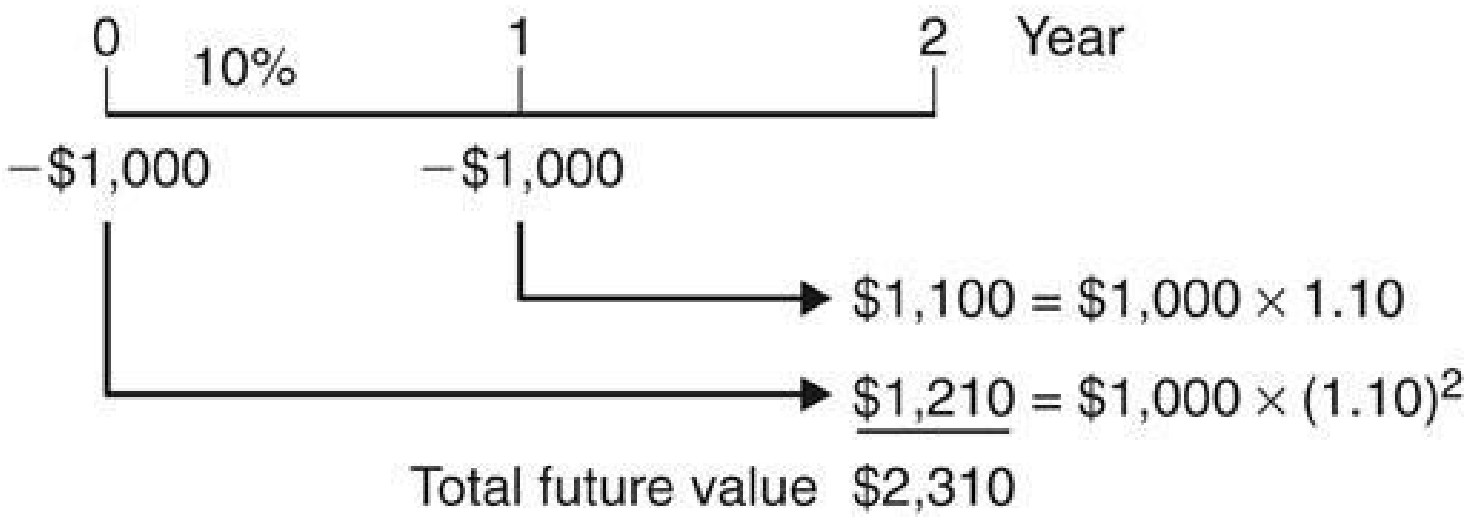


Exhibit 6.1 Future Value of Two Cash Flows
This exhibit shows a timeline for two cash flows invested in a savings account that pays 10 percent interest annually. The total amount in the savings account after two years is $2,310, which is the sum of the future values of the two cash flows.

Future Value of Three Cash Flows

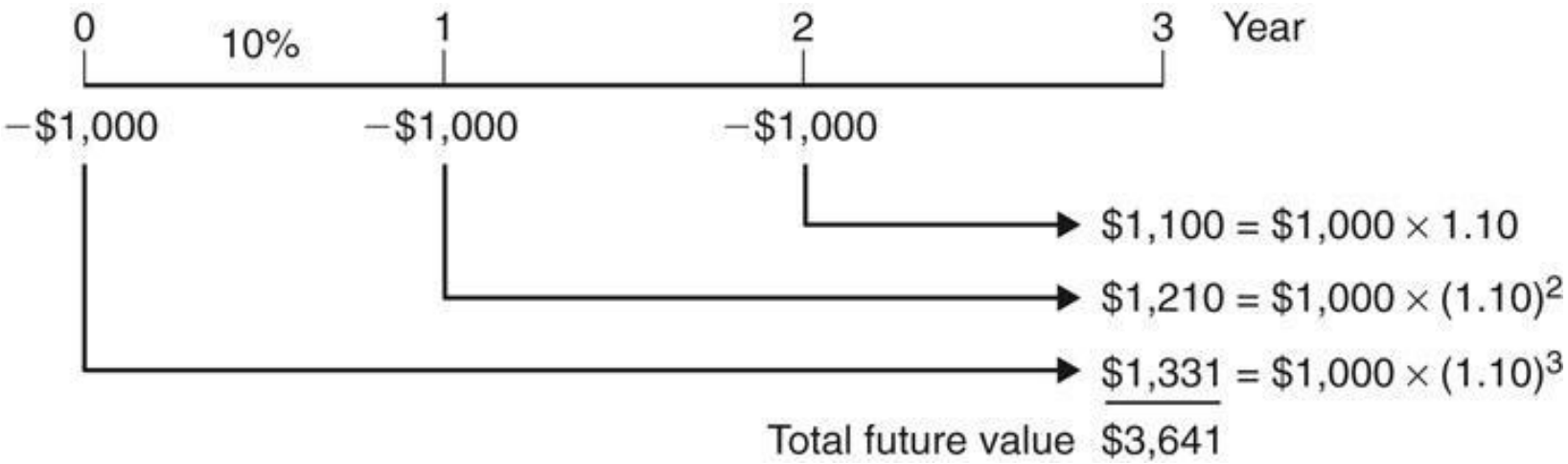


Exhibit 6.2 Future Value of Three Cash Flows
The exhibit shows a timeline for an investment program with a three-year horizon. The value of the investment at the end of three years is $3,641, the sum of the future values of the three separate cash flows.

Present Value of Three Cash Flows

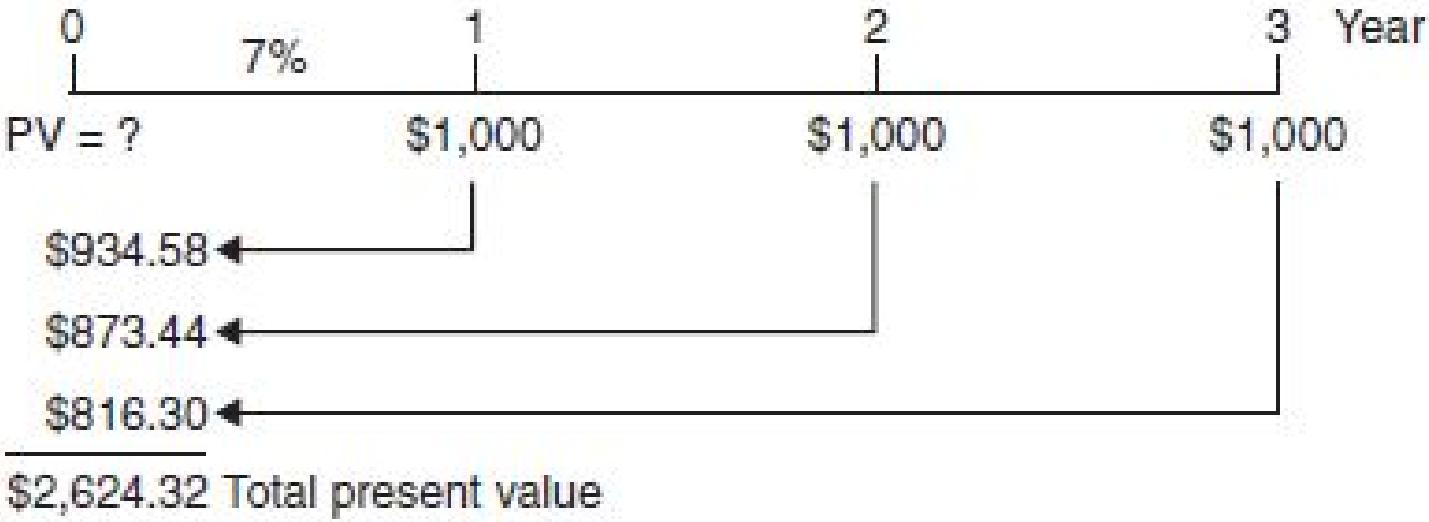


EXHIBIT 6.3

Present Value of Three Cash Flows

The exhibit shows the time line for a three-year loan with a payment of $1,000 at the end of each year and an annual interest rate of 7 percent. To calculate the value of the loan today, we compute the present value of each of the three cash flows and then add them up. The present value of the loan is $2,624.32.

Level Cash Flows: Annuities and Perpetuities

- ANNUITY
  - **A series of equally-spaced and level cash flows extending over a finite number of periods**
- PERPETUITY
  - **A series of equally-spaced and level cash flows that continue forever**

Level Cash Flows: Annuities and Perpetuities

- ORDINARY ANNUITY
  - **cash flows occur at the end of a period**

    mortgage payment

    interest payment to bondholder

    Exhibits 6.1, 6.2, and 6.3
- ANNUITY DUE
  - **cash flows occur at the beginning of a period**

    lease

    Exhibit 6.7

Level Cash Flows: Annuities and Perpetuities

- CALCULATE PRESENT VALUE OF AN ANNUITY

$$PVA_0 = CF \times PVFA$$

$$= CF \times \left[\frac{1 - PVFA}{i} \right]$$

$$= CF \times \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right] \quad (6.1)$$

Level Cash Flows: Annuities and Perpetuities

- CALCULATE PRESENT VALUE OF AN ANNUITY
  - To calculate a future value or a present value is to calculate an equivalent amount
  - The amount reflects an adjustment to account for the effect of compounding

Level Cash Flows: Annuities and Perpetuities

- CALCULATE PRESENT VALUE OF AN ANNUITY

  Present value of an annuity

  amount needed produce the annuity

  current fair value or market price of the annuity

  amount of a loan that can be repaid with the annuity

Level Cash Flows: Annuities and Perpetuities

- PRESENT VALUE OF AN ANNUITY EXAMPLE
  - **A contract will pay $2,000 at the end of each year for three years and the appropriate discount rate is 8%. What is a fair price for the contract?**

$$PVA^{3} = \$2000 \times \left[\frac{(1 - 1/(1+0.08)^{3}}{0.08} \right] = \$5{,}154.19$$

Level Cash Flows: Annuities and Perpetuities

- CALCULATOR EXAMPLE
  - **Present Value of Annuity**

Enter	3	8		2,000	0
	N	i	PV	PMT	FV
Answer			-5,154.19		

Level Cash Flows: Annuities and Perpetuities

- CALCULATE AN ANNUITY EXAMPLE
  - **You borrow $400,000 to buy a home. The 30-year mortgage requires 360 monthly payments at a monthly rate of 6.15%/12 or .51042%. How much is the monthly payment?**

$$\$400{,}000 = \text{Pmt} \times \left[\frac{(1 - 1/(1 + 0.0051042)^{360}}{0.0051042} \right]$$

$$\$400{,}000 / 164.5784 = \text{Pmt}$$

$$\$2{,}430.45 = \text{Pmt}$$

Present Value Annuity Factors

EXHIBIT 6.4 **Present Value Annuity Factors**

The table of present value annuity factors shows the present value of $1 to be received each year for different numbers of years and for different interest rates. To locate the desired PV annuity factor, find the row for the appropriate number of years and the column for the appropriate interest rate.

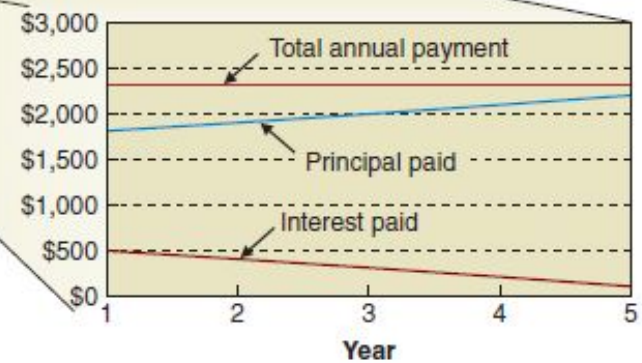
Number of Years	Interest Rate per Year						
	1%	5%	6%	7%	8%	9%	10%
1	$0.990	$0.952	$0.943	$0.935	$0.926	$0.917	$0.909
2	1.970	1.859	1.833	1.808	1.783	1.759	1.736
3	2.941	2.723	2.673	2.624	2.577	2.531	2.487
4	3.902	3.546	3.465	3.387	3.312	3.240	3.170
5	4.853	4.329	4.212	4.100	3.993	3.890	3.791
10	9.471	7.722	7.360	7.024	6.710	6.418	6.145
20	18.046	12.462	11.470	10.594	9.818	9.129	8.514
30	25.808	15.372	13.765	12.409	11.258	10.274	9.427

Level Cash Flows: Annuities and Perpetuities

- LOAN AMORTIZATION
  - How borrowed funds are repaid over the life of a loan
  - Each payment includes less interest and more principal; the loan is paid off with the last payment
  - Amortization schedule shows interest and principal in each payment, and amount of principal still owed after each payment

Amortization Table for a 5-Yr, $10,000 Loan at 5% Interest

Year	(1) Beginning Principal Balance	(2) Total Annual Payment[a]	(3) Interest Paid[b]	(4) Principal Paid (2)–(3)	(5) Ending Principal Balance (1)–(4)
1	$10,000.00	$2,309.75	$500.00	$1,809.75	$8,190.25
2	8,190.25	2,309.75	409.51	1,900.24	6,290.02
3	6,290.02	2,309.75	314.50	1,995.25	4,294.77
4	4,294.77	2,309.75	214.74	2,095.01	2,199.76
5	2,199.76	2,309.75	109.99	2,199.76	0.00



[a]The total annual payment is calculated using the formula for the present value of an annuity, Equation 6.1. The total annual payment is CF in $PVA_n = CF \times PV$ annuity factor.

[b]Interest paid equals the beginning balance times the interest rate.

EXHIBIT 6.5

Amortization Table for a Five-Year, $10,000 Loan at 5 Percent Interest

A loan amortization table shows how regular payments of principal and interest are applied to repay a loan. The exhibit is an amortization table for a five-year, $10,000 loan with an interest rate of 5 percent and annual payments of $2,309.75. Notice that the interest paid declines with each payment, while the principal paid increases. These relations are illustrated in the pullout graphic in the exhibit.

Using Excel – Loan Amortization Table

USING EXCEL

LOAN AMORTIZATION TABLE

Loan amortization tables are most easily constructed using a spreadsheet program. Here, we have reconstructed the loan amortization table shown in Exhibit 6.5 using Excel.

Notice that all the values in the amortization table are obtained by using formulas. Once you have built an amortization table like this one, you can change any of the input variables, such as the loan amount or the interest rate, and all of the other numbers will automatically be updated.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2								**Loan Amortization Table**					
3													
4		Loan amount		$10,000									
5		Interest rate		0.05									
6		Loan period		5									
7		**PMT**		**$2,309.75**									
8													
9		**Year**		**Beginning Balance**		**Total Annual Payment**		**Simple Interest Paid**		**Principal Paid**		**Ending Balance**	
10		1		$10,000.00		$2,309.75		$500.00		$1,809.75		$8,190.25	
11		2		8,190.25		2,309.75		409.51		1,900.24		6,290.02	
12		3		6,290.02		2,309.75		314.50		1,995.25		4,294.77	
13		4		4,294.77		2,309.75		214.74		2,095.01		2,199.76	
14		5		2,199.76		2,309.75		109.99		2,199.76		0.00	
15													
16	**Corresponding formulas:**												
17													
18		**PMT**		=PMT(D5, D6, -D4)									
19													
20		**Year**		**Beginning Balance**		**Total Annual Payment**		**Simple Interest Paid**		**Principal Paid**		**Ending Balance**	
21		1		=D4		=$D$7		=D10*$D$5		=F10-H10		=D10-J10	
22		2		=L10		=$D$7		=D11*$D$5		=F11-H11		=D11-J11	
23		3		=L11		=$D$7		=D12*$D$5		=F12-H12		=D12-J12	
24		4		=L12		=$D$7		=D13*$D$5		=F13-H13		=D13-J13	
25		5		=L13		=$D$7		=D14*$D$5		=F14-H14		=D14-J14	
26													

Using Excel – Calculating the Interest Rate for an Annuity

USING EXCEL

CALCULATING THE INTEREST RATE FOR AN ANNUITY

You can also solve for the interest rate using the =RATE function in Excel as illustrated below.

	A	B	C	D	E	F
1						
2		N		10		
3		PMT		$50,000		
4		PV		−$350,000		
5		FV		$0		
6						
7		Interest Rate		7.073%		
8		Formula		= RATE(D2,D3,D4,D5)		
9						

Level Cash Flows: Annuities and Perpetuities

- FINDING THE INTEREST RATE
  - **The present value of an annuity equation can be used to find the interest rate or discount rate for an annuity**
  - **To determine the rate-of-return for an annuity, solve the equation for *i***
  - **Using a calculator is easier than a trial-and-error approach**

Level Cash Flows: Annuities and Perpetuities

- CALCULATOR EXAMPLE
  - **Finding the Interest Rate**

    An insurer requires $350,000 to provide a guaranteed annuity of $50,000 per year for 10 years. What is the rate-of-return for the annuity?

Enter	10		-350,000	50,000	0
	N	i	PV	PMT	FV
Answer		7.073			

Level Cash Flows: Annuities and Perpetuities

- FUTURE VALUE OF AN ANNUITY
  - **The future value of an annuity equation is derived from Equation 6.1**

$$FVA_n = PVA_n \times \text{Future Value Factor}$$

$$= CF \times \frac{\text{Future Value Factor - 1}}{i}$$

$$= CF \times \left[\frac{(1+i)^n - 1}{i}\right] \qquad (6.2)$$

Future Value of 4-Yr Annuity: Colnago C50 Bicycle

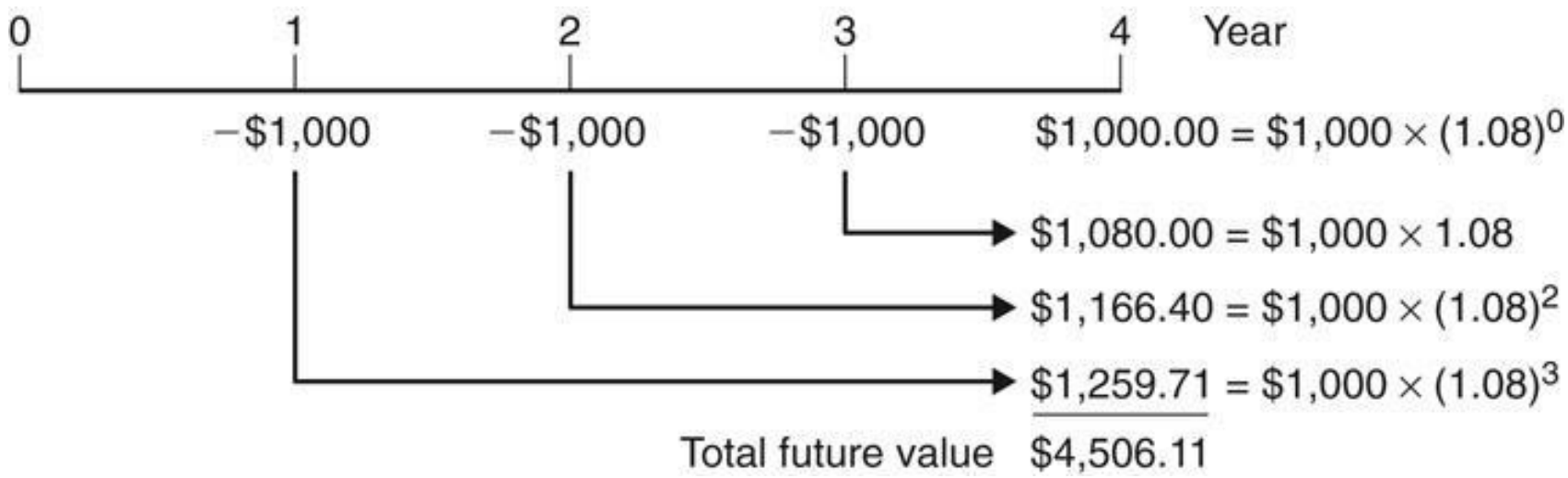


Exhibit 6.6
The exhibit shows a timeline for a savings plan to buy a Colnago C50 bicycle. Under this savings plan, $1,000 is invested at the end of each year for four years at an annual interest rate of 8 percent. We find the value at the end of the four-year period by adding the future values of the separate cash flows, just as in Exhibits 6.1 and 6.2.

Level Cash Flows: Annuities and Perpetuities

- CALCULATOR EXAMPLE
  - **Future Value of an Annuity**

    Colnago Bicycle C50

Enter	4	8	0	1,000	
	N	i	PV	PMT	FV
Answer					-4,506.11

Level Cash Flows: Annuities and Perpetuities

- PERPETUITY
  - **A stream of equal cash flows that goes on forever**
  - **Preferred stock and some bonds are perpetuities**
  - **Equation for the present value of a perpetuity can be derived from the present value of an annuity equation**

Level Cash Flows: Annuities and Perpetuities

- PRESENT VALUE OF A PERPETUITY

$$PVP_0 = CF \times \text{Present value factor for an annuity}$$

$$= CF \times \left[\frac{1 - \frac{1}{(1+i)^{\infty}}}{i} \right] = CF \times \frac{(1-0)}{i}$$

$$= \frac{CF}{i} \qquad (6.3)$$

Level Cash Flows: Annuities and Perpetuities

- VALUING PERPETUITY EXAMPLE
  - **Suppose you decide to endow a chair in finance. The goal of the endowment is to provide $100,000 of financial support per year forever. If the endowment earns a rate of 8%, how much money will you have to donate to provide the desired level of support?**

$$PVP_0 = \frac{CF}{i} = \frac{\$100{,}000}{0.08} = \$1{,}250{,}000$$

Level Cash Flows: Annuities and Perpetuities

- ORDINARY ANNUITY VERSUS ANNUITY DUE
  - **Present Value of Annuity Due**

    Cash flows are discounted for one period less than in an ordinary annuity.
  - **Future Value of Annuity Due**

    Cash flows are earn compound interest for one period more than in an ordinary annuity.

Level Cash Flows: Annuities and Perpetuities

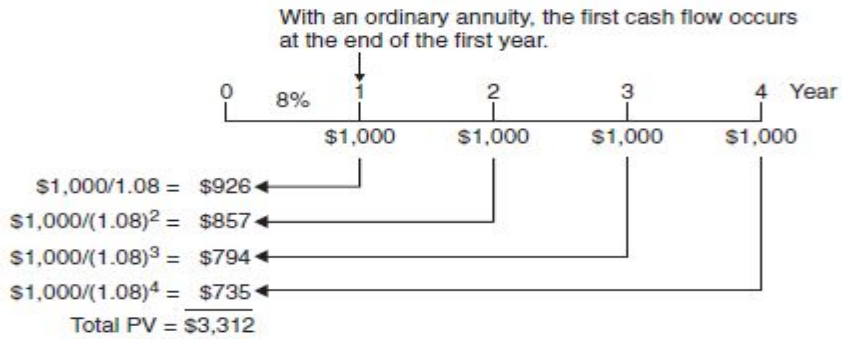
- ORDINARY ANNUITY VERSUS ANNUITY DUE
  - **The present value or future value of an annuity due is always higher than that of an ordinary annuity that is otherwise identical.**

$$PVA_{Due} = PVA \times (1+i)^1 \quad (6.4)$$

$$FVA_{Due} = FVA \times (1+i)^1$$

Ordinary Annuity versus Annuity Due

A. Ordinary Annuity (present value: four years at 8 percent)



B. Annuity Due (present value: four years at 8 percent)

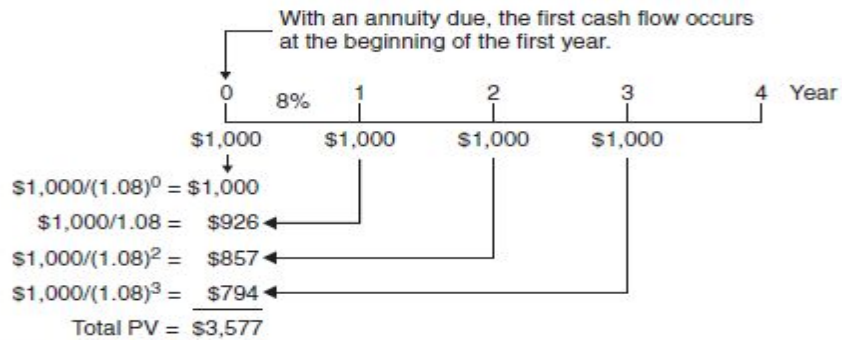


EXHIBIT 6.7
Ordinary Annuity versus Annuity Due

The difference between an ordinary annuity (part A) and an annuity due (part B) is that with an ordinary annuity, the cash flows take place at the end of each period, while with an annuity due, the cash flows take place at the beginning of each period. As you can see in this example, the PV of the annuity due is larger than the PV of the ordinary annuity. The reason is that the cash flows of the annuity due are shifted forward one year and thus are discounted less.

Cash Flows That Grow at a Constant Rate

- GROWING ANNUITY
  - equally-spaced cash flows that increase in size at a constant rate for a finite number of periods
- GROWING PERPETUITY
  - equally-spaced cash flows that increase in size at a constant rate forever

Cash Flows That Grow at a Constant Rate

- GROWING ANNUITY
  - **Multiyear product or service contract with periodic cash flows that increase at a constant rate for a finite number of years**
- GROWING PERPETUITY
  - **Common stock whose dividend is expected to increase at a constant rate forever**

Cash Flows That Grow at a Constant Rate

- GROWING ANNUITY
  - **Calculate the present value of growing annuity (only) when the growth rate is less than the discount rate.**

$$PVA_n = \frac{CF_1}{(i-g)} \times \left[1 - \left(\frac{1+g}{1+i}\right)^n\right] \qquad (6.5)$$

Cash Flows That Grow at a Constant Rate

- GROWING ANNUITY EXAMPLE
  - **A coffee shop will operate for fifty more years. Cash flow was $300,000 last year and increases by 2.5% each year. The discount rate for similar firms is 15%. Estimate the value of the firm.**

$$CF1 = \$300{,}000 \times (1 + 0.025) = \$307{,}500$$

$$PVA_0 = \frac{\$307{,}500}{0.15 - 0.025} \times \left[1 - \left(\frac{1.025}{1.15}\right)^{50}\right]$$

$$= \$2{,}460{,}000 \times 0.9968$$

$$= \$2{,}452{,}128$$

Cash Flows That Grow at a Constant Rate

- GROWING PERPETUITY
  - **Use Equation 6.6 to calculate the present value of growing perpetuity (only) when the growth rate is less than discount rate.**
  - **It is derived from equation 6.5 when the number of periods approaches infinity**

$$PVP_0 = \frac{CF_1}{(i - g)} \qquad (6.6)$$

Cash Flows That Grow at a Constant Rate

- GROWING PERPETUITY EXAMPLE
  - **A firm's cash flow was $450,000 last year. You expect the cash flow to increase by 5% per year forever. If you use a discount rate of 18%, what is the value of the firm?**

$$CF1 = \$450{,}000 \times (1 + 0.05) = \$307{,}500$$

$$PVP_0 = \frac{\$472{,}500}{0.18 - 0.05} = \frac{\$472{,}500}{0.13}$$

$$= \$3{,}634{,}615$$

The Effective Annual Interest Rate

- DESCRIBING INTEREST RATES
  - **The most common way to quote interest rates is in terms of annual percentage rate (APR). It does not incorporate the effects of compounding.**
  - **The most appropriate way to quote interest rates is in terms of effective annual rate (EAR). It incorporates the effects of compounding.**

The Effective Annual Interest Rate

- CALCULATE ANNUAL PERCENTAGE RATE (APR)
  - **APR = (periodic rate) x m**

    m is the # of periods in a year
  - **APR does not account for the number of compounding periods or adjust the annualized interest rate for the time value of money**
  - **APR is not a precise measure of the rates involved in borrowing and investing**

The Effective Annual Interest Rate

- ANNUAL PERCENTAGE RATE (APR) EXAMPLE
  - **Anna is charged 1% interest when she borrows $2000 for one week. What is the annual percentage interest rate (APR) on the loan?**

$$\text{APR} = (0.01) \text{ x } 52 = 0.52 \text{ or } 52\%$$

The Effective Annual Interest Rate

- EFFECTIVE ANNUAL INTEREST RATE (EAR)
  - **EAR accounts for the number of compounding periods and adjusts the annualized interest rate for the time value of money**
  - **EAR is a more accurate measure of the rates involved in lending and investing**

The Effective Annual Interest Rate

- EFFECTIVE ANNUAL RATE (EAR) EXAMPLE
  - **Anna is charged 1% interest when she borrows $2000 for one week. What is the effective annual interest rate (EAR)?**

$$\begin{aligned} EAR &= (1+0.01)^{52} - 1 \\ &= 1.6777 - 1 \\ &= 0.6777 \text{ or } 67.77\% \end{aligned}$$

The Effective Annual Interest Rate

- EFFECTIVE ANNUAL RATE (EAR) EXAMPLE
  - **Your credit card has an APR of 12 % (1% per month). What is the EAR?**

$$
\begin{aligned}
EAR &= (1 + 0.12/12)^{12} - 1 \\
&= (1 + 0.01)^{12} - 1 \\
&= 1.1268 - 1 \\
&= 0.1268 \text{ or } 12.68\%
\end{aligned}
$$

Consumer Protection and Information

- CONSUMER PROTECTION AND INFORMATION
  - **Truth-in-Lending Act (1968) requires that borrowers be told the actual cost of credit**
  - **Truth-in-Savings Act (1991) requires that the actual return on savings be disclosed to consumers**
  - **Credit Card Act (2009) limits credit card fees and interest rate increases, and requires better disclosure of contract details**