Fundamentals of Corporate Finance, 2/e

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Chapter 5: The Time Value of Money

Learning Objectives

1. EXPLAIN WHAT THE TIME VALUE OF MONEY IS AND WHY IT IS SO IMPORTANT IN THE FIELD OF FINANCE.
2. EXPLAIN THE CONCEPT OF FUTURE VALUE, INCLUDING THE MEANING OF THE TERMS PRINCIPAL, SIMPLE INTEREST AND COMPOUND INTEREST, AND USE THE FUTURE VALUE FORMULA TO MAKE BUSINESS DECISIONS.

Learning Objectives

3. EXPLAIN THE CONCEPT OF PRESENT VALUE, HOW IT RELATES TO FUTURE VALUE, AND USE THE PRESENT VALUE FORMULA TO MAKE BUSINESS DECISIONS.
4. DISCUSS WHY THE CONCEPT OF COMPOUNDING IS NOT RESTRICTED TO MONEY, AND USE THE FUTURE VALUE FORMULA TO CALCULATE GROWTH RATES.

The Time Value of Money

- EXCHANGING CONSUMPTION OPPORTUNITIES
  - **How does a manager determine the value of a future cash-flow, whether the cash-flow is a payment to be made or income to be received?**
  - **How much is a series of future cash-flows worth today?**
  - **The price/value today of cash-flows that occur in the future is determined by the time-value-of-money (TVM).**

The Time Value of Money

- CONSUME TODAY OR TOMORROW?
  - **TVM is based on the belief that people prefer to consume goods today rather than wait to consume the same goods tomorrow**

    An apple we can have today is more valuable to us than an apple we can have in one year.

    Money has a time value because buying an apple today is more important than buying an apple in one year.

The Time Value of Money

- CONSUME TODAY OR TOMORROW?
  - **A dollar someone has today can be spent for consumption or loaned to earn interest**
  - **A dollar loaned earns interest that increases wealth and the ability to consume**
  - **The rate of interest determines the trade-off between consumption today and saving (investing)**

The Time Value of Money

- TIMELINES AID PROBLEM SOLVING
  - **Timelines are an effective way to visualize cash flows**
  - **Present cash outflows as negative values**
  - **Present cash inflows as positive values**

Five-year Timeline for a $10,000 Investment

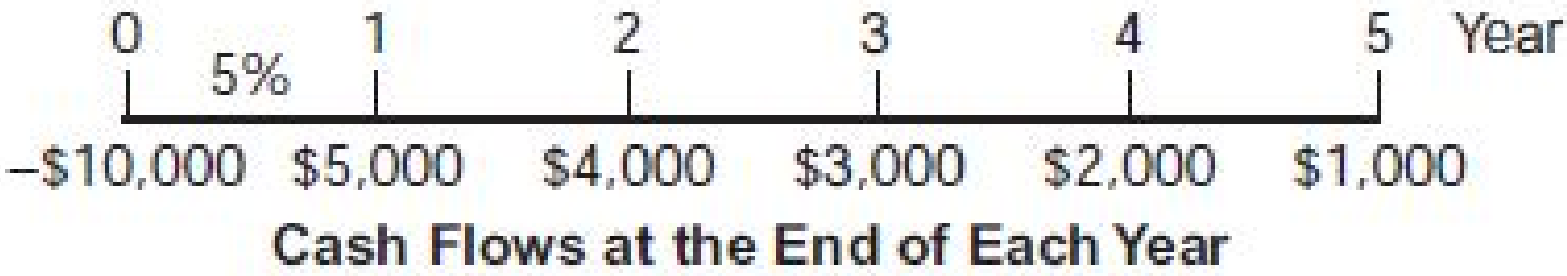


EXHIBIT 5.1

Five-year Time Line for a $10,000 Investment

Time lines help us to correctly identify the size and timing of cash flows—critical tasks in solving time value problems. This time line shows the cash flows generated over five years by a $10,000 investment in a situation where the relevant interest rate is 5 percent.

The Time Value of Money

- FUTURE VALUE VERSUS PRESENT VALUE
  - **Cash-flows are evaluated based on future value or present value**
  - **Future value measures what cash-flows are worth after a certain amount of time has passed**
  - **Present value measures what future cash-flows are worth before a certain amount of time has passed**

The Time Value of Money

- FUTURE VALUE VERSUS PRESENT VALUE
  - **Compounding is the process of increasing cash-flows to a future value**
  - **Discounting is the process of reducing future cash-flows to a present value**

Future Value of $100 at 10 Percent

EXHIBIT 5.2 **Future Value of $100 at 10 Percent**

With compounding, interest earned on an investment is reinvested so that in future periods, interest is earned on interest as well as on the principal amount. Here, interest on interest begins accruing in year 2.

		Interest Earned					
Year (1)	Value at Beginning of Year (2)	Simple Interest (3)		Interest on Interest (4)		Total (Compound) Interest (5)	Value at End of Year (6)
1	$100.00	$10.00	+	$ 0.00	=	$10.00	$110.00
2	110.00	10.00	+	1.00	=	11.00	121.00
3	121.00	10.00	+	2.10	=	12.10	133.10
4	133.10	10.00	+	3.31	=	13.31	146.41
5	146.41	10.00	+	4.64	=	14.64	161.05
Five-year total	$100.00	$50.00	+	$11.05	=	$61.05	$161.05

Future Value and Compounding

- SINGLE PERIOD LOAN
  - **We can determine the balance in an account at the end of a period if we know the interest rate earned on the principal**
  - **If principal of \$X is loaned for one period at the interest rate *i*, the account balance will increase to $\$X(1 + i)^1$**
  - **The term $(1+ i)^n$ is the future value interest factor or future value factor**

Future Value and Compounding

- TWO-PERIOD LOAN
  - **A two-period loan is two consecutive single-period loans**
  - **Interest earned is added to the account at the end of the first period and the new account balance is the amount that earns the interest rate *i* during the second period**
  - **The account balance is $\$X(1 + i)^1$ at the end of the first period and $\$X(1 + i)^2$ at the end of the second period.**

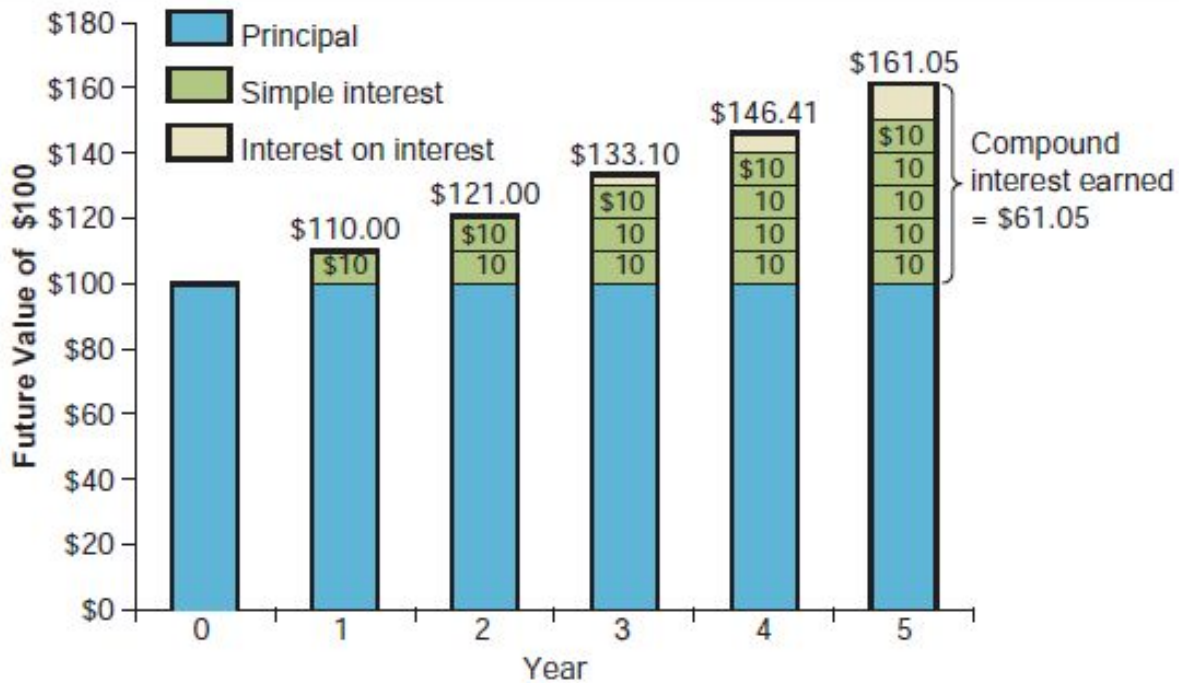
Future Value and Compounding

- TWO-PERIOD LOAN
  - The principal is the initial deposit or loan amount
  - Simple interest is paid on the original principal amount only
  - Compound interest consists of both simple interest and interest-on-interest

How Compound Interest Grows on $100 at 10 Percent

EXHIBIT 5.3
How Compound Interest Grows on $100 at 10 Percent

The amount of simple interest earned on $100 invested at 10 percent remains constant at $10 per year, but the amount of interest earned on interest increases each year. As more and more interest builds, the effect of compounding accelerates the growth of the total interest earned.



Future Value and Compounding

- FUTURE VALUE EQUATION
  - **The general equation to find a future value**

$$FV_n = PV \times (1+i)^n \qquad (5.1)$$

where:

FV_n = future value of investment at end of period n

PV = original principle (P0) or present value

i = the rate of interest per period

n = the number of periods, often in years

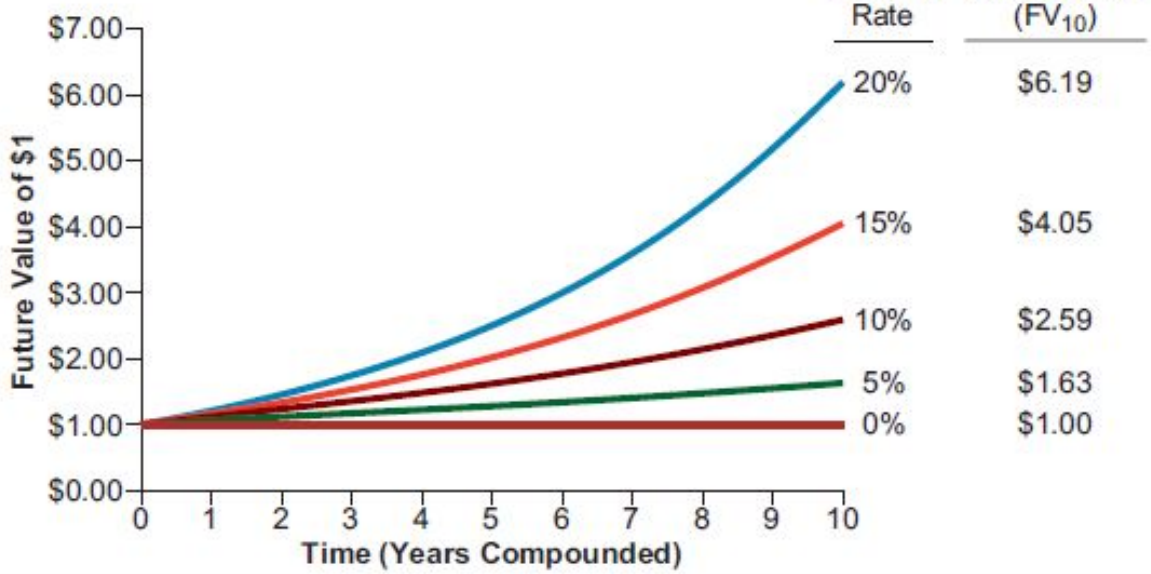
Future Value and Compounding

FUTURE VALUE EXAMPLE

You deposit $100 in a savings account earning 10% compounded annually for five years. How much is in the account at the end of that time?

$$
\begin{aligned}
FV_5 &= \$100 \times (1 + 0.10)^5 \\
&= \$100 \times (1.10)^5 \\
&= \$100 \times 1.6105 \\
&= \$161.05
\end{aligned}
$$

Future Value of $1 for Different Periods and Interest Rates



Interest Rate	Value of $1 after 10 years ($FV_{10}$)
20%	$6.19
15%	$4.05
10%	$2.59
5%	$1.63
0%	$1.00

EXHIBIT 5.4
Future Value of $1 for Different Periods and Interest Rates

The higher the interest rate, the faster the value of an investment will grow, and the larger the amount of money that will accumulate over time. Because of compounding, the growth over time is not linear but exponential—the dollar increase in the future value is greater in each subsequent period.

Future Value Factors

EXHIBIT 5.5 **Future Value Factors**

To find a future value factor, simply locate the row with the appropriate number of periods and the column with the desired interest rate. The future value factor for 10 years at 8 percent is 2.159.

Number of Years	Interest Rate per Year						
	1%	5%	6%	7%	8%	9%	10%
1	$1.010	$1.050	$1.060	$1.070	$1.080	$1.090	$1.100
2	1.020	1.103	1.124	1.145	1.166	1.188	1.210
3	1.030	1.158	1.191	1.225	1.260	1.295	1.331
4	1.041	1.216	1.262	1.311	1.360	1.412	1.464
5	1.051	1.276	1.338	1.403	1.469	1.539	1.611
10	1.105	1.629	1.791	1.967	2.159	2.367	2.594
20	1.220	2.653	3.207	3.870	4.661	5.604	6.727
30	1.348	4.322	5.743	7.612	10.063	13.268	17.449

Future Value and Compounding

- COMPOUNDING MORE THAN ONCE A YEAR
  - **The more frequently interest is compounded, the larger the future value of $1 at the end of a given time period**
  - **If compounding occurs *m* times within a period, the future value equation becomes**

$$FV_n = PV \times (1 + i/m)^{mn} \quad (5.2)$$

Future Value and Compounding

- COMPOUNDING WITHIN A PERIOD EXAMPLE
  - **You deposit $100 in an account that pays 5% annually with semi-annual compounding for two years. What is the ending account balance?**

$$FV_2 = \$100 \times (1+0.05/2)^{2\times 2}$$

$$= \$100 \times (1+0.025)^4$$

$$= \$100 \times (11038)$$

$$= \$110.38$$

Future Value and Compounding

- CONTINUOUS COMPOUNDING
  - **When compounding occurs on a continuous basis, the future value equation becomes**

$$FV^{n} = PV \times e^{i \times n} \qquad (5.3)$$

e = 2.71828, the base of the natural logarithm

Future Value and Compounding

- CONTINUOUS COMPOUNDING EXAMPLE
  - **Your grandmother wants to put $10,000 in a savings account. How much money will she have at the end of five years if the bank pays 5% annual interest compounded continuously?**

$$FV_n = \$10{,}000 \times e^{0.05 \times 5}$$

$$= \$10{,}000 \times (2.71828)^{0.05 \times 5}$$

$$= \$10{,}000 \times 1.284025$$

$$= \$12{,}840.25$$

Using Excel – Future Value and Compounding

USING EXCEL

TIME VALUE OF MONEY

Spreadsheet computer programs are a popular method for setting up and solving finance and accounting problems. Throughout this book, we will show you how to structure and calculate some problems using the Microsoft Excel spreadsheet program. Spreadsheet programs are like your financial calculator but are especially efficient at doing repetitive calculations. For example, once the spreadsheet program is set up, it will allow you to make computations using preprogrammed formulas. Thus, you can simply change any of the input cells, and the preset formula will automatically recalculate the answer based on the new input values. For this reason, we recommend that you use formulas whenever possible.

We begin our spreadsheet applications with time value of money calculations. As with the financial calculator approach, there are five variables used in these calculations, and knowing any four of them will let you calculate the fifth one. Excel has already preset formulas for you to use. These are as follows:

Solving for	Formula
Present Value	= PV(RATE, NPER, PMT, FV)
Future Value	= FV(RATE, NPER, PMT, PV)
Discount Rate	= RATE(NPER, PMT, PV, FV)
Payment	= PMT(RATE, NPER, PV, FV)
Number of Periods	= NPER(RATE, PMT, PV, FV)

To enter a formula, all you have to do is type in the equal sign, the abbreviated name of the variable you want to compute, and an open parenthesis, and Excel will automatically prompt you to enter the rest of the variables. Here is an example of what you would type to compute the future value:

1. =
2. FV
3. (

Here are a few important things to note when entering the formulas: (1) be consistent with signs for cash inflows and outflows; (2) enter the rate of return as a decimal number, not a percentage; and (3) enter the amount of an unknown payment as zero.

To see how a problem is set up and how the calculations are made using a spreadsheet, let's return to Learning by Doing Application 5.2.

Time Value of Money Calculations

Your grandmother wants to put $10,000 into a bank savings account for five years. Bank A pays 5 percent interest compounded quarterly, while Bank B offers 5 percent compounded daily. Which bank should your grandmother choose?

To answer the question, we need to solve for the future value.

Problem set-up and solution:

	Bank A	Bank B	Comment
Present value	($10,000)	($10,000)	Value given
Interest rate	0.01250	0.00014	Interest rate/# compounding periods per year
Number of periods	20	1825	# years × # compounding periods per year
Future value	**$12,820.37**	**$12,840.03**	See note below

The formula entered to calculate the future value for Bank A in cell B16 is =FV(B14, B15, 0, B13). Similarly, the formula to calculate the future value for Bank B in cell C16 is =FV(C14, C15, 0, C13). Since there are no payments, we enter PMT as zero. Also, notice that to be consistent with what we have said about cash inflows and outflows so far, the present value is entered as a negative number.

Future Value and Compounding

- CALCULATOR EXAMPLE
  - **Future Value**

    Suppose we lend $5,000 at 15% for 10 years. How much money will we have at the end of that time?

Enter	10	15	-5,000	0	
	N	i	PV	PMT	FV
Answer					20,227.79

Present Value and Discounting

- PRESENT VALUE EQUATION
  - General equation to find present value
- $$PV = \frac{FV_n}{(1+i)^n} \qquad (5.4)$$
  - This equation has the same elements as Equation 5.1, the future value equation. They differ only in the arrangement of the elements. Here, $(1 + i)^n$ is used for division and is called the present value factor or discount factor.

Present Value and Discounting

EXHIBIT 5.6 Tips for Using Financial Calculators

Following these tips will help you avoid problems that sometimes arise in solving time value of money problems with a financial calculator.

Use the Correct Compounding Period. Make sure that your calculator is set to compound one payment per period or per year. Because financial calculators are often used to compute monthly payments, some will default to monthly payments unless you indicate otherwise. You will need to consult your calculator's instruction manual because procedures for changing settings vary by manufacturer. Most of the problems you will work in other chapters of the book will compound annually.

Clear the Calculator Before Starting. Be sure you clear the data from the financial register before starting to work a problem because most calculators retain information between calculations. Since the information may be retained even when the calculator is turned off, turning the calculator off and on will not solve this problem. Check your instruction manual for the correct procedure for clearing the financial register of your calculator.

Negative Signs on Cash Outflows. For certain types of calculations, it is critical that you input a negative sign for all cash outflows and a positive sign for all cash inflows. Otherwise, the calculator cannot make the computation, and the answer screen will display some type of error message.

Putting a Negative Sign on a Number. To create a number with a negative sign, enter the number first and then press the "change of sign key." These keys are typically labeled "CHS" or "+/−".

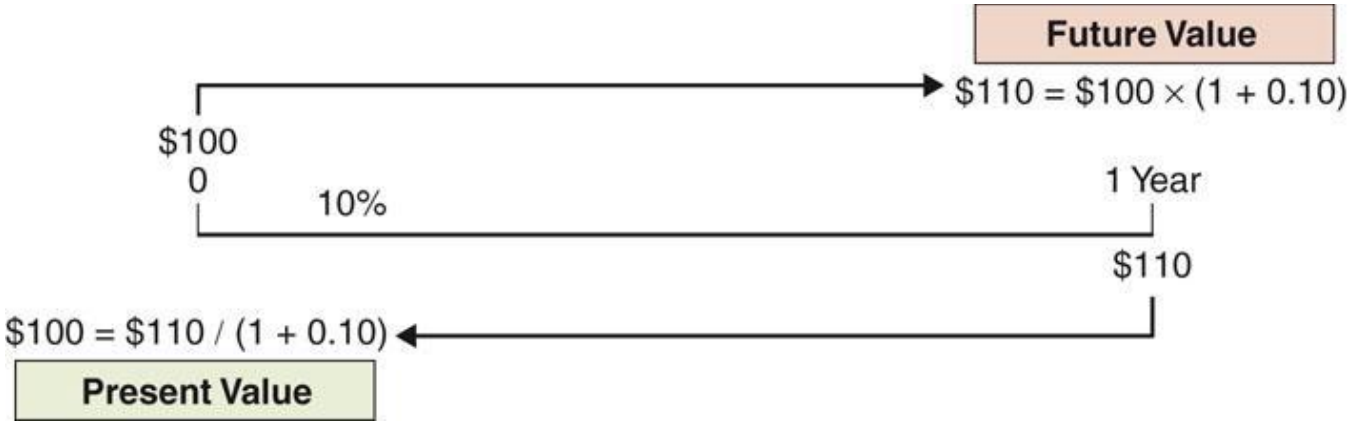
Interest Rate as a Percentage. Most financial calculators require that interest rate data be entered in percentage form, not in decimal form. For example, enter 7.125 percent as 7.125 and not 0.07125. Unlike nonfinancial calculators, financial calculators assume that rates are stated as percentages.

Rounding off Numbers. Never round off any numbers until all your calculations are complete. If you round off numbers along the way, you can generate significant rounding errors.

Adjust Decimal Setting. Most calculators are set to display two decimal places. You will find it convenient at times to display four or more decimal places when making financial calculations, especially when working with interest rates or present value factors. Again, consult your instruction manual.

Have Correct BEG or END mode. In finance, most problems that you solve will involve cash payments that occur at the end of each time period, such as with the ordinary annuities discussed in Chapter 6. Most calculators normally operate in this mode, which is usually designated as "END" mode. However, for annuities due, which are also discussed in Chapter 6, the cash payments occur at the beginning of each period. This setting is designated as the "BEG" mode. Most leases and rent payments fall into this category. When you bought your financial calculator, it was set in the END mode. Financial calculators allow you to switch between the END and BEG modes.

Comparing Future Value & Present Value Calculations



Present Value and Discounting

- PRESENT VALUE EQUATION
  - **A present value calculation takes end-of-the-period cash flows and reverses the effect of compounding to determine the equivalent beginning-of-the-period cash flows**

    This is discounting and the interest rate *i* is called the discount rate.

    Present value (PV) is often referred to as the discounted value of future cash-flows.

Present Value and Discounting

- PRESENT VALUE CALCULATION EXAMPLE
  - **You intend to buy a BMW 330 Sports Coupe one year from today. You predict the car will cost $40,000. If your bank pays 5% interest on savings, compounded annually, how much will you need to deposit today to have $40,000 after one year?**

$$PV = \frac{\$10,000}{1+0.05} = \$38,095.24$$

Present Value and Discounting

- PRESENT VALUE CONCEPTS
  - **Time and the discount rate affect present value**

    The greater the amount of time before a cash flow is to occur, the smaller the present value of the cash-flow.

    The higher the discount rate, the smaller the present value of a future cash-flow.

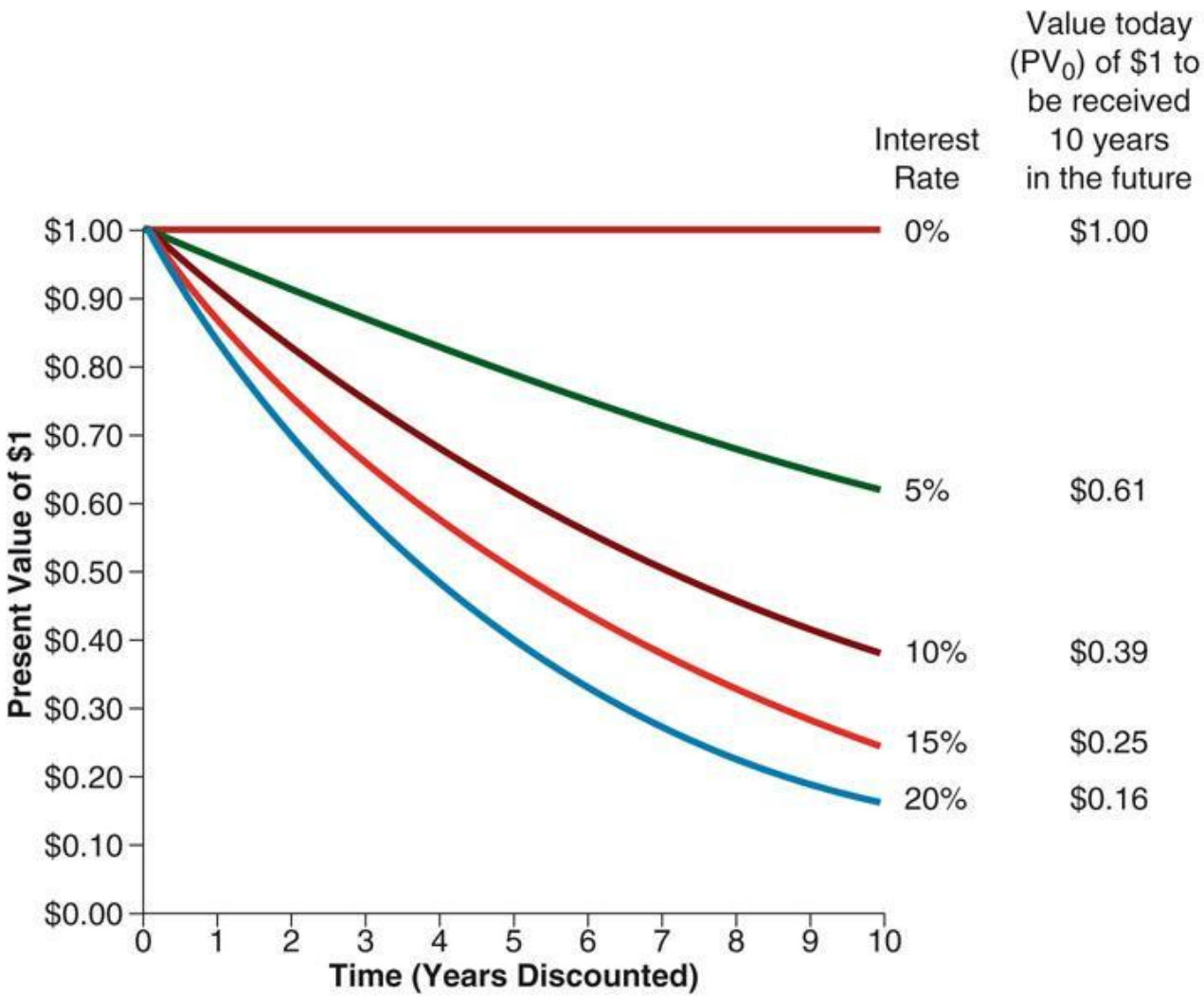
Present Value Factors

EXHIBIT 5.8 **Present Value Factors**

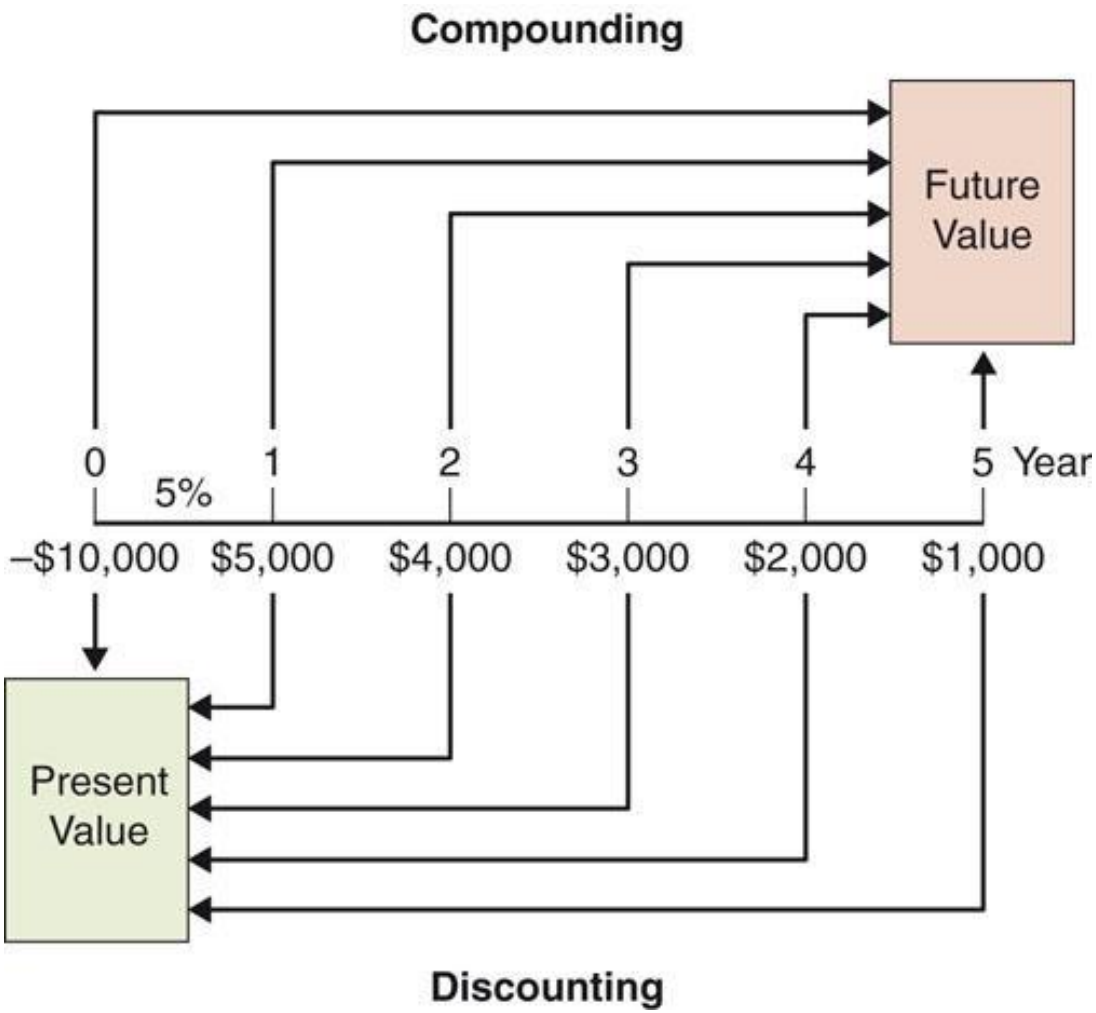
To locate a present value factor, find the row for the number of periods and the column for the proper discount rate. Notice that whereas future value factors grow larger over time and with increasing interest rates, present value factors become smaller. This pattern reflects the fact that the present value factor is the reciprocal of the future value factor.

Number of Years	Interest Rate per Year						
	1%	5%	6%	7%	8%	9%	10%
1	$0.990	$0.952	$0.943	$0.935	$0.926	$0.917	$0.909
2	0.980	0.907	0.890	0.873	0.857	0.842	0.826
3	0.971	0.864	0.840	0.816	0.794	0.772	0.751
4	0.961	0.823	0.792	0.763	0.735	0.708	0.683
5	0.951	0.784	0.747	0.713	0.681	0.650	0.621
10	0.905	0.614	0.558	0.508	0.463	0.422	0.386
20	0.820	0.377	0.312	0.258	0.215	0.178	0.149
30	0.742	0.231	0.174	0.131	0.099	0.075	0.057

Present Value of $1 for Different Periods and Discount Rates



Future Value and Present Value Compared



Present Value and Discounting

- CALCULATOR EXAMPLE
  - **Present Value**

    What is the present value of $1,000 to be received 10 years from now if the discount rate is 9%?

Enter	10	9		0	1,000
	N	i	PV	PMT	FV
Answer			-422.41		

Finding the Interest Rate

- TIME VALUE OF MONEY CALCULATIONS
  - **Many situations require using a time value of money calculation to determine a rate of change or growth rate**
  - **An investor or analyst may want**
    - the growth rate in sales
    - the rate-of-return on an investment
    - the effective interest rate on a loan

Compound Growth Rates

- CALCULATOR EXAMPLE
  - **Compound Growth Rate**

    A firm's sales increased from $20 million to $35 million in three years. What was the average annual growth rate in sales?

Enter	3		-20	0	35
	N	i	PV	PMT	FV
Answer		20.51			

Compound Growth Rates

- CALCULATOR EXAMPLE
  - **Compound Growth Rate**

    The house at 1245 Maple St. was appraised at $247,000 in 2006 and at $173,000 in 2011. What is the average annual change in its value?

Enter	5		-247000	0	173000
	N	i	PV	PMT	FV
Answer		-6.874			

The Rule of 72

- ESTIMATE THE NUMBER OF PERIODS
  - **The Rule of 72 is used to estimate the time (number of periods) it takes for an amount to double.**

    The time it takes for the amount to double is approximately equal to 72/*i*, where *i* equals the percentage earned each period.

    The Rule of 72 is fairly accurate for interest rates between 5% and 20%.

The Rule of 72

- CALCULATOR EXAMPLE
  - **Time required for an amount to double**

    If you can earn 8% compounded annually, how long will it take for your money to double?

Enter		8	-1	0	2
	N	i	PV	PMT	FV
Answer	9.006				

The Rule of 72

- CALCULATOR EXAMPLE
  - **Time required for an amount to double**

    If you can earn 8% compounded monthly (.667% /month), how many months will it take for an amount to double?

Enter		.667	-1	0	2
	N	i	PV	PMT	FV
Answer	104.32				